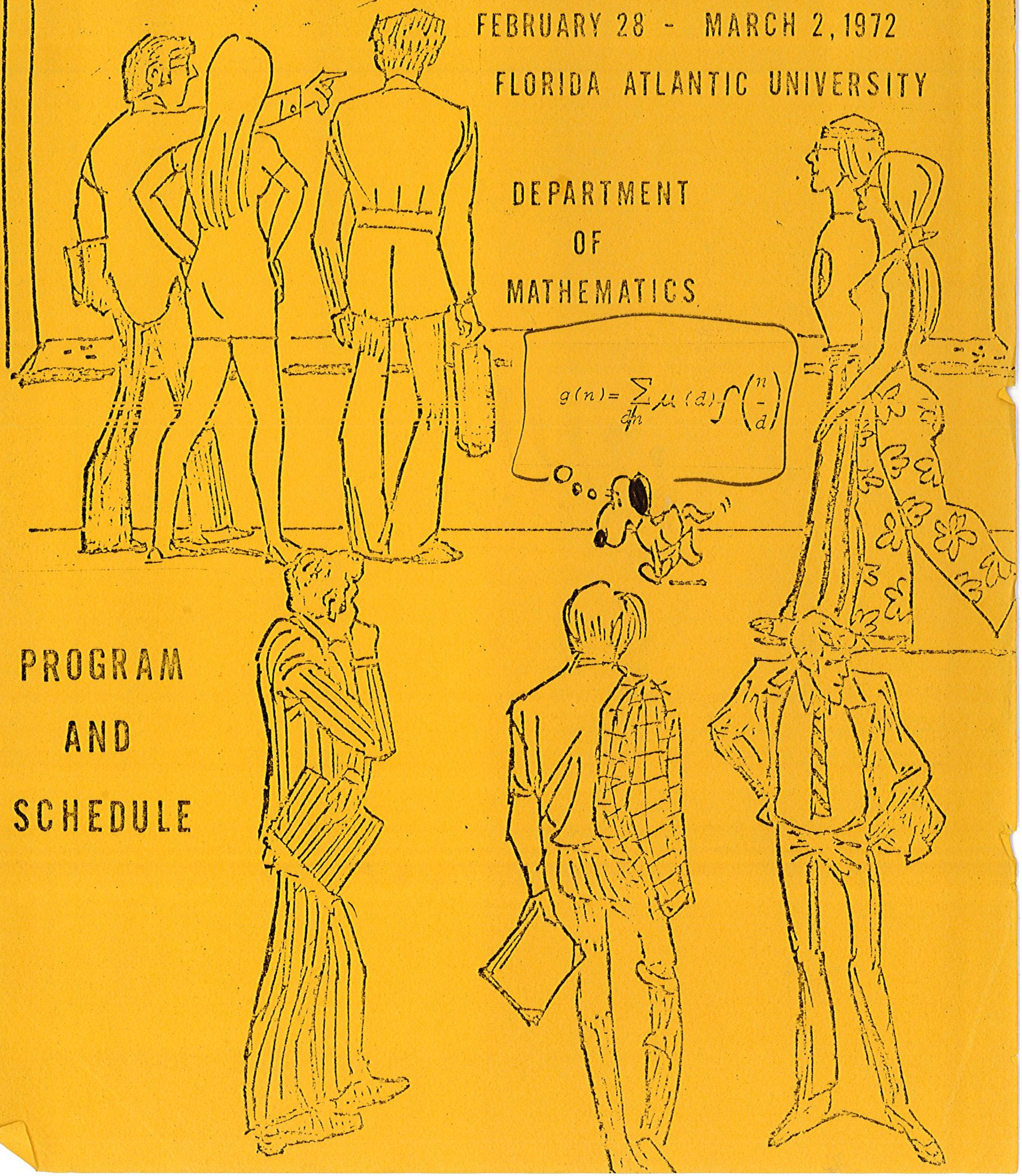
THIRD SOUTHEASTERN CONFERENCE ON COMBINATORICS, GRAPH THEORY AND COMPUTING

FEBRUARY 28 - MARCH 2, 1972

FLORIDA ATLANTIC UNIVERSITY

DEPARTMENT OF MATHEMATICS



PROGRAM AND SCHEDULE

A.M.	Monday		Tuesday		Wednesday		Thursday	
8:30			319 GCN	419 GCN	422 GCN	319 GCN	319 GCN	419 GCN
9:00			(10) W. W. Kuhn		(22) R.A. Kingsley R.G. Stanton	(25) P. C. Kainen	(35) C.C. Cadogan	(38) J.M. Freeman
9:30		117 GCS	(11) R. B. Levow	(14) G.E. McMaster F. J. Burkowski	(23) E.J. Cockayne S.T. Hedetniemi		(36) J. W. Di Paola	
10:00		10:15 WELCOMING REMARKS: Dr. C. Burnett	(12) L.O. James D.D. Cowan R.G. Stanton	(13) D.D. Cowan F.J. Burkowski	(24) D.T. Bean		(37) F.O. Hadlock	
10:30		A.J. Hoffman		117 GCS H. Wilf		117 GCS E. Berlekamp		
P.M. 1:30		A. J. Hoffman		H. Wilf		E. Berlekamp		
2:30	118 GCS (1) F. Hoffman	(4) B. Alspach	319 GCN (15) L. V. Quintas R. Frucht A. Gewirtz	(17) D. P. Roselle C.M. Cordes	118 GCS (26) D. Foata	(28) J. P. Dolch		
3:00	(2) R. Alter T.B. Curtz K.K. Kubota	(5) L. T. Ollmann	(16) B. Eisenberg		(27) L. Billard	(29) R.J. Collens P. H. Dirksen		
3:30	(3) B.D. Beach H. C. Williams	(6) J. G. Kalbfleisch	(18) K. B. Reid		(30) P.J. Schellenberg J.D. Horton R.C. Mullin	(33) W. D. Hoskins E. L. Albasiny		
4:00	(7) C. R. Zarnke H. C. Williams	(9) M. D. Plummer	(20) J. A. Bondy R. L. Hemminger	313 GCN (19) G. Berman R. N. Burns	(31) R.C. Mullin B. Gardner	(34) M. Doob		
4:30	(8) H. C. Williams		(21) R.L. Hemminger J. A. Bondy		(32) R.S.D. Thomas			
	Beer Party at Freeman's: 5:30 PM				Banquet at Boca Teca: 7:30 PM (Bar opens 6:30)			

110 GCN: Registration, Coffee Breaks
8:30 - 5:00 DAILY

1

Orbits Under Actions of Affine Groups

Frederick Hoffman
Florida Atlantic University

Algorithms for studying the actions of groups of affine transformations on the power sets of finite vector spaces are presented and applications given.

2

Remarks and Results on Congruent Numbers

*Ronald Alter, Thaddeus B. Curtz and K. K. Kubota
University of Kentucky
Lexington, Kentucky 40506

An integer a is said to be a congruent number if and only if there exist positive integer solutions to the system of Diophantine equations

$$x^2 + ay^2 = z^2 \quad \text{and} \quad x^2 - ay^2 = z^2. \qquad (1)$$

Since cn^2 is a congruent number if and only if c is a congruent number it suffices to study only square free congruent numbers. In 1915 A. Gérardin listed all square free congruent numbers $c < 1000$, for which $x < 3722$ in Equation (1). This was followed by a note of L. Bastien in which he listed all 25 square free incongruent numbers < 100. In this paper, new necessary and sufficient conditions for a number to congruent are established. Also, using a computer search, new congruent numbers < 1000 are found and all known square free congruent numbers < 1000 are exhibited in a table.

3

A NUMERICAL INVESTIGATION OF THE DIOPHANTINE EQUATION $x^2 - Dy^2 = -1$

B. D. Beach*
H. C. Williams
University of Manitoba

Several simple criteria for determining the solvability of the Diophantine equation

$$(1) \qquad x^2 - Dy^2 = -1$$

for certain forms of D are presented. From the list of all values of D ($< 10^6$) for which (1) is solvable are eliminated all values which can be predicted by any of the presented criteria. A table is presented of the remaining values of D.

4

Point-symmetric Graphs of Prime Degree and Transitive Permutation Groups of Prime Degree

Brian Alspach
Simon Fraser University

The automorphism group of any point-symmetric graph or digraph of prime order is explicitly determined in terms of its symbol. In addition, given a transitive permutation group G of prime degree p, the number of graphs or diagrams of order p having G as their group is determined.

5

MINIMAL n-CYCLE SATURATED GRAPHS

L. Taylor Ollmann
Louisiana State University

Let H be a graph. A graph G is H-saturated if G has no subgraph isomorphic to H but the addition of any new edge to G yields a graph with a subgraph isomorphic to H. G is said to be (H,n)-minimal if

1) G has n vertices
2) G is H-saturated
3) no graph G' with properties 1 and 2 has fewer edges than G.

We characterize the $(K_{2,2}, n)$ -- minimal graphs for all natural numbers n and show that the number of edges in a $(K_{2,2}, n)$ -- minimal graph is $[(3n - 5)/2]$ for $n > 4$.

6

Complete Subgraphs of Random Hypergraphs and Bipartite Graphs

J.G. Kalbfleisch, Dept. of Statistics, University of Waterloo

A random graph G_{np} is a graph on $n \geq 1$ vertices whose edges are added randomly and independently, with each of the $\binom{n}{2}$ possible edges having probability p of inclusion ($0 < p < 1$). Matula [1970] defined the following random variables:

K_{dnp} = number of d-membered complete subgraphs in G_{np}

D_{np} = largest number of vertices in any complete subgraph of G_{np}.

He obtained expressions for the mean and variance of K, and hence derived some interesting results concerning D. In particular, he found that the distribution of D is highly spiked, and that for n large, D is essentially a constant.

In the present paper, indicator variables and formulae for the mean and variance of a sum are used to simplify the derivation of Matula's results, and to obtain similar results for complete subgraphs of random hypergraphs and random bipartite graphs.

Reference: D.W. MATULA [1970], On the complete subgraphs of a random graph, Proc. 2nd Chapel Hill Conference on Combinatorial Mathematics and its Applications, 356-369.

7

COMPUTATION OF THE SOLUTIONS OF

THE DIOPHANTINE EQUATION

$x^2 - Dy^4 = 1$

H. C. Williams
C. R. Zarnke
University of Manitoba

A description is given of a computer programme which can be used to determine all solutions of the Diophantine equation

(1) $$x^2 - Dy^4 = 1 .$$

This programme determines the fundamental unit of the ring $Z[\sqrt{D}]$ by using the continued fraction algorithm and then applies the criteria of Ljunggren to determine the solutions of (1). A table of all solutions of (1) is given for various values of D.

8

SOME ALGORITHMS FOR SOLVING

$x^q \equiv a \pmod{p}$

H. C. Williams
University of Manitoba

Some algorithms are presented which can be used to solve the congruence

$$x^q \equiv a \pmod{p} ,$$

where p and q are primes and a is an integer such that $a^{(p-1)/q} \equiv 1 \pmod{p}$. For the most general of these algorithms, the number of operations necessary to find x is of order $q^3 \log p$. For some special cases ($q = 2, 3, 5$), algorithms are given which require fewer operations than the general algorithm.

9

On the (m^+,n^-) connectivity of 3-polytopes

M. D. Plummer
Vanderbilt University

A graph G is (m^+,n^-) connected if given any m+n points, $u_1,\ldots,u_m,v_1,\ldots v_n$, there is a path P in G which contains all the u_i's, but none of the v_j's. We shall obtain all values of m and n for which the following implication is true: "G is 3-polytopal (i.e. G is a 3-connected planar graph) $\Rightarrow$ G is (m^+,n^-) connected." Furthermore, arbitrarily large counterexamples are provided for each pair (m,n) for which the implication fails.

Brother Juniper

"—But we just RAN that play, Stupid."

10

A RANDOM GRAPH GENERATOR

William W. Kuhn
St. Joseph's College
Phila., Pa.

Graphs are generated by randomly selecting edges until all of the vertices are of the prescribed degree or no more edges are available. This could occur if in generating a regular graph of degree 2 on n vertices, we arrive at a graph that is regular of degree 2 on n - 1 vertices and has an isolated vertex.

To solve this problem, edges already in the graph are randomly selected until we arrive at an edge that is not incident to any of the deficient vertices and each of whose end points is not adjacent to all of the deficient vertices. For each end point the non-adjacent, deficient vertices are weighted by the number of edges missing, and one of these vertices is randomly selected to be joined to that end point. The process is then repeated until there are no more deficient vertices.

11

On Tutte's Algebraic Approach to the Theory of Crossing Numbers

Roy B. Levow
Florida Atlantic University

Some historical observations on Tutte's algebraic approach to the theory of crossing numbers are presented, along with observations concerning application of the method to testing for planarity and computing planar crossing numbers. An extension of the procedure to surfaces of higher genus is also discussed, and corresponding results are given.

12

GRAPH DECOMPOSITION

D. D. Cowan L. O. James* R. G. Stanton
University of Manitoba

It has proved very useful for algorithmic purposes to be able to represent lists as list structures. Some attempts have been made to create an analogous structure for graphs. In this paper, two types of graph structure are presented, one for directed and one for undirected graphs. Algorithms are presented for finding canonical decompositions of graphs which allow one to use similarities in structure of the graph to save memory.

13

The Numerical Solution of Integral Equations with Deviating Arguments

F. J. BURKOWSKI
DEPARTMENT OF COMPUTER SCIENCE
UNIVERSITY OF MANITOBA
WINNIPEG, MANITOBA

D. D. COWAN*
DEPARTMENT OF COMPUTER SCIENCE
UNIVERSITY OF WATERLOO
WATERLOO, ONTARIO

The theoretical existence of solutions of Fredholm integral equations involving deviating arguments, for example,

$$y(t) = f(t) + \int_a^b Q(s,t,y(g(s)))ds$$

has only recently received attention. This paper considers various types of problems related to such equations and presents some computational schemes designed to yield approximations to the solutions of such equations.

14

Quintic Splines and the Numerical Solution of a Differential-Difference Equation

F.J. BURKOWSKI G.E.McMASTER*
DEPARTMENT OF COMPUTER SCIENCE
UNIVERSITY OF MANITOBA
WINNIPEG, MANITOBA

This paper presents a fourth-order method for calculating the numerical solution of a differential-difference equation with retardation, having the form

$$y'(t) = F(t,y(g_1(t)),\ldots,y(g_n(t)))$$

where each $g_i(t) \leq t$ for all i and for $t \geq t_o$ where t_o is the initial instant. The propagated approximate solution is stored as a quintic spline with deficiency three. An error analysis is also given.

15

THE LEAST NUMBER OF EDGES FOR CONNECTED GRAPHS HAVING AUTOMORPHISM GROUP OF ORDER THREE

Roberto Frucht; Universidad Tecnica Federico
Santa Maria, Valparaiso, Chile

Allan Gewirtz; Brooklyn College, CUNY
Brooklyn, N.Y. 11210 U.S.A.

* Louis V. Quintas; Pace College
New York, N.Y. 10038 U.S.A.

Let $e_c(C_3;n)$ denote the least number of edges realizable by a connected graph having n vertices and automorphism group of order three. It is shown that:

$$e_c(C_3;n) = \begin{cases} \text{undefined} & \text{for } n < 9 \\ 15 & \text{for } n = 9 \text{ or } 10 \\ 16 & \text{for } n = 11 \\ 15 & \text{for } n = 12 \\ n+2 & \text{for } n = \begin{cases} 3k+1 \\ 3k+2 \\ 15 \end{cases} (k = 4, 5, \ldots) \\ n & \text{for } n = 3k \quad (k = 6, 7, \ldots). \end{cases}$$

16

AN INTERPRETATION OF THE CHROMATIC POLYNOMIAL FOR SOME NEGATIVE ARGUMENTS

Bernard Eisenberg
Kingsborough Community College C. U. N. Y.

Let $M_G(\lambda) = \sum_{r=1}^{n} (-1)^{n-r} a_r \lambda^r = \sum_{r=1}^{n} b_r \lambda^{(r)}$ be the chromatic polynomial of a graph G on N nodes in standard and factorial forms respectively, where $\lambda^{(r)} = \lambda(\lambda-1)(\lambda-2)\cdots\cdots(\lambda-r+1)$. It is shown that up to sign, $M_G(-1)$ is equal to the difference between the number of colorings of G (with color difference) using an odd number of colors and the number of such colorings using an even number of colors. An expression for $M_G(-2)$ is derived in terms of c_r, the number of colorings with color difference using exactly r colors. Lastly, it is shown that $\sum_{i\text{-even}} a_i = \sum_{i\text{-odd}} a_i = \frac{1}{2}|M_G(-1)|$.

17

ON GENERALIZED FRIEZE PATTERNS

by
Craig M. Cordes and D. P. Roselle *
Louisiana State University

An m-frieze pattern is an array of the form

$$\begin{array}{lllll} 1 & 0 & 0 & 0\ldots & \\ c_{11} & 1 & 0 & 0\ldots & \\ c_{21} & c_{22} & 1 & 0\ldots & \\ c_{31} & c_{32} & c_{33} & 1\ldots & \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \\ c_{n1} & c_{n2} & c_{n3} & c_{n4} & \ldots \\ 1 & c_{n+1,2} & c_{n+1,3} & c_{n+1,4}\ldots & \\ 0 & 1 & c_{n+2,3} & c_{n+2,4}\ldots & \\ & 0 & 1 & c_{n+3,4}\ldots & \\ & & 0 & 1 & \ldots \\ & & & 0 & \ldots, \end{array}$$

where the c_{ij} are defined according to the requirements that the $k \times k$ matrix with c_{ik} in the lower right hand corner have determinant $1 (2 \le k \le m)$ and the $m \times m$ matrix with c_{ij} in the lower right hand corner have determinant $1 (j \ge m)$. The periodicity of these arrays is proved for all $m \ge 2$.

18

On Hypotraceable Graphs

K. B. Reid
L. S. U.

A graph G is traceable if it contains a spanning path (often called a Hamiltonian path), a path that passes through each vertex exactly once. If v is a vertex of G, then G-v denotes the subgraph of G obtained from G by deleting the vertex v and all of its incident edges. G is hypotraceable if G has no spanning path, but G-v has a spanning path for each vertex v of G. The graph $\bar{K}_2$, two vertices and no edges, is hypotraceable. No other hypotraceable graph is known; the question of existence of another hypotraceable graph is unsolved. In this paper we discuss some necessary conditions if G is hypotraceable and cubic. A related problem concerning spanning cycles is also mentioned. Further, we introduce the notion of edge-saturated graphs (with respect to spanning paths) and make a conjecture on their structure. An affirmative solution to our conjecture implies the non-existence of any other hypotraceable graph.

19

A quadratic lattice algorithm for minimizing non-linear functions

Gerald Berman and R.N. Burns
University of Waterloo

A point $q = q(f, L)$ is associated with a real valued function f of n variables and a geometrical lattice L of E_n such that if f is a quadratic form, then f(q) is the minimum of f. A non derivative algorithm is presented for determining q, which when combined with the lattice approximation technique yields an effective method for approximating the minimum value of a unimodal function, provided the number of variables is not too large. Quadratic convergence is proved, and convergence is assured even for the example described by Zangwill which does not converge by Powell's method.

20

Reconstructing Infinite Graphs I: Disconnected Graphs

J.A. Bondy *
University of Waterloo, Ontario, Canada
R.L. Hemminger
Vanderbilt University, Nashville, Tennessee, U.S.A.

In the 1950's S.M. Ulam conjectured that any finite graph of order at least three can be reconstructed from its maximal vertex-deleted subgraphs. Formally (writing G_v for G-v) Ulam's Conjecture states: let G and H be finite graphs of order at least three such that there is a bijection $\sigma: V(G) \to V(H)$ with the property

$$G_v \cong H_{\sigma(v)} \quad \text{for all } v \in V(G).$$

Then $G \cong H$.

This conjecture has not been proved in general, although it is known to be true for disconnected graphs and trees and for several other classes of graph.

We here examine more closely Ulam's Conjecture for infinite graphs. We show that G and H must have the same finite components, occurring with the same multiplicity. Corollaries of this are that if G either has only finite components, or has some finitely occurring finite component, then $G \cong H$.

21

Reconstructing Infinite Graphs II: Trees

J. A. Bondy and R.L. Hemminger *

We prove Ulam's conjecture for m-coherent locally finite trees if m is finite and greater than one.

22

A SURVEY OF CERTAIN BALANCED INCOMPLETE BLOCK DESIGNS

R. A. Kingsley*
R. G. Stanton
University of Manitoba

A variety of methods for constructing balanced incomplete block designs with parameters (36, 15, 6), (45, 12, 3), and (56, 11, 2) are described. Partial results on isomorphisms among the designs are presented.

23

INTERPOLATION SYSTEMS

E. J. Cockayne*
University of Victoria

S. T. Hedetniemi
University of Iowa
University of Virginia

Let A be a finite set, let P be a property associated with subsets of A and let Q be a binary relation defined on the power set of A. A partition $\pi = \{A_1, A_2, \ldots, A_k\}$ of A, of order k, is complete if every block A_i of π has property P and every pair A_i, A_j of distinct blocks of π stand in the relation Q. In this paper we determine conditions on P and Q which will guarantee that if a set A has complete partitions of orders m and n, then for every k, $m < k < n$, A has a complete partition of order k.

24

Maximal Refinement of a Matroid.

D.T. Bean
York University

The circuit axioms are used to define a matroid. A refinement of a matroid $\mathcal{M}$ on a set E is a matroid $\mathcal{N}$ on the same set with the property that each atom of $\mathcal{M}$ is an atom of $\mathcal{N}$. A simple condition is given which ensures the existence of a proper maximal refinement $\mathcal{N}$ of $\mathcal{M}$ and the atoms of $\mathcal{N}$ are explicitly determined. The geometric lattice of $\mathcal{N}$ is the truncation lattice of the geometric lattice of $\mathcal{M}$.

25

ON THE CHROMATIC NUMBER OF CERTAIN 2-COMPLEXES

by

Paul C. Kainen
Case Western Reserve University

Let K be any 2-dimensional simplicial complex and let $G(K)$ denote the collection of all finite simple graphs which can be piecewise-linearly embedded in K. The chromatic number of K, $\chi(K)$, is defined to be

$$\sup_{G \in G(K)} \chi(G) \quad ,$$

where $\chi(G)$ denotes the ordinary chromatic number of a graph. It is easy to see that if K is non-compact, $\chi(K)$ may be infinite, so we shall deal only with finite (i.e., compact) complexes K. K is a quasi-manifold if every point has a neighborhood homeomorphic to the Euclidean plane or to a disjoint union of planes with their origins identified. If K is a compact quasi-manifold, $\nu(K)$ denotes

$$\max_{G \in G(K)} \; \min_{v \in V(G)} \delta(v)$$

where $V(G)$ is the set of vertices of G and $\delta(v)$ = degree of v. For example, $\nu(S^2) = 5$. We define another invariant $\mu(K)$ to be the maximum order of any complete graph which belongs to $G(K)$. The following theorem is then obtained. <u>Theorem</u>. If $\nu(K) \geq \mu(K)$, then $\chi(K) \leq \nu(K)$. <u>Corollary</u>. If $\nu(K) = \mu(K)$, then $\chi(K) = \nu(K)$. This includes a conjecture of Ringel that the chromatic number of a sphere in which two points have been identified is precisely 5.

26

Permutation profiles and Euler numbers.

Dominique Foata
University of Florida

Let $\mathfrak{S}'_n$ denote the set of all permutations σ of the set $[n] = \{1, 2, \ldots, n\}$ such that $\sigma(1) = n$ and let $\overline{d}, d, \overline{m}, m$ be four indeterminates. To each σ in $\mathfrak{S}'_n$ $(n \geq 2)$ is attached a monomial $V(\sigma) = v_1 v_2 \ldots v_n$ where for any $j = 1, \ldots, n$

$v_j = \overline{d}$ if $\sigma(j) > \sigma(j-1), \sigma(j+1)$ (by convention $\sigma(0) = \sigma(n)$ and $\sigma(n+1) = \sigma(1)$);

$v_j = d$ if $\sigma(j-1) > \sigma(j) > \sigma(j+1)$;

$= \overline{m}$ if $\sigma(j) < \sigma(j-1), \sigma(j+1)$;

$= m$ if $\sigma(j-1) < \sigma(j) < \sigma(j+1)$.

The polynomials $V_n(\overline{d}, d, \overline{m}, m) = \sum \{V(\sigma) : \sigma \in \mathfrak{S}'_n\}$ $(n \geq 2)$ have the following properties :

(i) $V_n(t, t, 1, 1) = t\, A_{n-1}(t)$ where $A_{n-1}(t)$ is the $(n-1)$-st Eulerian polynomial ;

(ii) There exist nonnegative integers $d_{n,k}$ such that $V_n(\overline{d}, d, \overline{m}, m) = \sum \{2^{k-1} d_{n,k} (\overline{d}\,\overline{m})^k (d+m)^{n-2k} : 0 < 2k \leq n\}$;

(iii) The integer $d_{n-1} = \sum d_{n,k}$ $(n \geq 2)$ is the coefficient of $u^{n-1}/(n-1)!$ in the expansion of $\sec u + \tan u$.

Accordingly property (iii) involves a new refinement of secant and tangent numbers, which can be interpreted combinatorially by using an abstract model called the André complex.

27

A SOLUTION OF A DIFFERENTIAL - DIFFERENCE EQUATION

L. Billard University of Waterloo

The probabilities of a general stochastic epidemic model are obtained as recursive solutions of a differential difference equation. By an appropriate partitioning of the associated matrix of coefficients, it is shown that the probabilities consist of factorial terms. Further, similarities between certain probabilities become evident.

28

NAMES AND ALIASES OF GRAPHS

John P. Dolch
Computer Science Department
University of Iowa
Iowa City, Iowa

This paper presents a computer method of obtaining a "NAME" for an arbitrary unlabelled, undirected graph based upon the concept of "the flat of a graph", which is simply a string of digits.

Although flattening a graph was originally derived as a method for computer manipulation of complex structures, this linear specification turned out to be an extremely convenient "short-hand" for human use, since anyone can readily construct "by hand" an ALIAS of a given graph. To determine from a given ALIAS the particular canonical form that is the NAME of the graph is, in general, a problem requiring a computer.

Since all undirected graphs can be assigned unique NAMES, it is possible to lexicographically order all such graphs; two graphs can be proven to be isomorphic by simply showing that they have the same NAME; the presence of an Hamiltonian path or cycle is always indicated by the NAME; and this coding scheme provides a realistic computer method of generating all NAMES of graphs for a relatively large given number of points.

29

An Analysis by Simulation of Hashing Function Performance

by

R.J. Collens* and P.H. Dirksen
University of Manitoba

The authors present performance results on three classes of hashing functions. The method used is to simulate activity in a symbol table for several different load factors for each class of hashing function. Performance is shown to primarily depend on the collision resolution technique used and on the statistical distribution of names used for entry into the table. Performance is affected to a lesser extent by the hashing function. Two resolution methods show relatively little performance degradation even with nearly full tables and highly clustered input distribution.

30

On the Existence of Room Designs of Side 8s + 5

J.D. Horton, R.C. Mullin, P.J. Schellenberg*
University of Waterloo

A construction for Room designs of side 8s + 5 , s > 0 , is established. It is then shown that for any value s > 0 , either the construction can be applied or a Room design of side 8s + 5 can be shown to exist by other constructions.

31

Asymptotic Behaviour of (r, λ)-Systems

*R. C. Mullin and B. Gardner
University of Waterloo

An (r, λ)-system is a finite set V of objects, whose elements are distributed into subsets called blocks, such that every pair of distinct elements occurs in precisely λ blocks, and every element occurs in precisely r blocks. With any such design one can associate the triple of integers (v, r, λ) , where |V| = v . We investigate the nature of the designs obtained for large values of one of these parameters when the other two are kept fixed. The main result obtained is that given fixed r and λ, r > λ > 0 , for sufficiently large v , every block of an (r, λ)-system has cardinality either 1 or v . The implications of this and some related bounds are discussed.

32

Partially Closed Braids

R. S. D. Thomas
University of Manitoba

Objects of combinatorial topology closely related to closed braids will be defined. These objects are equivalence classes of braids. It will be shown that each of these classes has a unique representation easily obtainable by an algorithm that makes use of the braid combing described at the Second Louisiana Conference.

33

Some Explicit Error Bounds for Polynomial Splines of Odd Order defined on a Uniform Set of Knots

E. L. ALBASINY
Division of Numerical Analysis and Computing, National Physical Laboratories.
W. D. HOSKINS*
Department of Computer Science, University of Manitoba.

The method of defining splines in terms of even derivatives derived Albasiny and Hoskins (1971), facilitates, in this paper, the development of theoretical explicit error bounds for the even derivatives of splines of any odd order. The corresponding bounds for the odd derivatives can be subsequently obtained from the bounds for the even derivatives.

34

USING GRAPH PRODUCTS TO CONSTRUCT ASSOCIATION SCHEMES, OR PLUS ÇA CHANGE PLUS ÇA LA MÊME CHOSE

M. Doob
University of Manitoba

Two class association schemes are identical in concept with strongly regular graphs, and hence it seems plausible that the Cartesian product of graphs might result in new higher class association schemes using the graph theoretic distance between vertices to determine association classes. Necessary and sufficient conditions will be given for this possibility to be realized as well as examples of new association schemes and partially balanced designs that result from these products.

35

DIFFERENTIAL OPERATORS OVER GRAPH-LATTICES

Charles C. Cadogan
University of the West Indies

Differential operators are derived and applied to counting polynomials over the elements of graph-lattices, which set up relations between the numbers of general graphs and multigraphs. The operators are applied in the first instance to cubic unlabelled graphs and later to graphs on trivalent as well as monovalent nodes of different kinds (colors). The paper ends with a section on the generalised operators which can be used to obtain results on multigraphs with a given partition.

36

GRAPH THEORY AND THE EXISTENCE CONJECTURES FOR BLOCK DESIGNS

Jane W. Di Paola, Florida Atlantic University

In a previous work (J. Comb. Th. 1(1966) 132-148) the author gave a graph theoretic interpretation to the existence problem for block designs by showing that the internal stability number of a binomial coefficient graph is determined by the existence of a block design with specified parameters. The general problem of determining the internal stability number of binomial coefficient graphs is now discussed in the light of recent work by Hanani, Ray-Chaudhuri and Wilson on the existence conjectures for block designs.

37

The Minimal 2-Coloration Problem

Frank O. Hadlock
Florida Atlantic University

We consider the problem of 2-coloring the vertices of a graph so as to minimize the number of edges with end points colored the same. Various formulations of the problem are posed. A heuristic scheme is described along with several algorithms for the special case when the graph is planar.

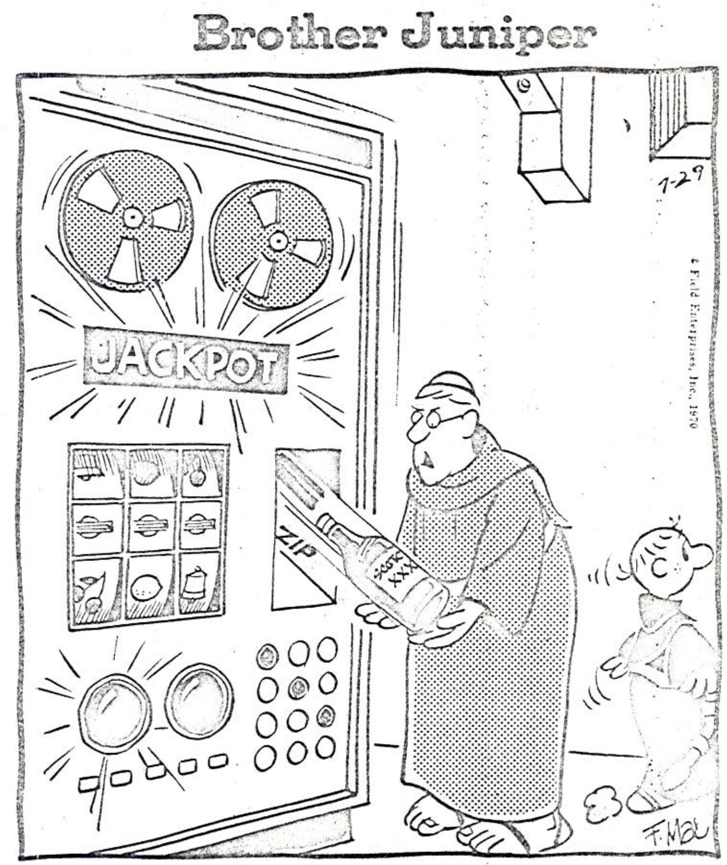
38

Spitzer's Formula for Non-Commutative Algebras

J.M. Freeman
Florida Atlantic University

Spitzer's formula equates the obvious iterative solution of the equation x = 1+T(ax) to a solution of exponential form. Here T is a Baxter operator on an algebra. Several approaches have been used in deriving this (Atkinson, Baxter, Rota), but all have depended on the assumption that the underlying algebra be commutative.

We present the Spitzer formula in form valid for non-commutative algebras, and indicate some applications.



"Say, what are you programming this computer with?"