k-Neighborhood Degrees of a Graph

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The k-neighborhood degree list of a vertex v in a graph G, denoted by $N_kDL(v)$, is the list of degrees of vertices at distance k from v, where $1 \leq k \leq diam(G)$. We will denote by $N_kDL(G)$ the lists of k-neighborhood degrees of every vertex of G. An N_kDL-unique graph is a labeled graph that is uniquely realizable by its $N_tDL(G)$ for $1 \leq t \leq k$. It is a generalization of NDL-unique graphs by Barrus and Donavan (Discrete Mathematics 341(2018)) and Bassler *et al* (New J. Phys. 17(2015)). We will present applications of $N_kDL(G)$ to the problem of determining the similarity of vertices in large graphs and some theoretical results relating N_kDL to diameter. A 2-switch operation that preserves degrees is the replacement of a pair of edges v_1v_2 and w_1w_2 such that $deg(v_1) = deg(w_1)$ and $deg(v_2) = deg(w_2)$ by the edges v_1w_2 and v_2w_1, given that v_1w_2 and v_2w_1 did not appear in the graph originally. We will prove that within the class of diameter 2 graphs, a 2-switch operation that preserves degrees also preserves $N_2DL(G)$. We will end with a generalization of the 2-switch operation and some conjectures.