The Even Cycle Spectrum

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Does this graph contain any cycles? Even cycles, or odd cycles? How many? How long are they? If you hang out with graph theorists often, you will hear these questions ad nauseum. Understanding the cycle structure of graphs has long been of great interest to graph theorists. For instance, it is elementary to show that every graph contains a cycle of length at least its minimum degree (this has been proven in every graph theory course ever taught). Adding a connectivity condition, though, gives much more: Dirac's Theorem says that every 2-connected graph of minimum degree d contains a cycle of length at least $\min\{2d, |G|\}$. In fact, such a connectivity condition guarantees not just long cycles, but cycles of many different lengths. And so, we must explore the cycle spectrum of 2-connected graphs: the set of cycle lengths appearing in the graph. In this talk, we'll tell a little bit of the history of the cycle spectrum, as well as present some new results on the even cycle spectrum of large 2-connected graphs (in particular, proving in a strong form the dense case of a conjecture of Faudree, Gould, Jacobson, and Magnant). This is joint work with Andrzej Czygrinow and Jangwon Yie.

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