Trees for Given Values of the Span, Caps and Icaps for $L(2,1)$-Colorings

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An $L(2,1)$-coloring of a graph is a labeling of the vertices using non-negative integers such that adjacent vertices differ in label by at least 2 and distance two vertices differ in label. The span of an $L(2,1)$-coloring is the smallest integer λ for a given graph such that there exists an $L(2,1)$-coloring of the graph using only non-negative integers less than or equal to λ. The invariant caps, denoted $\bar{\kappa}$, is the least number of color classes required to create an $L(2,1)$-coloring on a given graph. An $L(2,1)$-coloring of a graph is irreducible if reducing the label on any vertex violates an $L(2,1)$-coloring condition. The invariant icaps, denoted κ, is the least number of color classes required to create an irreducible $L(2,1)$-coloring on a given graph. For any tree T it is known that $\Delta+1 \leq \bar{\kappa} \leq \kappa \leq \lambda+1$ and $\lambda \in \{\Delta+1, \Delta+2\}$ where Δ is the maximum degree of the tree. Thus, there are only three possible values for $\bar{\kappa}$ and κ: $\Delta+1, \Delta+2, \Delta+3$. We prove that $\bar{\kappa} = \Delta+1$ for all trees. Then for each of the two possible values of λ , we consider the three possible values of κ, determine if there exists a tree with the two specified values of λ and κ, and provide a family of such trees if any exist.

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