s-Hamiltonicity in Line Graphs

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For an integer $s \geq 0$, a graph G is s-hamiltonian if for any vertex subset $S \subseteq V(G)$ with $|S| \leq s$, $G - S$ is hamiltonian, and G is s-hamiltonian connected if for any vertex subset $S \subseteq V(G)$ with $|S| \leq s$, $G - S$ is hamiltonian connected. Thomassen in 1984 conjectured that every 4-connected line graph is hamiltonian (see [J. Graph Theory, 10 (1986) 309-324]), and Kučzel and Xiong in 2004 conjectured that every 4-connected line graph is hamiltonian connected (see [J. Graph Theory 66 (2011), 152-173]). In [J. Graph Theory, 11 (1987), 399-407], Broersma and Veldman raised the characterization problem of s-hamiltonian line graphs. In [J. Graph Theory, 74 (2013), 344-358], it is conjectured that for $s \geq 2$, a line graph $L(G)$ is s-hamiltonian if and only if $L(G)$ is $(s+2)$-connected. In this paper we prove the following.
(i) For an integer $s \geq 2$, the line graph $L(G)$ of a claw-free graph G is s-hamiltonian if and only if $L(G)$ is $(s+2)$-connected.
(ii) The line graph $L(G)$ of a claw-free graph G is 1-hamiltonian connected if and only if $L(G)$ is 4-connected.

Keywords: claw-free graphs, line graphs, s-hamiltonian