

Cubic vertices in planar hypohamiltonian graphs

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A graph is *hypohamiltonian* if it is non-hamiltonian, yet all of its vertex-deleted subgraphs are hamiltonian. In 1976, Thomassen showed that planar hypohamiltonian graphs exist, settling a question of Chvátal. Two years later, Thomassen proved that every planar hypohamiltonian graph must contain a cubic vertex. He also gave a perhaps more appealing, equivalent formulation: a planar graph with minimum degree at least 4 in which every vertex-deleted subgraph is hamiltonian, must be itself hamiltonian. In this talk, we present three extensions of this result: (i) every planar hypohamiltonian graph contains at least four cubic vertices, (ii) every planar almost hypohamiltonian graph contains a cubic vertex which is not the exceptional vertex, and (iii) every hypohamiltonian graph with crossing number at most 1 contains a cubic vertex. An important tool in our proofs is a recent result due to Brinkmann and the speaker which strengthens a classic theorem of Tutte and work of Jackson and Yu.

Keywords: hypohamiltonian, planar, 3-connected