

A Sumset Problem in Abelian Groups

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Is it possible to partition a finite group G into three symmetric sets A , S , and F so that

- (i.) S is a subgroup of G ;
- (ii.) $F + F = G \setminus F$, i.e., F is maximal sum-free;
- (iii.) $A + A = G$;
- (iv.) $A + S = A \cup F$;
- (v.) $A + F = G \setminus \{0\}$;
- (vi.) $S + F = A \cup F$?

Conditions (ii.)-(vi.) basically just say that all the sumsets are as big as they can possibly be, subject to the constraint that S be a subgroup and F be sum-free.

We will discuss an approach to this problem where the ambient group G is $(\mathbb{Z}/2\mathbb{Z})^{3k+1}$ and

$$F = \{x \in (\mathbb{Z}/2\mathbb{Z})^{3k+1} : x \text{ has at least } 2k + 1 \text{ ones}\}.$$

S must be a “large” subgroup such that no element of S has more than $2k$ ones. Such subgroups have been constructed for $k = 3, 4$ but no general construction is known.

This work is motivated by some representation problems for finite relation algebras.

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