

Spanning 2-Forests and Resistance Distance in 2-Connected Graphs

Wayne Barrett*, Emily Evans, Mark Kempton, John Sinkovic, Department of Mathematics, Brigham Young University; Amanda Francis, Mathematical Reviews, Ann Arbor, Michigan

A spanning 2-forest separating vertices u and v of an undirected connected graph is a spanning forest with 2 components such that u and v are in distinct components. Aside from their combinatorial significance, spanning 2-forests have an important application to the calculation of resistance distance or effective resistance. The resistance distance between vertices u and v in a graph representing an electrical circuit with unit resistance on each edge is also the number of spanning 2-forests separating u and v divided by the number of spanning trees in the graph. It is well-known that the number of these spanning 2-forests separating u and v in a graph is equal to the determinant of the matrix obtained from the combinatorial Laplacian matrix of the graph by deleting the rows and columns corresponding to u and v . For most interesting graphs, neither of these quantities can be easily found. The calculation of these quantities simplifies in the special case that the graph has a cut vertex. We show that there are also reduction formulae for any connected graph with a 2-separator. These formulae will be briefly illustrated by applying them to linear 2-trees.

Keywords: spanning 2-forest, 2-connected graph, 2-separator, 2-tree, resistance distance