List Colorings: Some Refinements of Hall's Condition.

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Let *C* be an infinite set of symbols. A function *L* is a list assignment to a graph *G* if *L* assigns to each vertex of *G* a non-empty finite subset of *C*, called a list. A proper *L*-coloring of *G* is an assignment of "colors" to the vertices of *G*, from their lists, so that adjacent vertices are colored with different colors. Interpreted as a theorem about proper list colorings of complete graphs, P. Hall's theorem on systems of distinct representatives inspires a generalization, a necessary condition for proper colorings, known as Hall's Condition (HC). We present some refinements of HC, and explore some conditions for proper *L*-colorings of the graph G.

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