Asymptotics for recursions with algebraic generating functions

Robin Pemantle* and Kaitian JIN, University of Pennsylvania
Yuliy Barynishkov, University of Illinois

Enumeration problems satisfying tree-like recursions often
have generating functions that are algebraic over the
rational numbers. Examples include: trees with prescribed
out-degrees, lattice paths, context-free grammars, dissections
of polygons, diagonals of rational multivariate generating
unctions and many more.

In 1990, Flajolet and Odyzko gave an automated method
for determining asymptotics from a univariate algebraic
function from its behavior near its dominant singularities
(so-called "singularity analysis"). Recently, automatic
asymptotics for algebraic functions has been extended to
multivariate generating functions via ACSV, a multivariate
generalization of univariate complex contour methods.

This talk presents an elementary and self-contained
method to compute asymptotic behavior of multivariate
algebraic functions. More precisely, we show how to
manipulate the complex contour integral so as to
automatically extract asymptotics for algebraic generating
functions while avoiding the topological machinery of ACSV.