Graphs whose generalized adjacency matrix has few distinct eigenvalues

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The generalized adjacency matrix of a graph Γ is defined as $M(x,y,z) := xI+yA+z(J-I-A)$, where x, y and z are real numbers satisfying $y \neq z$ and J (resp. I) is the all-ones matrix (resp. identity matrix) of suitable dimension. For a real number h, we define the matrix B_h by, $B_h := A + h(J - I)$. This implies that M is an affine transformation of B_h. Note that the Seidel matrix S of a graph defined as $S := J - I - 2A$, then $S = -2B_{-\frac{1}{2}}$. The adjacency matrix of a graph is $A = B_0$ and adjacency matrix of its complement is $\overline{A} := J - I - A$ and $\overline{A} = -B_{-1}$. Note that the B_h matrix is always irreducible if $h \notin \{0, -1\}$. In this talk, I will present some results regarding graphs with few distinct B_h-eigenvalues which we have proven recently. Graphs with two distinct B_h-eigenvalues have been characterized. Various results on connected graphs with three distinct B_h-eigenvalues have been proven. Disconnected graphs with three distinct B_h-eigenvalues are also characterized. As a by-product, we obtain the first example of non-regular non-bipartite graphs with three distinct distance eigenvalues.

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