Hall t-chromatic spectra and weak Hall t-chromatic spectra of the Petersen Graph and wheels with odd numbers of spokes

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A color demand function on a graph G is a function $\kappa : V(G) \to N$. A proper (t, κ)-coloring of G is a function ϕ assigning each vertex of G a subset of $[t] = \{1, 2, \ldots, t\}$ so that for each $v \in V(G)$, $|\phi(v)| = \kappa(v)$ and for each $uv \in E(G)$, $\phi(u) \bigcap \phi(v) = \emptyset$. $\alpha(G)$ is the vertex independence number of G. G and κ satisfy Hall's t-condition if and only if for each subgraph H of G

$$t\alpha(H) \geq \sum_{v(H)} \kappa(v)$$

It is clear that Hall's t-condition is necessary for the existence of a proper (t, κ)-coloring of G. If it is sufficient (i.e. If G is properly (t, κ)-colorable for every color demand κ on G such that G and κ satisfy Hall's t-condition) then G is Hall t-chromatic. If Hall's t-condition with the equation $t\alpha(G) = \sum_{v(G)} \kappa(v)$ suffice for the existence of a proper (t, κ)-coloring of G, then G is weakly Hall t-chromatic.

We show that the Petersen graph is Hall 3-chromatic and determine the weak Hall t-chromaticity of wheels with odd numbers of spokes.

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