Beta-Packing Sets in Graphs

Benjamin M. Case*, Renu C. Laskar, and Evan Haithcock, Clemson University

A set $S \subseteq V$ is α-*dominating* if for all $v \in V - S$, $|N(v) \cap S| \geq \alpha|N(v)|$. The α-*domination number* of G equals the minimum cardinality of an α-dominating set S in G. Since being introduced by Dunbar, et al. in 2000, α-domination has been studied for various graphs and a variety of bounds have been developed. In this paper, we propose a new parameter derived by flipping the inequality in the definition of α-domination. We say a set $S \subset V$ is a β-*packing set* of a graph G if S is a proper, maximal set having the property that for all vertices $v \in V - S$, $|N(v) \cap S| \leq \beta|N(v)|$ for some $0 < \beta \leq 1$. The β-*packing number* of G (β-pack(G)) equals the maximum cardinality of a β-packing set in G. The single greatest interest of studying β-packing sets, as with α-dominating sets, is finding the value of β-pack(G) for some graph. In this research, we set out to find β-pack(G) for different types of graphs. We focused particular attention on $1/2$ β-packing sets; that is, where we set $\beta = 1/2$.

Keywords: α-domination, β-packing, graph theory, graph parameters