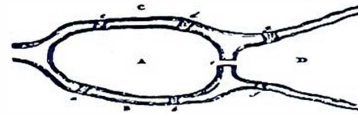


Thirty-Fifth Southeastern International Conference on

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Combinatorics, Graph Theory & Computing

Florida Atlantic University

March 8 - 12, 2004

Program and Abstracts

Invited Speakers

35th Southeastern International Conference
on Combinatorics, Graph Theory, and Computing

MONDAY, MARCH 8, 2004

9:40 AM and 2:00 PM

John F. Dillon

National Security Agency

**p-ary Perfect Sequences and Cyclic Differences Sets
with Singer Parameters: A Survey in Two Parts**

TUESDAY, MARCH 9, 2004
9:00 AM

Carsten Thomassen
Technical University of Denmark

On Hajos' Conjecture

Hajos' conjecture says that every graph of chromatic number k contains a subdivision of the complete graph on k vertices. The conjecture was disproved by Catlin in 1979 for all $k > 6$. We relate this conjecture to other aspects of graph theory, namely Ramsey theory, perfect graphs, and the maximum cut problem, and point that well known classical results provide a variety of explicit counterexamples. On the other hand, Catlin proposed a general conjecture for producing counterexamples, and we point out that this conjecture is not true.

TUESDAY, MARCH 9, 2004
10:20 AM

Ann N. Trenk
Wellesley College

Tolerance Graphs - A Retrospective via Hierarchies

Tolerance graphs were introduced in 1982 by Golumbic and Munma as a generalization of the class of interval graphs. A graph $G = (V, E)$ is a *tolerance graph* if each vertex $v \in V$ can be assigned a real interval I_v and a positive tolerance $t_v \in \mathbb{R}$ so that $xy \in E(G)$ iff $|I_x \cap I_y| \geq \min\{t_x, t_y\}$.

Golumbic and Trenk have recently completed a book on Tolerance Graphs and related subjects. In this talk we summarize some of the main ideas in the book using hierarchy diagrams as a unifying theme.

TUESDAY, MARCH 9, 2004
2:00 PM

Carsten Thomassen
Technical University of Denmark

Graphs and Surfaces

It is well-known that the property of being locally connected simplifies the structure of a metric space considerably. Nevertheless, a complete description of the locally connected compact metric spaces seems hopeless. However, a complete description becomes possible if we add the condition that the space does not contain an infinite complete graph and if we also strengthen the local connectivity condition to *local 2-connectedness*, that is, for every element x in the space, and every neighborhood U of x , there exists a neighborhood V of x contained in U such that both V and $V \setminus \{x\}$ are connected. Surprisingly, such a space must be locally 2-dimensional, that is, it is contained in a compact 2-dimensional surface. Some applications to graph embeddings will be given. The result depends on and includes the classification of the compact 2-dimensional surfaces. A very short proof of that will be given, too.

WEDNESDAY, MARCH 10, 2004
11:00 AM

Herbert S. Wilf
University of Pennsylvania

0-1 Matrices and Automated Discovery of Fibonacci-ish Identities

After a brief genealogical digression, we'll discuss two new results. The first is an exact count of the number of $n \times n$ 0-1 matrices whose eigenvalues are real and positive. This result is due to McKay, Oggier, Royle, Sloane, Wanless and myself, and proves a conjecture of Weisstein. Next we show how to automate the discovery and proof of identities involving sums of products of numbers that satisfy certain recurrences, the prototype being the Fibonacci numbers.

WEDNESDAY, MARCH 10, 2004
2:00 PM

Richard A. Brualdi
University of Wisconsin

The Class $A(R, S)$ of $(0, 1)$ -matrices

THURSDAY, MARCH 11, 2004
9:00 AM

Richard A. Brualdi
University of Wisconsin

Bigraphs, Digraphs, SNS-matrices, Tilings, and Aztec Diamonds

THURSDAY, MARCH 11, 2004
10:00 AM

Frank Harary
New Mexico State University

Some Fascinating New Games on Graphs

Monday, March 8, 2004

8:00am	Registration - Grand Palm Room Available until 5:00pm			
9:00am	Opening Ceremonies & Welcome by: President Frank Brogan, Interim Provost Kenneth Jessell, & Dean Nathan Dean Grand Palm Room			
9:40am	Invited Speaker: Dillon - Grand Palm Room			
10:40am	Coffee			
	Live Oak Pavillion			
	Room A	RoomB	Roome	RoomD
11:00am	001: A Abueida	002: H Harborth	003: D. Erwin	004: H Tapia-Recillas
11:20am	005: Y. Zou	006: M. Kasinadhuni	007: A. Raju	008: L. Gionfriddo
11:40am	009: A. Starling	010: W. Gasarch	011: D. VanderJagt	012: O. Abu Ghneim
12:00pm	Lunch (on your own)			
2:00pm	Invited Speaker: Dillon - Grand Palm Room			
3:00pm	Coffee			
3:20pm	013: P. Li	014: E. Cheng	015: G. Rinaldi	016: D. McIntyre
3:40pm	017: D. Genova	018: A. Farley	019: D. Froncek	020: D. Kountanis
4:00pm	021: U. Peled	022: M. Plantholt	023: T. Kovarova	024: I. Taksa
4:20pm	025: H. Uehara	026: F. Harris, Jr.	027: J. Klerlein	028: W. El-Hajj
4:40pm	029: D. Kephart	030: T. Adachi	031: D. Hoffman	032: A. Lee
5:00pm	033: J. Ellis-Monaghan	034: H. Escudro	035: E. Mitzer	036: N. Miyamoto
5:45pm	Reception at the Baldwin House			

Coffee and refreshments will be served in the registration area of the Grand Palm Room.

1 Cycle extendability and Hamilton cycles in chordal graph classes

Atif Abueida* and R. Sritharan, University of Dayton

A cycle C in a graph is *extendable* if there exists a cycle C' such that $V(C) \subset V(C')$ and $|V(C')| = |V(C)| + 1$. A graph is *cycle extendable* if every non-Hamiltonian cycle in the graph is extendable. An unresolved question is whether or not every Hamiltonian chordal graph is cycle extendable. We show that Hamiltonian graphs in classes such as interval, split, and some subclasses of strongly chordal, are cycle extendable. We also address efficiently finding a Hamilton cycle in some cases. A unifying theme to our approach is the use of appropriate vertex elimination orders.

Keywords: cycle, Hamiltonian, extendability, chordal graph

2 Vertex Turian Numbers for Cube Graphs

Reiko Harborth, Technische Universität Braunschweig

The Turian number for given graphs G and H is the maximum number of edges chosen in G such that no subgraph H in G contains chosen edges only. The classical Turian numbers for $G = K_n$ and $H = K_m$ are known since 1940. If G is the cube graph Q_n and $H = Q_2$ then we have an unsolved problem of P. Erdős. Here we propose to consider vertices instead of edges for $G = Q_n$ and $H = Q_m$ and we give first results for $m = 2$.

(Common work with Hauke Nienborg)

3 α -labelings of graphs

Gary Chartrand, Ping Zhang, Western Michigan University; David Erwin*, Trinity College; and Donald VanderJagt, Grand Valley State University

For a connected graph G of order n and size m , a α -labeling of G is a one-to-one function $f: V(G) \rightarrow \{0, 1, 2, \dots, m\}$ that induces a labeling $J: E(G) \rightarrow \{1, 2, \dots, m\}$ of the edges of G defined by $J(e) = |J(u) - J(v)|$ for each edge $e = uv$ of G . Hence, a graceful labeling is a α -labeling J having the property that J is one-to-one. For a α -labeling J , define $\text{val}(J) = \sum_{e \in E(G)} J(e)$, and, for a graph G , $\text{val}_{\min}(G) = \min\{\text{val}(f)\}$ and $\text{val}_{\max}(G) = \max\{\text{val}(f)\}$, where the minimum and maximum are taken over all α -labelings f of G . We discuss $\text{val}_{\min}(G)$ and $\text{val}_{\max}(G)$ for several classes of graphs G .

Keywords: labeling

4 Some partial difference sets over the Galois ring $GR(p^2, m)$

H. Tapia-Recillas, Universidad Autónoma Metropolitana-I

Difference sets are related to a number of topics which include bent functions and error detecting-correcting linear codes. Several authors have studied these sets in relation to some mathematical objects over the binary field. Recently some authors have been studied a class of difference sets over the Galois ring $GR(4, m)$ and its image under the Gray map, providing binary difference sets. In this talk some results on partial difference sets over the Galois ring $GR(p^2, m)$, where p is any prime, will be discussed, some of which generalize those for the case $p = 2$.

5 Decycling of Cartesian Product of Cycles

David A. Pike and Yubo Zou, Memorial University of Newfoundland

The decycling number $\nabla(G)$ of a graph G is the smallest number of vertices which can be removed from G so that the resultant graph contains no cycles. The family of graphs which we consider is the Cartesian product of two cycles, sometimes called a toroidal grid.

In this paper, we completely solve the problem of determining the decycling number of Cartesian product of two cycles, $C_m \times C_n$, for all m and n , and we find the corresponding decycling set. Moreover, we find a vertex set T that yields a maximum induced tree in $C_m \times C_n$.

Keywords: decycling number, Cartesian product, maximum induced tree

6 Multiple Genomic Representations That Self Adapt A Genetic Algorithm

Maheswara Kasinadhuni, Michael L. Gargano and Joseph DeCicco, Pace University

Using multiple genomic representations we make a genetic algorithm more efficient through self adaptation. A specific example and results will be discussed.

Keywords: multiple genomic representation, genetic algorithm, adaptation

7 Computational Complexity of k-stratified Graph Construction

Anupama Raju, Wasim El-Hajj and Dionysios Kountanis, Western Michigan University

A graph $G(V, E)$ is a k -stratified graph if V is partitioned into $V_1, V_2, \dots, V_k$ classes (strata). Let G_i be the subgraph of G induced by V_i ($1 \leq i \leq k$) and $G_{i,j}$ be the bipartite subgraph with V_i and V_j as its two partites and $E_{i,j} = \{e(u, v) \in E \mid u \in V_i, v \in V_j\}$. If d is the degree of a vertex $v \in V$, then a k -stratification of G defines for d a k -vector of integers $(d_1, d_2, \dots, d_k)$, where d_i ($1 \leq i \leq k$) indicates the number of vertices in V_i incident to v . Note that $d = \sum_{i=1}^k d_i$. If a multiset of n k -vectors is given, does a k -stratified graph exist that defines these vectors?

The computation complexities of this problem and a number of its subproblems are investigated. It is proven that the above problem is NP-Complete. It remains NP-Complete even if the cardinalities of $V_1, V_2, \dots, V_k$ are known. The problem becomes polynomial if the identities of vertices in each V_i ($1 \leq i \leq k$) are known.

Keywords: k -stratified graphs, NP-complete, complexity, integer vectors

8 Strong difference families over arbitrary graphs

Marco Buratti and Lucia Gionfriddo, Universita degli Studi di Catania

Given a graph Γ , we define a (n, Γ, μ) strong difference family (SDF in short) to be a collection of maps from $V(\Gamma)$ to Z_n such that the list of differences $\{a - (x) - o - (y) \mid a \in E, \{x, y\} \in E(\Gamma)\}$ covers all of Z_n (0 included!) exactly μ times. When $r = K_k$ (the complete graph on k -vertices) this concept is equivalent to the concept of a (v, k, μ) -SDF as introduced in [M. Buratti, *Old and new designs via difference multisets and strong difference families*, J. Combin. Des. 7 (1999), 406-425]. In that paper it is shown how a (v, k, μ) -SDF gives rise to cyclic $(k, >)$ group divisible designs of type n^m for many values of m and $>$ with $m \leq k$ and $>$ a divisor of μ . We show, more generally, that a (n, Γ, μ) -SDF allows to get cyclic decompositions of $xK_m \times n$ (the $>$ -fold complete m -partite graph with parts of size n) into copies of Γ .

Keywords: Group divisible design, complete m -partite graph, (cyclic) graph decompositions

9 Mixed Modulus Torus Connected n -Cycle Graphs

A Gregory Starling, and Gordon Beavers, University of Arkansas

We introduce a generalization of the Cube Connected n -Cycle graph, CCC_n , and call the new graph $MMTC_n = (V, E)$ with vertex set $V = \mathbb{Z}_n \times (\mathbb{Z}_{m_n-1} \times \mathbb{Z}_{m_n-2} \times \dots \times \mathbb{Z}_{m_n-1})$ and edges defined as follows. Let $\Gamma = \{(1, (0 \dots 0)), (0, (10 \dots 0)), (0, (010 \dots 0)), \dots, (0, (0 \dots 01))\}$ be a set of generators for the standard group on V . There is an edge from $(i, (a_{n-1} \dots a_i \dots a_0))$ to $(i, (a_{n-1} \dots a_{i+1} \dots a_0))$, where a_{i+1} is computed modulo m_i , and there is an edge from $(i, (a_{n-1} \dots a_0))$ to $(i+1, (a_{n-1} \dots a_0))$, where $i+1$ is computed modulo n . There are no other edges in the graph. The graph, $MMTC_n$ thus described is a subgraph of the Cayley Graph $\text{Cay}(V, \Gamma)$. We describe several properties of these graphs including an optimal routing algorithm.

Keywords: Cube Connected Cycles Graph, routing, Cayley Graph

10 Finding Large Sets Without Arithmetic Progressions of Length Three

Fawzi Emad, William Gasarch, University of Maryland; James Glenn, Loyola University; and Tasha Inniss, Spelman University

How big can a set $A \subseteq \{1, \dots, n\}$ be and still not have an arithmetic progression of size 3. There are several constructions of such sets in the literature. We survey these constructions and code them up to see how well they do. Of particular interest is to see for which n an asymptotic method begins doing better than an easier method.

11 Homogeneous Embeddings of Stratified 5-Cycles

Gary Chartrand, Ping Zhang, Western Michigan University; and Donald W. VanderJagt, Grand Valley State University

A stratified graph G is a graph whose vertex set has been partitioned into subsets, called the strata or color classes of G . A stratified graph with k strata is a k -stratified graph. Two k -stratified graphs G and H are isomorphic if there exists a color-preserving isomorphism ϕ from G to H . A k -stratified graph G is said to be homogeneously embedded in a k -stratified graph H if for every vertex x of G and every vertex y of H where x and y are colored the same, there exists an induced k -stratified subgraph H' of H containing y and a color-preserving isomorphism ϕ from G to H' such that $\phi(x) = y$. A k -stratified graph F of minimum order in which G can be homogeneously embedded is called a frame of G and the order of F is called the framing number of G . The framing numbers are determined for all stratifications of the 5-cycle.

Keywords: stratified graph, homogeneous embedding

12 On Nonabelian McFarland Difference Sets

Omar A. Abu Ghneim, Central Michigan University

We provide one negative and one positive result for the existence of nonabelian $(96, 20, 4)$ difference sets. We show if G is a group of order $q^2(q+2)$ which has two normal subgroups U, U' with nontrivial intersection and $|U| = |U'| = q$ such that G/U and G/U' are cyclic, then G does not admit a $(q^2(q+2), q(q+1), q)$ difference set. This generalizes a result of Arasu, Davis, Jedwab and Ma for nonabelian case. We use our result to show that any group of order 96 which has $Z_{24} \times Z_2$ or $4 \times Z_2$ or $(Z_3 \rtimes Z_8) \times Z_2$ or $\mathbb{D}_{16}$ as a factor subgroup does not have a $(96, 20, 4)$ difference set.

Finally we use GAP to construct $(96, 20, 4)$ difference sets in all groups of order 96 which have an elementary abelian normal subgroup of order 16.

Keywords: Difference set, Nonabelian, GAP

13 A Parallel Implementation of an Algorithm for Computing Covering Numbers

Pak Ching Li, University of Manitoba

A parallel implementation of an exact algorithm for finding covering designs will be presented. This algorithm is based on a branch-and-bound algorithm which incorporates isomorphism rejection techniques and the Schoheim bound. The message passing interface (MPI) was used to implement the parallel program. Using this algorithm, we were able to improve three covering numbers.

14 Extended fault-diameter of star graphs

Eddie Cheng, Ray Kleinberg, Will Lindsey, Dan Steffy, Oakland; Marc J. Lipman, IPFW

An important issue in computer communication networks is fault-tolerant routing. A popular graph topology for interconnection network is the class of star graphs (Akers, Harel and Krishnamurthy). The star graph S_n is a $(n-1)$ -regular graph with $n!$ vertices and connectivity $n-1$. It is known that if up to $2n-4$ vertices are deleted from S_n , the resulting graph has a single large component and at most two components of size at most two. This talk will discuss routing in the large component and also show that its diameter in the faulty star graph is bounded by $\text{diam}(S_n) + 9$.

Keywords: Interconnection Networks, Routing, Diameter

15 Nilpotent one-factorization of the complete graph

Gloria Rinaldi, University of Modena and Reggio Emilia

In 1985 Hartman and Rosa proved that each cyclic group of even order $2n$ can be realized as a sharply vertex transitive automorphism group of a one-factorization of the complete graph of order $2n$, except the case $2n = 2^m$, $2^m \geq 8$. In 2001 M. Buratti generalized this result to each Abelian group of even order. We consider other classes of groups, trying to generalize the result to the class of Nilpotent groups.

16 Exploiting "Unused" States to Efficiently Encode Canonical Huffman Decode Trees

D. R. McIntyre, Cleveland State University; and F. G. Wolff, Case Western Reserve University

Given an information source defined by a pair (S, F) of source symbols $S = \{s_1, s_2, \dots, s_n\}$ and a set of corresponding nonnegative frequencies $F = \{f_1, f_2, \dots, f_n\}$, the well-known static Huffman compression algorithm can be used to construct a binary tree with n leaf nodes and $n-1$ internal nodes. Huffman's tree has the minimum value of $I = \sum_{i=1}^n f_i l_i$ over all such binary trees, where l_i is the level at which symbol s_i with frequency f_i occurs in the tree. Binary trees with n leaves are in a one-to-one correspondence with sets of n strings on $\{0, 1\}$ that form a "minimal prefix code" (strings in which no string is a proper prefix of another). The correspondence between trees and codes is simply to represent the path from the root to each leaf node as a string of 0's and 1's, where 0 corresponds to a left branch and 1 to a right branch. Huffman codes are clearly information source dependent and hence decode information must be stored along with the compressed source. We introduce a more space efficient representation of canonical trees to store the decode information contained in the Huffman tree. This is achieved by taking advantage of "unused" decode states in the implementation of canonical trees.

Keywords: canonical trees, compression, decode, data, Huffman, prefix codes

17 Forbidding-enforcing families of languages

Daniela Genova, University of South Florida

Forbidding-enforcing systems (fe-systems) were introduced by A. Ehrenfeucht and G. Rozenberg in "Forbidding-enforcing systems" Theoretical Computer Science 292 (2003) 611-638. Fe-systems define new classes of languages (fe-families) based on boundary conditions. Fe-families are topologically closed and compact spaces and as such completely different from the well-known Chomsky classes of languages. In this paper we investigate some algebraic properties and graph-theoretical aspects of fe-families.

Keywords: Forbidding and enforcing, Formal language theory, Theory of computation

18 Betweenness Centers of Trees and Unicyclic Graphs

Arthur M Farley, University of Oregon

A vertex v of an undirected graph G is *between* two other vertices x and y of G if and only if there exists a shortest path connecting x and y that passes through v . The *betweenness* of a vertex v , $b(v)$, is the number of pairs of vertices that v is between. The *betweenness center* of G is the set of vertices with maximum betweenness. We consider betweenness centers of trees and unicyclic graphs, determining the range of betweenness values such central vertices can assume. We present a linear-time algorithm for determining the betweenness center of a tree and an $O(n^2)$ algorithm for a unicyclic graph.

Keywords: trees, unicyclic graphs, centers, betweenness

19 Factorizations of complete graphs into short caterpillars

Dalibor Froncek, Tereza Kovarova, University of Minnesota Duluth; and Michael Kubesa, Technical University Ostrava

Although the factorization of complete graphs into isomorphic spanning trees is a very natural problem, it has not received much attention. One reason may be that the labeling methods that are often used for isomorphic decompositions of complete graphs K_{2n+1} into $2n+1$ copies of trees with $n+1$ vertices (like graceful labeling) do not work for factorizations unless they are modified. Such a modification is not hard to discover, as we show in this talk when we define a *blended labeling* and a *2m-cyclic blended labeling*. However, it is much harder to prove the existence of a blended labeling even for very simple classes of trees. For instance, it is well known that all caterpillars are graceful. On the other hand, it is still not completely known which caterpillars of diameter 5 admit a blended labeling.

We present the complete characterization of caterpillars of diameter 4 that factorize complete graphs K_{4k+2} and some partial results on caterpillars of diameter 5. Notice that complementing results for factorizations of K_{4k} will be given along with more labeling methods in the talk *New techniques for decomposition of complete graphs into isomorphic spanning trees* by Tereza Kovarova.

Keywords: Graph factorization, spanning trees, graph labeling

2Q Minimum Steiner Tree Approximation using Binary Image Skeletons

Dionysios Kountanis", Wasim El-Hajj and Anupama Raju, Western Michigan University

The problem of finding a Minimum Steiner Tree (*MSTT*) for a set of points S on a rectilinear space has been proven to be NP-Complete. A binary image is constructed on the rectilinear space, which includes all points in S . The skeleton of the binary image is used to develop a polynomial approximation algorithm (*AMSTT*) for the *MSTT*. The skeleton defines a tree with vertices $V = S \cup SP$, where SP is the set of Steiner points and a set of edges E that connect the vertices in V . Each edge $e = (v_i, v_j) \in E$ has a cost $c(e) = |x_i - x_j| + |y_i - y_j|$, where (x_i, y_i) and (x_j, y_j) are the coordinates of the vertices v_i and v_j respectively. Then, the cost of the *AMSTT* is:

$$c(AMSTT) = \sum_{e \in E} c(e)$$

The cost $c(AMSTT)$ is compared with the cost $c(MST)$ of the Minimum Spanning Tree (*MST*) of the set S . The difference between the $c(MST)$ and $c(MSTT)$ is well known, therefore the proximity of *AMSTT* to *MSTT* can be measured.

Keywords: Minimum Steiner tree, minimum spanning tree, skeleton, binary image, rectilinear space

21 Computation of Entropy in Statistical Mechanics and Information Theory

Shmuel Friedland and Uri N. Peled', University of Illinois at Chicago

We outline the most recent theory for the computation of the exponential growth rate of the number of configurations on a multi-dimensional grid. As a demonstration, we compute the monomer-dimer constant for the plane lattice correct to 8 decimal digits.

22 An Improved Well-Spread Halving of Multigraphs

Stacey Butler, Mike Plantholt• and Shailesh Tipnis, Illinois State University

By using alternating edges in an euler circuit, we can obtain a "halving" for any connected multigraph G with each vertex degree even and an even number of edges. That is, we get a decomposition into multigraphs G_1 and G_2 where for each vertex v , $\deg(v, G_1) = \deg(v, G_2)$. It is similarly easy to see that if the edges of G with odd multiplicity induce no connected components with an odd number of edges then we can obtain such a halving that is "well-spread", meaning that for each edge uv , with multiplicity denoted $m(uv)$, we have $|m(uv, G_1) - m(uv, G_2)| \leq 1$. Extending previous results, we give improved conditions under which we can add/delete a hamilton cycle to/from G and get such a well-spread halving, even when the original graph G itself has no such well-spread halving. We also discuss application of this result to obtaining sports schedules such that multiple encounters between teams are well-spread throughout the season.

23 New techniques for decomposition of complete graphs into isomorphic spanning trees

Tereza Kovaiova, University of Minnesota Duluth

We introduce the concept of new types of labeling, namely fixing blended labeling and swapping labeling. We show that there is a decomposition of K_{2nk} , *for* k odd, into nk isomorphic copies of a graph G with $2nk - 1$ edges if G has a fixing blended labeling. This new technique is demonstrated by completing the classification for factorization of K_{2nk} into caterpillars with diameter $d = 4$.

Also we show that there is a decomposition of K_n into $2n$ copies of a graph G with $n - 1$ edges if G has a swapping labeling. The method based on swapping labeling covers also the case when the number of vertices is a power of two, for which a fixing blended labeling does not exist.

Keywords: decomposition, spanning tree, blended labeling, swapping labeling, complete graph

24 The Number of Subqueries Required by an Underlying Search Engine

Isak Taksa, Baruch College, CUNY

This research investigates the algorithmic development and complexity analysis of finding similar documents on the web to a submitted long query. This is accomplished by using an underlying search engine and querying it with a series of subqueries (each using k out of the n significant terms found in the user's original query) and merging all of the results of the query into one coalesced list where the documents that have appeared in the most subquery result sets will be listed first with their corresponding weight.

Although this problem is inherently combinatorial, in practice only a polynomial number of queries are actually formed. This is due to three assumptions that search engines use:

- (1) the maximum number of documents returned by a given query is fixed;
 - (2) a dictionary of significant terms with their corresponding weights in the collection of documents has been formed a priori and,
 - (3) the criterion of similarity between documents is met if the number of significant terms that appear in two documents is at least k for some predetermined fixed k .
- Upper bounds on the number of subqueries required were explored and algorithmic efficiency considerations were thus suggested.

25 A positive clone detecting algorithm for DNA library screening

Hiroaki Uehara, Keio University

Group testing was proposed by Robert Dorfman in the 1940s and has recently found new applications in fields such as DNA library screening. In this application, pooling designs are used to specify the positive clones of some given clone library. The goal of a pooling design is to test as many clones as possible by a small number of tests. However, the existence of errors which occur in experiments of a pooling design cannot be disregarded. Therefore, in an experiment of pooling design it is required that positive clones can be detected with a high probability even the existence of errors.

In this talk, we propose a new positive clone detecting algorithm.

26 A Low-Cost Parallel Queuing System for Computationally Intensive Problems

Sean C. Martin, Bei Yuan, Judith R. Fredrickson and Frederick C. Harris, Jr.,
University of Nevada

With the availability of inexpensive computer clusters it is now economically feasible to attack computationally intensive problems using parallel processing. This paper presents an easily adaptable parallel work queue for solving these types of problems, many of which lie in graph theory. This system lifts the user above the details of processor communication (message synchronization, work load balancing) and allows them to focus on the problem to be solved without having to become an expert in parallel processing details. Initial results are presented using this system on two graph theory problems: calculating the Minimum Crossing Number for complete graphs and the Traveling Salesman Problem.

Keywords: parallel, minimum crossing number, complete graph, TSP

27 Hamiltonian Cycles in $C_n \times C_m - T_k$

Edward C. Carr, North Carolina A&T State University; Joseph B. Klerlein*, Western Carolina University; and A. Gregory Starling, University of Arkansas

Let $C_n \times C_m$ be the Cartesian product of two directed cycles C_n and C_m , and let T_k be the k -th triangular number. To form the directed graph $C_n \times C_m - T_k$ one removes T_k vertices in a triangular pattern from the upper right corner of directed graph $C_n \times C_m$ and then the cycles are reconnected. That is, viewing $C_n \times C_m$ as a grid with n rows and m columns where each row is a cycle of length m and each column is a cycle of length of length n , $C_n \times C_m - T_k$ is formed by deleting the vertex from column $m - k$ which is in row n , the cycle of column $m - k$ is reconnected forming a cycle of length $n - 1$. The two vertices from column $m - k - 1$ which are in row $n - 1$ and row n are deleted, this cycle is reconnected to form a cycle of length $n - 2$. The deletion is continued in this fashion until k vertices are deleted from column m , the remaining vertices are reconnected to form a cycle of length $n - k$. In like manner the row cycles are also reconnected. In this paper we investigate the hamiltonicity in the directed graphs $C_n \times C_m - T_k$. In addition we consider the hamiltonicity of directed graphs formed from $C_n \times C_m$ by deleting triangles from the upper left corner and of those directed graphs formed by deleting squares from the corners.

Keywords: Digraph, Hamilton cycle, Cartesian product

28 Network Congestion Measurement and Control

Wasim El-Hajj", Anupama Raju, and Dionysios Kountanis, Western Michigan University

Given a network $G = (V, E)$, a set of projected communication loads l_{ij} between any two sites $v_i, v_j \in V$ and a static routing strategy R then the congestion for an $e \in E$ is defined as:

$$c(e) = \sum_{i=1}^M \sum_{j=1}^M l_{ij} \times B_{ij}(e)$$

where $B_{ij}(e) = 1$ if l_{ij} is in the path P_{ij} defined by R and $B_{ij}(e) = 0$ otherwise. The congestion of the network is defined as: $c(G) = \sum_{e \in E} c(e)$. The objective of the problem is to find a routing R which minimizes the function $(\sigma + L)$, where σ is the standard deviation on the set $\{c(e) | e \in E\}$ and L is the total load of the network i.e. $L = \sum_{e \in E} c(e)$. The problem is proven to be NP-Complete. A number of subproblems and their complexities are investigated. A polynomial approximation algorithm is developed to design a routing strategy R which reasonably controls the congestion of the network G .

Keywords: Network, congestion, NP-Complete, routing, algorithm

29 CodeGen: The Generation and Testing of DNA Codewords

David E. Kephart and Jeff Lefevre, University of South Florida

With this paper we present algorithms to generate and test DNA code words that avoid unwanted cross hybridizations. Methods from the theory of codes based on formal languages are employed. These algorithms are implemented in user-friendly software, CodeGen, which contains a collection of language-theoretic objects adaptable to various related tasks. Lists of code words may be stored, viewed, altered and retested. Implemented in Visual Basic 6.0, its interface allows for lists of code words to be assembled at varying levels of acceptability from a single main window. Now fully implemented, this is the development of a version of software first presented at the 33rd Conference on Combinatorics, Graph Theory, and Computing in March 2002.

30 Constructions of wrapped t_i -labelings leading to cluttered orderings for the complete bipartite graph

Tomoko Adachi, Toho University

The desire to speed up secondary storage systems has lead to the development of redundant arrays of independent disks (RAID) which incorporate redundancy utilizing erasure code. To minimize the access cost in RAID, Cohen, Colbourn and Fritchman (2001) introduced (d, f) -cluttered orderings of various set system for positive integers d, f . In case of a graph this amounts to an ordering of the edge set such that the number of points contained in any d consecutive edges is bounded by the number f . For the complete graph, Cohen et al. gave some cyclic constructions of cluttered orderings based on wrapped p -labelings.

Muller, Adachi and Jimbo (2003) investigated cluttered orderings for the complete bipartite graph. RAID utilizing two-dimensional parity code can be modeled by the complete bipartite graph. Müller et al. adapted the concept of wrapped t_i -labelings to the bipartite case instead of wrapped p -labelings, and gave the explicit construction of several infinite families of wrapped t_i -labelings.

Here, we investigate constructions of more generalized infinite families of wrapped t_i -labelings leading to cluttered orderings for the corresponding bipartite graphs. In this talk, we will give constructions of wrapped t_i -labelings for such cases.

31 An f -factor theorem for signed graphs

Heather Gavlas, Illinois State University; and D.G. Hoffman, Auburn University

We extend Tutte's f -factor theorem to signed graphs, and use it to characterize degree sequences of simple signed graphs.

32 Empirical Tests for Pseudorandom Number Generators

Andrew C. Lee, University of Louisiana at Lafayette

Pseudorandom number generators (PRNG) are commonly used in computational experiments. The validity of conclusion(s) drawn from these experiments often depends on the performance of the pseudorandom number generators used. In practice, the evaluation of the performance of a PRNG are done by applying empirical tests to the PRNG. While empirical tests cannot provide a proof of performance, a PRNG that passes a significant number of empirical tests may provide to the practitioners some confidence to use the PRNG.

In this talk, we will describe a class of empirical tests that are based on Marsaglia's overlapping m -tuple test. Relationships between this class of empirical tests and De Bruijn graphs will be demonstrated.

Keywords: Pseudorandom Number Generators, Empirical Tests, De Bruijn Graphs

33 The generalized transition polynomial, cycle double cover conjecture, and biomolecular computing

Jo Ellis-Monaghan, St. Michael's College

The generalized transition polynomial provides a highly adaptable generalization of the transition polynomial of Jaeger and embeds it in an algebraic structure. As such, it provides a unifying framework for the Martin (or circuit partition) polynomials, the Penrose polynomial, and the Kauffman bracket. We show that it also provides a generating function formulation for an application from biomolecular computing (assembling graphs from strands of DNA) that requires a double covering of the edges of a graph. Although the biomolecular construction is less restrictive than a cycle double cover (an edge may be covered twice by the same "cycle" for example), results from cycle double covers inform the original problem from biomolecular computing.

Keywords: graph polynomials, transition systems, Eulerian graphs, cycles, cycle double cover conjecture, biomolecular computing, DNA strands

34 Detour Distance in Graphs

Gary Chartrand, Henry Escudro, and Ping Zhang, Western Michigan University

For two vertices u and v in a connected graph G , the detour distance $D(u, v)$ from u to v is defined as the length of a longest u - v path in G . The detour eccentricity of a vertex v in G is the maximum detour distance from v to a vertex of G . The detour radius of G is the minimum detour eccentricity among the vertices of G , while the detour diameter of G is the maximum detour eccentricity among the vertices of G . Some results on detour distance are presented.

Keywords: distance, detour distance, detour eccentricity

35 Randomly Decomposable Graphs in $K_m P_e$

Erin Mitzer, Central Michigan University

Given a graph H , we say a graph G is randomly H -decomposable if any subgraph isomorphic to H is part of an H -decomposition. The set of all randomly H -decomposable graphs is denoted by $RD(H)$. We examine $RD(H)$ where H is a graph constructed by identifying a vertex of the complete graph on m vertices with the end-vertex of a path of length e .

Keywords: graph decomposition, randomly decomposable

36 Mutually M -intersecting (k, d) -arcs and Optical Orthogonal Codes

Nobuko Miyamoto, Tokyo University of Science; and Satoshi Shinohara, Meisei University

A (k, d) -arc in $PG(2, q)$ is a set K of k points such that some line of the plane meets K in d points but such that no line meets K in more than d points, where $d \geq 2$. Let M be a set of nonnegative integers. A family of (k, d) -arcs is called a *Mutually M -intersecting (k, d) -arcs* if any pair of (k, d) -arcs in the family has m points in common, where m is an integer from M .

In this talk we have Mutually M -intersecting $(q+1, 2)$ -arcs, where $M = \{0, 1, 2\}$ and Mutually M -intersecting $(q+2, 2)$ -arcs, where $M = \{0, 1, 2, 3\}$. These families can be applied to obtain optical orthogonal codes with the correlation 2 or 3, which are used for fiber optical code division multiple access (CDMA) communications.

Keywords: (k, d) -arc, optical orthogonal code

Tuesday, March 9, 2004

8:00am	Registration - Grand Palm Room Available until 3:00pm			
	<i>Live Oak Pavillion</i>			
	<i>Room A</i>	<i>RoomB</i>	<i>Roome</i>	<i>RoomD</i>
8:20am	037: M. Gargano	038: D. Stewart	039: J. Georges	040: J. Young
8:40am	041: J. Malerba	042: V. Grolmusz	043: J. Factor	044: E. McMahon
9:00am	Invited Speaker: Thomassen - Grand Palm Room			
10:00am	Coffee			
10:20am	Invited Speaker: Trenk - Grand Palm Room			
11:30am	045: W. Trotter	046: R. Sulanke	047: K. Factor	048: A. Kemnitz
11:50am	049: A. Finbow	050: A. Nkwanta	051: R. Gera	052: M. Albertson
12:10pm	053: G. Isaak	054: D. Chopra	055: J. Villalpando	056: P. Slater
12:30pm	Lunch (on your own)			
2:00pm	Invited Speaker: Thomassen - Grand Palm Room			
3:00pm	Coffee			
3:20pm	057: M. Stern	058: S. Horton	059: N. Cavenagh	060: H. Balakrishnan
3:40pm	061: S. Kim	062: D. Siewert	063: S. Hurd	064: K. Shafique
4:00pm	065: M. Golumbic	066: Y. Wang	067: G. van Rees	068: S. Lee
4:20pm	069: J. Lundgren	070: J. Watkins	071: A. Street	072: E. Leung
4:40pm	073: M. Stern	074: S. Merz	075: K. Delamar	076: F. Saba
5:00pm	077: D. Brown	078: C. Wallis	079: M. Ferencak	080: E. Yera
5:20pm	081: P. Zhang	082: D. Steffy	083: A. Cami	084: D. Herscovici
6:00pm	Reception at the Visual Arts Patio			

Coffee and refreshments will be served in the registration area of the Grand Palm Room.

37 Angiogenesis As A Random Graph Process

Michael L. Gargano[•], Pace University; Louis V. Quintas and Eric M. Wahl, NY
Institute for Bioengineering and Health Sciences

Graph theory can be used to model vascular networks. Here we introduce a random graph process to model network growth.

Keywords: angiogenesis, random graph process, discrete medical modeling

38 Fully Indecomposable Tournament Matrices

J. Richard Lundgren and Dustin J. Stewart[•], University of Colorado at Denver

A tournament matrix is the adjacency matrix of a tournament. If the rows and columns of an $n \times n$ matrix M can be independently permuted so that M has an $r \times s$ sub-matrix of zeros with $r + s = n$, then M is said to be partly decomposable, and fully indecomposable if not. If the rows and columns of M can be permuted so that M can be written as the direct sum of two smaller matrices, we say M is separable.

In this talk, we classify the tournament matrices which are fully indecomposable, partly decomposable, and separable. We then use this information to give some linear algebra and graph theoretic results. In particular, we show when any arc is contained in a cycle factor of a tournament.

Keywords: tournament, tournament matrix, fully indecomposable, separable, cycle factors

39 On the $L(j,k)$ -number of Prisms and Related Graphs

John P. Georges[•] and David W. Mauro, Trinity College

Let j and k be positive integers, $j \geq k$. For graph G $L: V(G) \rightarrow \mathbb{Z}$ is an $L(j, k)$ -labeling of G if and only if the labels of vertices adjacent in G differ by at least j , and the labels of vertices that are distance two apart in G differ by at least k . The $L(j, k)$ -number of G is the minimum span over all $L(j, k)$ -labelings of G . In this paper, we investigate the behavior of the $L(j, k)$ -number of n -prisms ($C_n \times K_2$), as well as of the infinite ladder ($P_{\infty} \times K_2$) and the Petersen graph.

Keywords: $L(j, k)$ -labeling, n -prism, infinite ladder, Petersen graph

40 Some New Families of Edge-Magic Cubic Graphs

Sin-Min Lee, Maki Kitagaki and Joseph Young, San Jose State University

If G is a (p, q) -graph in which the edges are labeled $1, 2, 3, \dots, q$ so that the vertex sums are constant, mod p , then G is said to be *edge-magic*. If G is a (p, q) -graph in which the edges are labeled $1, 2, 3, \dots, q$ so that the vertex sums are distinct, mod p , then G is said to be *edge-graceful*. It is conjectured by Lee that any simple cubic graph with $p \equiv 2 \pmod{4}$ vertices, is *edge-magic*. Lee and Shiu showed that the conjecture is not true for disconnected cubic graphs and multigraphs. Two of them are disconnected simple graphs and one is a connected simple graph. Three new families of edge-magic and edge-graceful graphs are exhibited.

Kitagaki and Young investigated whether connected cubic graphs with $p = 4n$ vertices are *edge-magic* and *edge-graceful*.

41 Paintable Graphs-II

Michael L. Gargano, Joseph F. Malerba, Pace University; and Marty Lewinter, Purchase College

A nontrivial graph G of order n is called paintable if its vertex set can be labeled with the natural numbers $\{1, 2, \dots, n\}$ such that for each $i = 1, 2, \dots, n - 1$, we have edge $(i)(i + 1)$ in the edge set of G . We give a complete discussion of the paintable characteristics of trees and product graphs.

42 Defying Dimensions Modulo 6

Vince Grolmusz, Eotvos University

We consider here a certain modular representation of polynomials. The well-known modulo 6 representation of polynomial g is just polynomial $g + 6e$. The 1-a-strong representation of g modulo 6 is polynomial $g + 3f + 4h$, where no two of g, f and h has common monomials. In some cases the retrieval of g from its 1-a-strong representation is not hard: e.g., from $x_1 + 3x_2 + 4x_3$ one can get back x_1 simply by running through the values of x_i on the set $\{0, 1, 2, 3, 4, 5\}$, and noticing that only x_1 has period 6, ($3x_2$ has period 2, $4x_3$ has period 3).

Using this representation, we describe some surprising applications: we show that $n \times n$ matrices can be converted to $n^{o(1)} \times n^{o(1)}$ matrices (Quantity $o(1)$ here denotes a positive number which goes to 0 as n goes to infinity) and from these tiny matrices we can retrieve 1-a-strong representations of the original ones, also with linear transformations. We call this phenomenon a dimension-defying property of 1-a-strong representations. We also show that a 1-a-strong representation of the matrix-product can be computed with only $n^{o(1)}$ multiplications.

43 Internal Chords of $\text{Pdom}(U;t)$ and the Arcs that Produce Them

James D. Factor, Marquette University

The regular rotational tournament U_n has domination graph $\text{dom}(U_n) = C_n$. Certain sets of arcs can be added to U_n , creating $U;t$, that generate a specific internal chord in C_n . These sets of arcs are a well-defined collection. Part of this collection generates only the specific internal chord, while the others generate additional chords as well. The form and number of these collections of arcs and their generated internal chords are characterized and enumerated.

Keywords: extended tournament, tournament, regular rotational tournament, m-tie internal chord, domination graph, partial domination graph

44 Color-Consistent Automorphisms of Cayley Graphs

Melanie A. Albert, Timothy A. Fargus, Elizabeth W. McMahon, and Jaren A. Smith, Lafayette College

It is well known that the group color-preserving automorphisms (those which send edges corresponding to a given generator to other edges corresponding to the same generator) of the Cayley graph of a group G is isomorphic to G . We define automorphisms of Cayley graphs which are color-consistent, that is, those which (at most) permute the edges corresponding to various generators. The group of color-preserving automorphisms of a Cayley graph can be larger than the original group, but it is restricted in how much larger it can be. This paper examines the structure of this larger automorphism group. We give general results as well as specific results for certain examples.

Keywords: Cayley graphs, graph automorphisms

45 Hamiltonian Cycles and Symmetric Chain Partitions

William T. Trotter, Georgia Institute of Technology.

We show that for each $n \geq 2$, there is a hamiltonian cycle in the n -cube which parses into a symmetric chain partition of the subset lattice. The motivation for this work is rooted in a pair of well known conjectures. The first of these is the Middle Levels Conjecture first posed by Ivan Havel: There is a hamiltonian cycle in the bipartite graph formed by the k and $(k + 1)$ -element subsets of a set of size $2k + 1$. The second is the Montone Hamiltonian Path Conjecture posed by Felsner and Trotter: For each $n \geq 1$, there is a hamiltonian path $A_1, A_2, \dots, A_{2^n}$ in the lattice subset of $\{1, 2, \dots, n\}$ so that if $A_i \subset A_j$, then $i \leq j + 1$.

This second conjecture in turn has its origins in the problem of determining the maximum chromatic number of the diagram of an interval order of height h .

46 Generalizing Narayana and Schroder Numbers to Higher Dimensions

Robert A. Sulanke, Boise State University

Let $C(d, n)$ denote the set of d -dimensional lattice paths using the steps $X_1 := (1, 0, \dots, 0)$, $X_2 := (0, 1, \dots, 0)$, $\dots$, $X_d := (0, 0, \dots, 1)$, running from $(0, 0, \dots, 0)$ to $(n, n, \dots, n)$, and lying in $\{(x_1, x_2, \dots, x_d) : 0 \leq x_1 \leq x_2 \leq \dots \leq x_d\}$. On any path $P := p_1 p_2 \dots p_n \in C(d, n)$, define the statistics $asc(P) := |\{i : p_i p_{i+1} = X_l X_t, j < l\}|$ and $des(P) := |\{i : p_i p_{i+1} = X_l X_t, i > l\}|$. Define the generalized Narayana number $N(d, n, k)$ to count the paths in $C(d, n)$ with $asc(P) = k$. We derive a formula for $N(d, n, k)$, implicit in MacMahon's work. We examine other statistics for $N(d, n, k)$ and show asc and $des-d+1$ to be equidistributed. We use Wegschaider's algorithm, extending the Sister Celine (Wilf-Zeilberger) method to multiple summation, to obtain recurrences for $N(3, n, k)$. We introduce the generalized large Schroder numbers $(2^{d-1} \sum_k N(d, n, k) 2^k)_{n \geq 1}$ to count constrained paths using step sets which include diagonal steps.

Keywords: Lattice paths, Catalan numbers, Narayana numbers, Schroder numbers, Sister Celine (Wilf-Zeilberger) method

47 Domination and Network Stability: An Application

Amit S. Arora and Kim A.S. Factor, Marquette University

Given a digraph D , the domination graph of D , $dom(D)$, is constructed where $\{u, v\} \in E[dom(D)]$ when $(u, w) \in A(D)$ or $(v, w) \in A(D)$ for any $w \in V(D)$. Here, our digraph represents a network. A dominant vertex a is removed from the model. For each of its adjacent vertices (b) in $dom(D)$, a distance measure is applied to determine an appropriate transmitter replacement for the missing vertex. The (3)-stability of the network is defined, and then examined for networks represented by certain classes of tournaments.

Keywords: domination graph, tournament, stability, network

48 Circular Chromatic Numbers of Certain Planar Graphs

Arnfried Kemnitz, Technische Universitat Braunschweig

A (k, d) -coloring $(k, d \in \mathbb{N}, k \geq 2d)$ of a graph G is an assignment c of k colors $\{0, 1, \dots, k-1\}$ to the vertices of G such that $d \leq |c(v_i) - c(v_j)| \leq k-d$ whenever two vertices v_i and v_j are adjacent. The circular chromatic number $X_c(G)$ (sometimes also called star chromatic number) is defined by $X_c(G) = \inf\{k/d : G \text{ has a } (k, d)\text{-coloring}\}$.

Since a $(k, 1)$ -coloring of G is a k -coloring of G - which implies $X_c(G) \leq x(G)$ -, the circular chromatic number is a refinement of the chromatic number $x(G)$ and therefore contains more information about the structure of the graph G .

We determine $X_c(G)$ for some classes of planar graphs such as outerplanar graphs, Platonic solid graphs, Archimedean solid graphs, Archimedean prism graphs and for all graphs of order at most 7.

Keywords: Circular chromatic number, star chromatic number

49 Tree Tolerance Representations of $K_{2,n}$

Art Finbow', Saint Mary's University; and Robert Jamison, Clemson University

We are interested in graphs which are representable as intersection graphs of subtrees of a tree where adjacency is determined by having sufficiently many nodes t in common. The number t is called the *tolerance*.

These graphs form classes which are a natural extension of the classes of chordal graphs and interval graphs. By putting various restrictions on t , on the tree, and on the subtrees one obtains various classes.

In this work, we fix t and determine the maximal n such that $K_{2,n}$ has a so called "leaf generated orthodox $[3, 3, t]$ representation".

50 A note on Riordan matrices and combinatorial sums

A. Nkwanta, Morgan State University

When given a combinatorial array a natural question is what are the combinatorial sums of the array? In this note we survey various combinatorial arrays and consider the arrays as Riordan matrices. We then use a Riordan group method to compute the row sums, weighted row sums, and diagonal sums of the matrices. As a result of this method, we obtain combinatorial sums involving the Fibonacci, Catalan, Fine, triangular, and rook numbers and other counting numbers that are contained in Sloane's on-line database of integer sequences.

51 On F-Domination in Stratified Graphs

Ralucca Gera and Ping Zhang, Western Michigan University

A graph G is 2-stratified if its vertex set is partitioned into two classes, the red vertices and the blue vertices. Let F be a 2-stratified graph rooted at some blue vertex v . The F-domination number of a graph G is the minimum number of red vertices of G in a red-blue coloring of the vertices of G such that every blue vertex v of G belongs to a copy of F rooted at v . Some results concerning F-domination are presented.

Keywords: stratified graphs, F-domination

52 Coloring Vertices and Faces of Embedded Graphs

Michael O. Albertson, Smith College; and Bojan Mohar, University of Ljubljana

If G is an embedded graph, $c : V(G) \cup F(G) \rightarrow \{1, 2, \dots, r\}$ is called a *vertex-face r -coloring* if elements are assigned different colors whenever they are either adjacent or incident. Let $X_{v,f}(G)$ denote the minimum r such that G has a vertex-face r -coloring. Ringel conjectured that if G is planar, then $X_{v,f}(G) \leq 6$. A graph G is said to be I -embedded in S_9 if every edge crosses at most one other edge. Borodin proved that if G is I -embedded in the plane, then $x(G) \leq 6$. This result implies Ringel's conjecture. Ringel also stated a Heawood style theorem for I -embedded graphs. We prove a slight generalization of this result. We also consider locally planar graphs. If G is I -embedded in $S_{9\bullet}$, let $w(G)$ denote the *width* of G viz the length of a shortest non-contractible cycle in G . We show that if G is I -embedded on $S_{9\bullet}$ and $w(G)$ is large enough, then $X_{v,f}(G) \leq 8$. We do not know any locally planar I -embedded graph that requires more than six colors.

53 Staircase graphs and t-split interval orders

Garth Isaak, Lehigh University

Tolerance graphs can be viewed as intersection graphs of parallelograms on two parallel lines. We review this perspective for variations on tolerance graphs. Generalizing from the unit bitolerance case we get to intersections graphs of staircases, or t-split interval graphs. We discuss results about these from the order theoretic perspective: an order is t-split if there are increasing strings $(x_1, x_2, \dots, x_t)$ such that $x < y$ in the order if and only if $x_2 < y_1, x_3 < y_2, \dots, x_t < y_{t-1}$. 2-split orders are interval orders and 3-split orders have been called split interval orders. All orders are t-split for some t so they yield an alternate notion of dimension of an order.

54 On the nonexistence of some orthogonal arrays

D.V. Chopra, Wichita State University; and R. Dios, New Jersey Institute of Technology

A balanced array (B-array) of strength t with m constraints ($m \geq t$), N runs, and with two symbols (say, 0 and 1) is merely a matrix T of size $(m \times N)$ such that in every submatrix $(t \times N)^*$ of T , every $(t \times 1)$ vector a with i ($0 \leq i \leq t$) ones in it appears a constant number μ_i (say) times. The vector $(\mu_0, \mu_1, \dots, \mu_t)$ is called the index set of the B-array. If $\mu_i = \mu$ for each i , then B-array is called an orthogonal array (0-array). We consider here the nonexistence of some 0-arrays with $t = 4$.

Keywords: constraints, balanced arrays, orthogonal arrays, strength of an array

55 Parameters Concerning Cycle-Domination

J. Villalpando, Gonzaga University; and R. Laskar, Clemson University

Let G be a graph with vertex set V and edge set E , and let H be a set of graphs. As defined by Cockayne et al., a set $S \subseteq V$ is H-independent if (S) contains no subgraph in H . Using maximality and minimality conditions an H-dominating set and H-irredundant set can be defined.

Domination parameters can be generalized for H-independence, H-dominating, and H-irredundance. The domination chain holds for these generalized domination parameters. In this paper we study H-domination and its associated parameters when H is the set of cycles, called cycle-domination. We prove results concerning conditions and implications of cycle-independence, cycle-domination, and cycle-irredundance. We prove that the difference between any of two consecutive cycle-domination parameters can be made arbitrarily large. We also prove bounds of the cycle-domination parameter.

Keywords: H-independent, H-dominating, H-irredundant, cycle-domination

56 Colored problem for graphs

Suk J. Seo, Middle Tennessee State University; and Peter J. Slater, University of Alabama in Huntsville

We consider problems in which the vertex and/or edge set is partitioned into color classes. The solution set is required to contain either all or none of the elements in each color class. Problems of this type include finding a longest cycle, an independent set, or a dominating set. We primarily consider the "coupled domination" problem in which the vertex set is partitioned into subsets of order at most two.

Keywords: coupled domination, colorings

57 The k -edge intersection graphs of paths in a tree

Martin Charles Golumbic, Marina Lipshteyn and Michal Stern, University of Haifa

We consider a generalization of edge intersection graphs of paths in a tree. Let P be a collection of nontrivial simple paths in a tree T . We define the k -edge ($k \geq 1$) intersection graph $fk(P)$, such that the vertices correspond to the members of P , and we join two vertices by an edge if the corresponding members of P share k edges in T . An undirected graph G is called a k -edge intersection graph of paths in a tree, and denoted by $k\text{-EPT}$, if $G = f_k(P)$ for some P and T . It is known that the recognition of the 1-EPT graphs is NP-complete. We prove that the recognition of the $k\text{-EPT}$ graphs is also NP-complete for any fixed $k \geq 2$. We show that the family of 1-EPT graphs is contained, but not equal, to the family of $k\text{-EPT}$ graphs for any fixed $k \geq 2$. We show that there exists an infinite family of graphs that are not $k\text{-EPT}$ graph for every $k \geq 1$. The edge intersection graphs are used in network applications. Scheduling undirected calls in a tree is equivalent to coloring an edge intersection graph of paths in a tree.

Keywords: intersection graphs, paths, trees, NP-completeness

59, A note on the completion of partial latin squares

Nicholas J. Cavenagh, G.H.J. van Rees and Diane Donovan, The University of Queensland

A *partial latin square* is an n by n array in which entry of $N = \{1, 2, \dots, n\}$ occurs at most once in each row and once in each column. The problem of determining whether a partial latin square has a unique completion to a latin square is NP-complete. Nevertheless there is potential to improve algorithms that determine whether a partial latin square has unique completion. In this talk we introduce a new test by which the completion of a partial latin square to a latin square may be "forced".

Keywords: latin square, partial latin square, unique completion

58 Broadcast Domination Algorithms for Interval Graphs, Series-Parallel Graphs, and Trees

Jean R. S. Blair, Steve Horton, United States Military Academy; Pinar Heggernes and Fredrik Manne, University of Bergen

A *broadcast domination* of a graph G is an assignment of an integer value $J(u) \geq 0$ to each vertex u such that every vertex u with $j(u) = 0$ is within distance $f(v)$ from a vertex v with $f(v) > 0$. We can regard the vertices v with $f(v) > 0$ as broadcast stations, each having a transmission power that might be different from the transmission power of other stations. The optimal broadcast domination problem seeks to minimize the sum of the costs of the broadcasts assigned to the vertices of the graph, where the costs are taken to be the $j(v)$ values. We present dynamic programming algorithms that solve the optimal broadcast domination problem for the first classes of graphs with non-trivial solutions: interval graphs, series-parallel graphs, and trees. We also show that optimal broadcast domination is equivalent to optimal domination on proper interval graphs.

Keywords: domination, algorithms, interval graphs, series-parallel graphs

60 Implementation and Analysis of a Parallel Algorithm for Radiocoloring

Hemant Balakrishnan and Narsingh Deo, University of Central Florida

A *radiocoloring* of a graph G is a function f from the vertex set $V(G)$ to the set of all non negative integers such that $|f(x) - f(y)| \geq 2$ if $d(x, y) = 1$ and $|f(x) - f(y)| \geq 1$ if $d(x, y) = 2$ where $d(x, y)$ represents the distance between the vertices x and y . The radiochromatic number, $X(G)$ is the smallest number k such that G is radiocolored with $\max\{f(v) : v \in V(G)\} = k$. This paper presents a parallel heuristic for this NP-complete problem and the result of the experiments performed with an implementation of our approximate parallel algorithm for radiocoloring using MPI. Besides the expected reduction in execution time, it was observed that there exists a threshold (graph size) only after which the fruits of parallelization can be obtained. We also perform an analysis of the execution time, which is heavily dependent on the data structure.

Keywords: Radiocoloring, $E(2, 1)$ labelling, Parallel algorithm

61 Coloring the complements of intersection graphs of geometric figures

Seog-Jin Kim and Kittikorn Nakprasit, University of Illinois at Urbana-Champaign

Let $\bar{G}$ be the complement of the intersection graph G of a family of translations of a compact convex figure in $\mathbb{R}^n$. When $n = 2$, we show that $\chi(\bar{G}) = \min \{3\alpha(G) - 2, 6 - \gamma(G)\}$, where $\gamma(G)$ is the size of the minimum dominating set of G . The bound on $\chi(\bar{G}) \leq 6\gamma(G)$ is sharp. For higher dimension we show that $\chi(\bar{G}) \leq f_2(n^2 \cdot (n+1)^{1/2}) - 1$ for $n \geq 3$. We also study the chromatic number of the complement of the intersection graph of homothetic copies of a fixed convex body in $\mathbb{R}^n$.

62 Matrix Ranks of the Reduced Adjacency Matrices of Trees

Daluss J. Siewert, Black Hills State University

The biclique cover and partition numbers of bipartite graphs are related to several matrix ranks. These ranks include the boolean rank, nonnegative integer rank, and real rank. In recent years there has been considerable research directed towards the problem of finding classes of bipartite graphs where the biclique cover number and biclique partition number are equal or classes where two or more of the related matrix ranks are equal. In this talk, some of the known results will be discussed and it will be shown that the boolean rank, nonnegative integer rank, term rank, and real rank of the reduced adjacency matrices of trees are all equal. In addition, best possible lower and upper bounds on the ratio of the value of these ranks to the size of the tree will be given.

Keywords: bipartite graph, tree, biclique cover, biclique partition, boolean rank, nonnegative integer rank, and real rank

63 Pair-Resolvability And Tight Embeddings For Path Designs

Spencer P. Hurd and Dinesh G. Sarvate, Citadel & College of Charleston

We define a new type of resolvability for designs called *a-pair-resolvability* in which each point appears in each resolution class as a member of a-pairs. This concept is intended for path designs or other designs in which the role of points in blocks is not uniform or for designs that are not balanced. We determine the necessary conditions for path designs and show they are sufficient for $k = 3$ and $a = 2, 3$ ($a > 1$ is necessary in every case). We also consider near a-pair-resolvability and show the necessary conditions are sufficient for $a = 2, 4$. We consider under what conditions is it possible for the ordered blocks of a path design to be considered as unordered blocks and thereby create a triple system (a tight embedding) and there also show the necessary conditions are sufficient. We show it is always possible to embed maximally unbalanced path designs $\text{Path}(v, 3, 1)$ into $\text{Path}(v + s, 3, 1)$ for admissibles, and to embed any $\text{Path}(v, 3, 2A)$ into a $\text{Path}(v + s, 3, 2A)$ for any s .

Keywords: pair-resolvable, path designs, tight-embedding, triple system

64 On X-Free Covers

Khurram Shafique and Ronald D. Dutton, University of Central Florida

For an arbitrary graph $G = (V, E)$, let X be a graphical property that can be possessed, or satisfied by the subsets of V . For example, being a clique (maximal complete subgraph), a maximal independent set, an edge, a closed neighborhood, a minimal dominating set, etc. Let $Sx = \{A \mid A \text{ possesses or satisfies property } X\}$. A set S is an X -cover (or X -free) if Ans (or $An(V - S)$) is not empty for every $A \in Sx$. Further, S is an X -free cover when S is both X -free and an X -cover. Many vertex-partitioning problems can be viewed as that of finding an X -free cover. In this paper, we present properties of X -free covers and investigate the conditions for their existence in some special cases.

Keywords: vertex cover, edge cover, independent set, vertex-partition

65 Cycle-bicolorable Graphs and Triangulating Chordal Probe Graphs

Anne Berry, Universite Blaise Pascal; Martin Charles Golumbic and Marina Lipshteyn, University of Haifa

A graph $G = (V, E)$ is chordal probe if its vertices can be partitioned into two sets P (probes) and N (non-probes) where N is a stable set and such that G can be extended to a chordal graph by adding edges between non-probes.

We give several characterizations of chordal probe graphs, first in the case of a fixed given partition of the vertices into probes and non-probes, and second in the more general case where no partition is given. In both of these cases, our results are obtained by introducing new classes, namely, N -triangulatable graphs and cycle-bicolorable graphs. We give polynomial time recognition algorithms for each class.

N -triangulatable graphs have properties similar to chordal graphs, and we characterize them using graph separators and using a vertex elimination ordering. Cycle-bicolorable graphs are shown to be perfect, and any cycle-bicoloring of a graph renders it N -triangulatable.

The corresponding recognition complexity for chordal probe graphs, given a partition of the vertices into probes and non-probes, is $O(|P||N|)$, thus also providing an interesting tractible subcase of the chordal graph sandwich problem. If no partition is given in advance, the complexity of our recognition algorithm is $O(|V|^2 |E|)$.

Keywords: chordal graph, cycle-bicolorable graphs, probe graphs, graph separators, minimal triangulation, chordal sandwich problem

66 The competition-acquisition number of a graph

Peter J. Slater and Yan Wang, University of Alabama in Huntsville

Competitive optimization parameters in graphs are those associated with the formation of a set S of elements of a graph G where the set S is formed by two players alternately selecting elements of S . The players are trying to maximize, respectively, minimize, the order of S . Cases included are those where S must be independent, enclaveless, a packing, or an acquisition set.

Keywords: acquisition, optimization, graphical game theory

67 $(v, k, 2)$ -Covering Designs

M. Greig, P.C. Li and G.H.J. van Rees, University of Manitoba

A (v, k, t) -covering design is a set of k -subsets of a set of v elements such that every set of t elements occurs in at least one block. The minimum number of blocks in any such design is denoted by $C(v, k, t)$.

We find all v and k so that $C(v, k, 2) = 13$ and give all the details and proofs. Further, we determine that $C(28, 9, 2) = C(41, 13, 2) = 14$.

Keywords: covering design, minimum subsets, pairs

68 On the Edge-graceful spectra of the cycles with one chord and dumbbell graphs

Sin-Min Lee, Kuo-Jye Chen and Jack Yung-Chin Wang, San Jose State University

Let G be a (p, q) -graph and $k > 0$. The graph G is said to be k -edge graceful if the edges can be labeled by $k, k+1, \dots, k+q-1$ so that the vertex sums are distinct, mod p . We denote the set of all k such that G is k -edge graceful by $egl(G)$. The set is called the *edge-graceful spectrum* of G . In this paper the problem of what sets of natural numbers are the edge-graceful spectra of two types of $(p, p+1)$ -graphs is studied.

Keywords: Composition, Palindromic compositions, Carlitz compositions, partitions, generating functions

69 Bipartite Probe Interval Graphs

David E. Brown and J. Richard Lundgren, University of Colorado at Denver

A graph is a probe interval graph (PIG) if its vertices can be partitioned into probes and nonprobes with an interval assigned to each vertex so that vertices are adjacent if and only if their corresponding intervals overlap and at least one of them is a probe. Probe interval graphs are used as a generalization of interval graphs in physical mapping of DNA. PIGs have been characterized by forbidden subgraphs in the cycle-free case by Sheng, but finding forbidden subgraph characterizations in more general cases has proved to be very difficult. Here we consider the case where the PIGs are bipartite. First, we consider the case where for a bipartite graph $G = (X, Y, E)$, the probes and nonprobes correspond to the bipartition. We show that this happens exactly when G is an interval point bigraph and when the reduced adjacency matrix for G has the consecutive ones property for either rows or columns. In addition, we show that the complements of these graphs are two-clique circular arc graphs with a particular structure. We also give a long list of forbidden subgraphs, which may or may not be complete. For the general case of bipartite PIGs we give another list of forbidden subgraphs which also may not be complete.

Keywords: Probe interval graph, bipartite, circular arc graph, consecutive ones property

70 A Miscellany of Chessboard Problems: An Update

John J. Watkins, Colorado College

Last year I gave a talk on chessboard problems and the way in which standard old problems about knight's tours, domination, and independence can be taken into new and somewhat unfamiliar territory. In this talk I will give an update and describe several specific results in some detail. For example, we now know that any $a \times b \times c$ box has a knight's tour, a result which generalizes a tour of the surface of an $8 \times 8 \times 8$ cube originally given as a puzzle by H. E. Dudeney. Ian Stewart gave a 3-dimensional tour of an $8 \times 8 \times 8$ cube by touring one layer of the cube at a time and then splicing these individual tours together into a single tour. We propose doing a 3-dimensional tour of the cube using a new move for the knight, one that is itself genuinely 3-dimensional. Finally, it is relatively easy to see that it takes n bishops to dominate an $n \times n$ chessboard. I will discuss what happens to bishops domination when the chessboard is placed on a Klein bottle.

71 Classification of Partitions of All Triples on Ten Points into Copies of Fano and Affine Planes

Rudolf Mathon, University of Toronto; Anne Penfold Street, The University of Queensland; and Greg Gamble, Curtin University of Technology

We continue our study of partitions of the full set of $\mathcal{D}$ triples chosen from a v -set into copies of the Fano plane $PG(2, 2)$ (Fano partitions) or copies of the affine plane $AG(2, 3)$ (affine partitions) or into copies of both of these planes (mixed partitions). The smallest cases for which such partitions can occur are $v = 8$ where Fano partitions exist, $v = 9$ where affine partitions exist, and $v = 10$ where both affine and mixed partitions exist. The Fano partitions for $v = 8$ and the affine partitions for $v = 9$ and $v = 10$ have been fully classified, into eleven, two and 77 isomorphism classes respectively. Here we classify:

- (1) the sets of i pairwise disjoint affine planes for $i = 1, \dots, 7$; and
- (2) the mixed partitions for $v = 10$ into their 22 isomorphism classes.

We also consider how these partitions relate to the large sets of $AG(2, 3)$.

Keywords: partitioning sets of blocks into designs, Fano and affine planes

72 On Super Vertex-graceful Trees

Sin-Min Lee and Elo Leung, San Jose State University

Lee introduced the concept of super vertex-graceful graphs. For the graph G with vertex set $V(G)$ and edge set $E(G)$ with $p = |V(G)|$ and $q = |E(G)|$, G is said to be super vertex-graceful (in short SVG), if there exists a function pair (f, j^+) which assigns integer labels to the vertices and edges; that is, $f: V(G) \rightarrow P$, and $j^+: E(G) \rightarrow Q$, such that f is onto P and j^+ is onto Q , and $j^+(u, v) = f(u) + f(v)$ where $(u, v) \in E(G)$.

$$Q = \begin{cases} \{\pm 1, \dots, \pm n\} & \text{if } q \text{ is even} \\ \{0, \pm 1, \dots, \pm n\} & \text{if } q \text{ is odd} \end{cases}$$

$$P = \begin{cases} \{\pm 1, \dots, \pm n\} & \text{if } p \text{ is even} \\ \{0, \pm 1, \dots, \pm n\} & \text{if } p \text{ is odd} \end{cases}$$

We determine here trees of diameter at most 5 which are super vertex-graceful graph. Moreover, a conjecture is proposed.

73 The Stars-Clustering-Optimization-Problem

Ephraim Korach, Ben-Gurion University of the Negev; and Michal Stern, University of Haifa

We consider the following problem: Let (V, S) be a given hypergraph where V is a ground set and S is a simple collection of subsets of V . Let $G = (V, E)$ be a complete graph with a cost on every edge. Find in G a spanning tree T such that each subset of S induces a star in T . Under these conditions T has a minimum possible cost. We call this problem the *stars-clustering-optimization-problem* and denote it by *SCOP*. The complexity of this general case is still open and we consider here some restricted cases. For the case where each subset induces a complete star minus two we prove that the problem is NP-hard. Next we consider the case where the intersection graph of the subsets is connected and each subset induces a complete star in such a way that no two subsets have the same center. We call this case the *complete-stars-clustering-optimization-problem* and denote it by *CSCOP*. For this case we present a structure theorem that describes all feasible solutions. Based on the structure theorem we present a polynomial algorithm for finding an optimal solution. We consider another version of the *CSCOP* problem where the objective is to find in G a tree T such that each subset of the collection induces a complete star in T and the sum of all costs of the complete stars is minimum. We call this problem the *complete-stars-clustering-median-optimization-problem* and denote it by *CSCMOP*. We show that a tree is optimal for *CSCOP* if and only if it is optimal for *CSCMOP*. Motivation for these problems comes from communication where we would like to construct a minimum cost communication tree network for a collection of non-disjoint groups of customers. First we require that each group induces a star so that all communications between group members will be within that star - we call this property the "group privacy". Second we require that each group is not affected by failures of communication outside of the group - we call this property the "group fault tolerance".

Keywords: combinatorial optimization, stars, clustering spanning trees, hypergraphs, polynomial graph algorithms, NP-hardness

74 Directed Tree Satisfying a Gallai-type Domination Theorem

Jason Albertson, Audene Harris, Larry Langley, and Sarah Merz, University of the Pacific

The lower domination number of a digraph D , denoted by $\gamma_-(D)$, is the least number of vertices in a set S , such that $O[S] = V(D)$. A Gallai-type theorem has the form $\alpha(D) + \gamma(D) = n$, where α and γ are parameters defined on digraph D and n is the number of vertices in the digraph. We characterize directed trees satisfying $\gamma_-(D) + \delta_+(D) = n$, where $\delta_+(D)$ is the maximum outdegree of a vertex in the directed graph.

Keywords: domination, directed tree, digraph, Gallai

75 The intersection problem for bipartite star designs

Kathryn L. DeLamar, LaGrange College and D.G. Hoffman, Auburn University

We determine those integers a, b, m and k for which there are two m -star designs on $K_{a,b}$ having exactly k stars in common.

76 On The Integer -Magic Spectra of the Graphs that are Two-vertex sum of Trees

Sin-Min Lee, San Jose State University; and Farrokh Saba, University of Nevada

For $k > 0$, we call a graph $G = (V, E)$ as Z_k -magic if there exists a labeling $l: E(G) \rightarrow \mathbb{Z}_k$ such that the induced vertex set labeling $l^+: V(G) \rightarrow \mathbb{Z}_k$

$$l^+(v) = L(l(u, v) : (u, v) \in E(G))$$

is a constant map. We denote the set of all k such that G is k -magic by $IM(G)$. We call this set as the *integer-magic spectrum* of G . We investigate the integer-magic spectra for graphs that are two-vertex sum of trees.

77 Interval Tournaments

David E. Brown• and J. Richard Lundgren, University of Colorado at Denver

A directed graph is an *interval digraph* (ID) if each vertex v corresponds to an ordered pair of intervals (S_v, T_v) such that $u \rightarrow v$ if and only if $S_u \cap T_v \neq \emptyset$. Das et. al. introduced interval digraphs as an analogue of interval graphs, and gave various characterizations. A *tournament* is a complete directed graph. We explore tournaments that are interval digraphs and call them *interval tournaments* (ITs). A bipartite graph is an *interval bigraph* (IBG) if it is the intersection graph of two families of intervals with vertices adjacent if and only if their corresponding intervals intersect and each interval belongs to a distinct class. Recently, IBGs have been characterized in various ways by Hell, Huang, Brown, Lundgren, and Flink and we use these results together with the prior results for IDs, (and the fact that the models are equivalent) to describe the structure of ITs. In particular, we explore which ITs are arc-traceable, the nature of the vertex ranking given by the interval representation, and which upset tournaments are ITs. Note: this abstract is preliminary, actual contents of the talk may vary.

Keywords: Interval digraph, interval bigraph, tournament, upset tournament

78 Domination in the Graph of the T_3 Association Scheme and Related Chessboard Graphs

Charles Wallis-, Western Carolina University; and Renu Laskar, Clemson University

The T_3 association scheme, also known as the tetrahedral association scheme, consists of unordered triples of distinct elements on n symbols, two triples being called "first associates" if they share exactly two symbols in common, and "second associates" if they share exactly one symbol in common. The graph of the T_3 association scheme, in which the vertices correspond to unordered triples and the edges correspond to first associates, can be visualized as a portion of a three-dimensional chessboard. Domination parameters of these chessboard graphs will be discussed, and several open problems will be mentioned.

Keywords: domination, independence, association scheme, chessboard graph

79 Outline and amalgamated transitive triple systems: The case when two vertices are amalgamated

Michael N. Ferencak", University of Pittsburgh at Johnstown; and Anthony J.W. Hilton, Reading University

Define $\triangleright D_n$ to be the directed multigraph on n vertices where each distinct pair of vertices is joined by $\triangleright$ edges in each direction. A transitive triple system of order n and index $\triangleright$, denoted by $TT(n, \triangleright)$, is thought of as a decomposition of $\triangleright D_n$ into edge disjoint acyclic directed copies of K_3 (directed triangles).

Very informally, an amalgamated transitive triple system H of order n and index $\triangleright$, or an $ATT(n, \triangleright)$, is derived from a $TT(n, \triangleright)$ G by identifying various of the vertices (but without identifying any directed edges) of G so that a vertex v in $V(G)$ becomes $\mathbb{Z}/\ell$ (say) in $V(H)$, and assigning the edges to vertices in $V(H)$ so that an edge incident with a vertex $v \in V(G)$ corresponds to an edge in H incident with $\mathbb{Z}/\ell(v)$. Edges between amalgamated vertices become loops on the new vertex. Orientations of edges are not changed. The decomposition of G yields a decomposition of H into directed multigraphs, each an amalgamation of an acyclic directed K_3 .

An outline transitive triple system F of order n and index $\triangleright$ can be thought of as a multigraph on $k \leq n$ vertices that possess the characteristics sufficient for it to have been the amalgamation of some complete transitive triple system of order n and index $\triangleright$.

A definition of an outline transitive triple system will be given in the special case when $\triangleright = 1$ and precisely two vertices has been amalgamated and a recent result showing that every outline transitive triple system in this special case is an amalgamated transitive triple system of the same order and index. A definition of an outline transitive triple system will be given in the special case when $\triangleright = 1$ and precisely two vertices has been amalgamated and a recent result showing that every outline transitive triple system in this special case is an amalgamated transitive triple system of the same order and index.

80 On Super Edge-graceful $(p, p+1)$ -Graphs

Sin-Min Lee, Edward Chen, Emmanuel R. Yera• and Ling Wang, San Jose State University

Let G be a (p, q) -graph in which the edges are labeled $1, 2, 3, \dots, q$ so that the vertex sums are distinct, mod p , then G is called edge-graceful. J. Mitchem and A. Simoson introduced the concept of super edge-graceful graphs which is a stronger concept than edge-graceful for some classes of graphs. We show here some $(p, p+1)$ -graphs are super edge-graceful. but not edge-graceful and some are edge-graceful but not super edge-graceful. Moreover, some conjectures are proposed.

81 Hamiltonian Colorings of Graphs

Gary Chartrand, Ping Zhang, Western Michigan University; and Ladislav Nebesky, Charles University

For vertices u and v in a connected graph G of order n , the length of a longest $u - v$ path in G is denoted by $D(u, v)$. A hamiltonian coloring c of G is an assignment c of colors (positive integers) to the vertices of G such that $D(u, v) + |c(u) - c(v)| \leq n - 1$ for every two distinct vertices u and v of G . We present some results on hamiltonian colorings.

Keywords: distance, coloring, longest paths

82 University examination timetabling problem

E. Cheng, R. Kleinberg, S. Kruk, W. Lindsey and D. Steffy, Oakland University

Our problem is: Given a sets of rooms, exams, students and timeslots, we must assign each exam to a room and timeslot. Each room has certain features, such as class size, and media equipment. Each exam requires certain features, and each student has a set list of exams they are required to take. From this information we must generate a timetable that satisfies all hard constraints, and satisfies as many soft constraints as possible. Hard constraints include such obvious restrictions as no student can take multiple exams at once; soft constraints include attempting to avoid giving students multiple successive exams and minimizing the number of exams in the last timeslot of the day. This problem is NP-hard. We solve this problem in stages using stable sets, weighted bipartite matching, paths in hypergraphs and max flow. In this talk, we will discuss our methods, implementation and results.

Keywords: Timetabling, Scheduling, Feasibility

83 Compression of Vertex Transitive Graphs

Aurel Cami, Narsingh Deo, University of Central Florida; and Bruce Litow, James Cook University

We consider the lossless compression of vertex transitive graphs. A graph $G = (V, E)$ is called vertex transitive if for every pair of vertices (i, j) , there exists an automorphism u of G such that $u(i) = j$. Vertex transitivity is a natural condition required of interconnection networks, both for computing and for data communications because the network 'looks the same' from every vertex. Given a vertex transitive graph G , we propose as the compressed form of G a finite presentation (X, R) . This approach is based on a theorem by G. Sabidussi, which states that for every vertex transitive graph G there exists a positive integer m , such that the graph mG is a Cayley graph. Employing a conjecture, we show that for a large subfamily of vertex transitive graphs, the original graph G can be completely reconstructed from its compressed representation (X, R) . At this early stage we are mainly concerned with how much compression is possible for vertex transitive graphs. Although we do not focus on the computational complexity of the proposed compression method, we show that all the steps involved can be easily computed.

Keywords: Graph compression, vertex transitive graph, finite presentation

84 Generalizations of Graham's Pebbling Conjecture

David S. Herscovici, Quinnipiac University

We investigate generalizations of pebbling numbers and of Graham's pebbling conjecture that $J(G \times H) \leq f(G)J(H)$, where $f(G)$ is the pebbling number of the graph G . We show that certain conjectures imply others that initially appear stronger. We also show that the most general versions of Graham's conjecture are true in the variant of optimal pebbling.

Keywords: Pebbling, optimal pebbling, Graham's conjecture

Wednesday, March 10, 2004

8:00am	Registration - Grand Palm Room Available until 5:00pm			
	<i>Live Oak Pavillion</i>			
	<i>Room A</i>	<i>RoomB</i>	<i>Roome</i>	<i>RoomD</i>
8:20am	085:	086:	087:	088:
8:40am	089: D. West	090: D. Leach	091: V. Voloshin	092: D. Narayan
9:00am	093: S. Flink	094: M. Xie	095: B. Montagah	096: L. Fisher Macon
9:20am	097: K. Bogart	098: R. Brigham	099: E. Salehi	100: D. Boutin
9:40am	101: B. Balof	102: J. Siagiova	103: P. Johnson	104: L. Langley
10:00am	The Institute for Combinatorics and its Applications			
10:40am	Coffee			
11:00am	Invited Speaker: Wlf - Grand Palm Room			
12:00pm	Conference Photo			
12:15pm	Lunch (on your own)			
2:00pm	Invited Speaker: Brualdi - Grand Palm Room			
3:00pm	Coffee			
3:20pm	105: K. Smith	106: R. Eggleton	107: V. Saenpholphat	108: M. Emamy-K.
3:40pm	109: P. Lam	110: J. Anderson	111: B. Wei	112: L. Uribe
4:00pm	113: A. Busch	114: J. Fredrickson	115: K. Markstrom	116: M Tsuchiya
4:20pm	117: J. Laison	118: D. Moazzami	119: M. Walsh	120: J. Pfalz
4:40pm	121: J. Kratochvil	122: M Axenovich	123: I. Schiermeyer	124: R. Grimaldi
5:00pm	125: J. Kratochvil	126: D. Ferrero	127: M El-Hashash	128: P. Chinn
5:20pm	129:	130:	131:	132:

Coffee and refreshments will be served in the registration area of the Grand Palm Room.

89 Total Interval Number: A Survey

Douglas B. West, University of Illinois

A *multi-interval representation* of a graph G assigns each vertex a union of intervals on the real line so that vertices are adjacent if and only if the corresponding sets intersect. The *interval number* $i(G)$ of G is the minimum t such that G has a multi-interval representation in which each vertex is assigned a union of at most t intervals. Instead of minimizing the maximum number of intervals assigned to a vertex, we may want to minimize the average number. The *total interval number* $I(G)$ is the minimum t such that G has a multi-interval representation in which $\sum_{v \in V(G)} |I(v)| \leq t|V(G)|$ where v is assigned a union of $|I(v)|$ intervals. This talk will survey known results about the total interval number of graphs, including aspects of complexity and bounds in terms of other parameters. Many of these results are old but not yet published. One aim is to stimulate renewed interest in settling questions that remain open about the parameter.

90 Hamilton Decompositions of $K_{n,m}$ with n odd and a 2-factor Removed

Darryn Bryant, University of Queensland; David Leach, State University of West Georgia; and Chris Rodger, Auburn University

We show that for any 2-factor U of $K_{n,m}$ with n odd, there exists a hamilton decomposition of $K_{n,m} - E(U)$ with a 1-factor leave. This result completes the set of necessary and sufficient conditions for the existence of a hamilton decomposition of any complete multipartite graph with the edges of any 2-factor removed.

Keywords: hamilton cycle, hamilton decomposition, bipartite graph

91 Coloring Mixed Hypergraphs: a survey

V. Voloshin, Troy State University

Mixed hypergraph is a triple $H = (X, C, D)$ with vertex set X and two families of subsets, C and D , called C -edges and D -edges respectively. Proper k -coloring of H is a mapping from X into a set of k colors in such a way that every C -edge has two vertices of the same color and every D -edge has two vertices of different colors. Mixed hypergraph is called colorable if it admits at least one proper coloring and uncolorable otherwise. Maximum and minimum number of colors over all proper colorings which use all k colors is called the upper and lower chromatic numbers respectively. Mixed hypergraph has a continuous chromatic spectrum if proper colorings exist using all numbers of colors between the lower and upper chromatic numbers; otherwise it has a broken chromatic spectrum. Mixed hypergraph is called planar if it can be embedded in the plane in such a way that edges intersect only at the respective neighborhoods of common vertices. Mixed hypergraph is called perfect if, in any induced subhypergraph, the upper chromatic number coincides with the maximum independent set of vertices with respect to C -edges. Many further details may be found at Mixed Hypergraph Coloring Web Site <http://math.net.md/voloshin/mh.html>.

We survey some results and open problems on the chromatic spectrum, planar and perfect mixed hypergraphs.

Keywords: graph coloring, mixed hypergraphs, colorability, chromatic numbers, chromatic spectrum, planar graphs and hypergraphs, perfect graphs and hypergraphs

92 Minimal Rankings of Cycles

Victor Kostyuk and Darren A. Narayan, Rochester Institute of Technology

Given a graph G , a function $f: V(G) \rightarrow \{1, 2, \dots, k\}$ is a k -ranking of G if $J(u) = j(v)$ implies every $u-v$ path contains a vertex w such that $f(w) > J(u)$. A k -ranking is *minimal* if reducing any label larger than 1 violates the described ranking property. The *a-rank number* of G , denoted $\alpha_k(G)$ equals the largest k such that G has a minimal k -ranking. We consider the case when G is the cycle on n vertices C_n , and give necessary conditions for a given ranking to be minimal. We also present a monotonicity property for cycles, showing that $\alpha_k(C_n) \leq \alpha_k(C_m)$ for $n \leq m$. Finally, we investigate methods for calculating the *a-rank number* of a cycle.

Keywords: Vertex labeling, minimal ranking, ordered coloring problem

93 Interval homomorphisms, interval k-graphs and related partial orders

Stephen C. Flink, University of Colorado at Denver

Interval k-graphs (IKGs) are a natural extension of both interval bigraphs and probe interval graphs. I will describe containment relations between interval k-graphs and these other classes in terms of interval homomorphisms, and show some general properties of interval homomorphisms.

The relationship between interval k-graphs and cocomparability graphs (graphs whose complements admit a transitive orientation) is explored, and a partial characterization is given in terms of the interval representation. In particular, the partial orders of width < 4 whose cocomparability graphs are IKGs are completely described. We compare this with the problem of determining which cocomparability graphs are probe interval graphs. In addition, a representation of cocomparability IKGs as pure intersection graphs is outlined.

Keywords: Probe interval graph, interval bigraph, interval k-graph, tube order, graph homomorphism

94 The radio numbers for square paths and square cycles

Daphne Liu and Melanie Xie, California State University

For a connected graph G , let $dc(u, v)$ denotes the distance between vertices u and v , and $diam(G)$ denotes the diameter of G . A radio labelling (as motivated by the Radio Channel Assignment Problem) for G is a function J that assigns to each vertex a non-negative integer such that $|J(u) - J(v)| \geq diam(G) - dc(u, v) + 1$ holds for any $u, v \in V(G)$. The span of f is the maximum label assigned to a vertex of G . The radio number of G is defined as the minimum span over all radio labellings of G . The radio number for paths and cycles were studied by Chartrand et al., and were determined by Liu and Zhu recently.

In this talk, we extend the results to the square of paths and the square of cycles. The radio numbers for the square of paths and for the square of even cycles are determined. For the radio number of the square of odd cycles, we obtain a lower bound, and show that the bound is achieved by some cases.

Keywords: radio labelling, radio number

95 Ramsey-like theorems for balanced colourings

Jonathan Cutler, University of Memphis; and Balazs Montag, University of Memphis

We investigate some Ramsey-like problems under the additional condition that the edge-colouring of the underlying graph K_n is balanced, but the required colouring on some copy of K_k can be other than monochromatic. For example, we estimate the maximal value of n for which K_n is 2-colourable such that each colour appears on at least $1/2 - c/\log n$ edges, and there is no copy of K_k in which the edges of any colour form exactly one $K_{k/2}$ or two vertex-disjoint $K_{k/2}$. The gap between our lower and upper bound is essentially the same as it is between the current bounds for the classical, diagonal Ramsey-numbers.

Keywords: Generalized Ramsey theory

96 GraphML: An XML Representation for Graphs Rendered in SVG Using the W3C DOM

Lisa Fischer Macon, University of Central Florida

We present GraphML, an XML representation for graphs along with a component XML Schema. The Document Object Model (DOM) standardized by the World Wide Web Consortium (W3C) is used in conjunction with JavaScript to translate graphs into Scalable Vector Graphics (SVG), which can be rendered in a Web Browser. All implementations comply with W3C Recommendations and Standards.

Keywords: XML, SVG, DOM, graph rendering

97 Triangle Graphs and Triangle Orders

Ken Bogart^{*} and Stephen Ryan, Dartmouth College

An *interval graph* is a graph isomorphic to the intersection graph of a family of intervals on the real line. An interval graph is *proper* if it is isomorphic to an intersection graph of a family of intervals no one of which is a proper subset of any other, and is *unit* if it is the intersection graph of a family of intervals of unit length. Roberts showed that proper and unit interval graphs are the same. A *trapezoid graph* is a graph isomorphic to the intersection graph of a family of trapezoids whose bases are on two parallel lines. A trapezoid graph is unit if it is the intersection graph of a family of trapezoids of unit area, and proper if it is the intersection graph of a family of trapezoids no one of which is a proper subset of any other. Bogart, Mohring, and Ryan showed proper and unit trapezoid graphs aren't the same. A triangle graph is one isomorphic to the intersection graph of a family of triangles which have a base and the opposite vertex on two parallel lines. (not all bases need be on the same line). All triangle graphs are proper. The non-unit proper trapezoid graph is a non-unit triangle graph. In this talk I will summarize what we do know and what it would be nice to know about triangle graphs and the closely related triangle orders, and introduce free triangle graphs and orders. In the next talk, Barry Balof will show free triangle graphs aren't all unit (though they are all proper).

Keywords: Interval Graph, Interval Order, Trapezoid Graph, Trapezoid Order, Triangle Graph, Triangle Order, Unit, Proper, Comparability Invariant

98 A Sharp Lower Bound on the Powerful Alliance Number of $C_m \times C_n$

Robert C. Brigham^{*}, Ronald D. Dutton, University of Central Florida; and Steven T. Hedetniemi, Clemson University

Let $G = (V, E)$ be a graph and S be a nonempty subset of V . The *boundary* of S is $\partial S = N[S] - S$ where $N[S]$ is the closed neighborhood of S . $S \subseteq V$ is a *defensive alliance* if, for every $v \in S$, $|N[v] \cap S| \geq |N[v] - S|$, an *offensive alliance* if, for every $v \in \partial S$, $|N[v] \cap S| > |N[v] - S|$, and a *powerful alliance* if $|N[v] \cap S| \geq |N[v] - S|$ for all $v \in N[S]$. Thus S is a powerful alliance if it is both a defensive and an offensive alliance. The size of a smallest powerful alliance of G is denoted $a_p(G)$. We show $a_p(C_m \times C_n) \geq \frac{1}{2}|V|$ and this bound is sharp.

Keywords: grid graphs, alliances

99 On IC-Coloring and IC-indices of Graphs

Ebrahim Salehi^{*}, University of Nevada Las Vegas; Sin-Min Lee, San Jose State University; and Mahdad Khatirinejad, Simon Fraser University

Given a coloring $f: V(G) \rightarrow \mathbb{N}$ of graph G and any subgraph $H \subseteq G$ we define $f_s(H) = \sum_{v \in V(H)} f(v)$. In particular, we denote $J_{\leq k}(G)$ by $S(f)$. The coloring f is called an IC-coloring if for any integer $k \in [1, S(f)]$ there is a connected subgraph $H \subseteq G$ such that $f_s(H) = k$. Also, we define the maximum index of G to be

$$M(G) = \max\{S(f) : f \text{ is an IC-coloring of } G\}.$$

In this paper we will examine some well-known classes of graphs and will determine their maximum indices.

Keywords: IC-coloring, IC-indices

100 Graphs Embedded with all Symmetries Displayed

Debra Boutin, Hamilton College

It's natural to want graphs drawn with all their symmetries displayed. This paper proves that every graph has an embedding in Euclidean space in which the isometry group of the embedded vertices induces precisely the automorphism group of the graph. Such an embedding is called isometric. The paper also considers the minimum dimension necessary for an isometric embedding and looks at which graphs have such an embedding in the plane.

101 Unit and Non-Unit Free Triangle Graphs

Barry Balor, Whitman College; and Ken Bogart, Dartmouth College

This talk will expand on the idea of a free triangle graph (introduced in Ken Bogarts talk). A unit free triangle graph is one which has a representation by unit triangles. All triangles lie between two parallel baselines, with each triangle intersecting each baseline in one vertex. We will look at two examples of non-unit free triangle graphs (a free triangle graph for which no unit free triangle representation exists), one which is finite, and one which is infinite with the interesting property that no finite subgraph is non-unit. We will also discuss our progress to date in proving that being a unit free triangle order is a comparability invariant.

Keywords: Free Triangle Graph, Free Triangle Order, Unit, Proper, Comparability Invariant

102 Covalences of infinite planar vertex-transitive and Cayley maps

Jana Siagiova and Mark E. Watkins, Syracuse University

Planar tessellations of various levels of symmetry have received considerable attention in the literature. In this talk we will focus on infinite planar maps with the same cyclic sequence of covalences (i.e., face lengths) around each vertex.

We will present a characterization of covalence sequences of infinite, planar, 1-ended, 3-connected vertex-transitive maps and Cayley maps. One of the consequences is that for each such sequence the number of its "realizations" by vertex-transitive maps or Cayley maps is finite. Covalence sequences containing exactly 2 different entries will be discussed in some detail. We conclude by relating our results to maps on arbitrary orientable surfaces.

Keywords: Graph, map, covalence, planar, vertex-transitive, Cayley, surface, covering

103 The Hall ratio, the fractional chromatic number, and the lexicographic product

Peter Johnson, Auburn University

It is elementary that the chromatic number of a graph is at least as large as the order of the graph divided by the vertex independence number of the graph-and therefore the chromatic number is at least as large as the maximum of those ratios taken over all subgraphs of the graph. This maximum is called the Hall ratio of the graph, denoted $HR(G)$ in this abstract.

It has been noted for a number of years that $\chi(G)/HR(G)$ can be arbitrarily large (look at the Kneser graphs); but HR is also a lower bound of frx , the fractional chromatic number, and the search for graphs G such that $frx(G)/HR(G)$ is large has been none too successful. Until recently, in the presenter's experience, the largest of these ratios known was $6/5$. The lexicographic product is a promising tool in the search, because $frx(G \text{ lex } H) = frx(G)Jrx(H)$ -if only the Hall ratio were similarly multiplicative! Well, it's not, but a theorem has been stumbled upon that says that it is sometimes (exactly the wrong times for the purpose of making frx/HR large): if $HR(G) = frx(G)$, then $HR(G \text{ lex } H) = HR(G)HR(H)$ for all H .

Keywords: Hall ratio, fractional chromatic number, vertex independence number, lexicographic product

104 Alliances in Directed Graphs

Larry J. Langley, University of the Pacific

Alliances in undirected graphs were introduced by Hedetniemi, Hedetniemi, and Kristiansen. We generalize these definitions of alliances to directed graphs. We examine examples of alliances on classes of directed graphs and consider bounds for the size of alliances and strong alliances.

Keywords: alliance, offensive alliance, defensive alliance, strong alliance, directed graph

105 On Graph Summability

Ken Smith, Sivaram Narayan, Josh Whitney, Raj Doshi, and Janae Eustice, Central Michigan University

Label the vertices of a graph with positive integers. Assign to any induced subgraph the sum of the labels of the vertices of the subgraph and require that the labels of all connected induced subgraphs cover the interval $[1 \dots n]$. Given a graph G , how large can we make n ? How close is n to the total number of connected induced subgraphs?

This graph labeling problem was introduced by Stephen Penrice and appeared in a column of Doug West. Students in summer undergraduate research programs at Central Michigan University attacked this problem. We will give some preliminary results.

Keywords: vertex labeling, graph summability, induced subgraphs, orderings, probe interval graphs

106 The structure of the graph posets of orders 4 to 8

Roger B. Eggleton, Illinois State University; Peter Adams, University of Queensland; and James A. MacDougall, University of Newcastle

The simple graphs of small order are the fundamental objects of graph theory, so it is important to have their structural relationships explicitly determined and available for reference. The poset $G(n)$ comprises the unlabeled simple graphs of order n , with partial ordering $G \leq H$ whenever G is a spanning subgraph of H . We define a modified Steinbach numbering of the graphs in $G(n)$, apply this to each $G(n)$ with $n \leq 8$, and use it to tabulate the Hasse diagram structure of each $G(n)$ with $4 \leq n \leq 8$, along with key aspects of the independence structure of these posets. In particular, the Hasse diagram of $G(8)$ is a directed graph of order 12,346 and size 125,066; the poset $G(8)$ has 51,952,895 independent pairs and 96,775,426,396 independent triples. We present descriptive data for each $G(n)$ with $4 \leq n \leq 8$, the data for order 8 being the main contribution of this work.

Keywords: simple graphs, graph poset, Hasse diagram, independence structure

107 On Resolvability in Graphs

Varaporn Saenpholphat, Srinakharinwirot University; and Ping Zhang, Western Michigan University

For an ordered set $W = \{w_1, w_2, \dots, w_k\}$ of vertices and a vertex v in a connected graph G , the code of v with respect to W is the k -vector

$$cw(v) = (d(v, w_1), (v, w_2), \dots, d(v, w_k)),$$

where $d(x, y)$ represents the distance between the vertices x and y . The set W is a resolving set for G if distinct vertices of G have distinct codes with respect to W . A resolving set containing a minimum number of vertices is a basis for G . The dimension $\dim(G)$ is the number of vertices in a basis for G . We present some results in this area.

Keywords: distance, resolving set

108 Projective Functionals and a Geometric Connection to Threshold Logic

M.R. Emamy-K., University of Puerto Rico

To present an introduction to the theory of convex polytopes, a new approach and methodology will be introduced. This new approach is used to construct an interaction between polytopes and threshold logic. We shall show that in this presentation, projective functionals are the desired objects and play a significant role in the geometric connection to threshold logic.

The described approach contributes to the proofs of many basic facts of convex polytopes. For instance, we present a new proof for the fact that every compact convex set in the Euclidean space has a supporting hyperplane passing through a given relative boundary point.

109 On Distance Two Labelling Characterization of Chordal Graphs

Peter Che Bor Lam and Guohua Gu, Hong Kong Baptist University

An $L(2, 1)$ -labelling of a graph G is an assignment of nonnegative integers to the vertices of G such that vertices at distance at most two get different numbers and adjacent vertices get numbers which are at least two apart. The $L(2, 1)$ -labelling number of G , denoted by $\lambda(G)$, is the minimum range of labels over all such labellings. In this paper, we completely characterize the unit interval graphs G of order $2\chi(G) + 1$ with $\lambda(G) = 2\chi(G) - 2$ or with $\lambda(G) = 2\chi(G) - 1$, where $\chi(G)$ is the chromatic number of G . Finally, we discuss some structures of unit interval graphs G with more than $2\chi(G) + 1$ vertices and $\lambda(G) = 2\chi(G) - 2$ or $2\chi(G)$.

Keywords: $L(2, 1)$ -labelling, chordal graph, unit interval graph

110 An extremal problem for contractible edges in 3-connected graphs

Joe Anderson and Haidong Wu, University of Mississippi

Let G be a simple 3-connected graph with at least five vertices. Tutte showed that G has at least one contractible edge. Thomassen gave a simple proof of this fact and showed that contractible edges have many applications. Contractible edges have been studied by many authors. Recently Wu proved that a simple 3-connected graph G with at least five vertices has at most $|V(G)|$ vertices that are not incident to contractible edges. In this paper, we characterize all simple 3-connected graphs with exactly $|V(G)|$ vertices that are not incident to contractible edges.

Keywords: contractible edges, 3-connected graphs

111 Every Connected, Locally Connected, Claw-Free Graph is Path Extendable

Bing Wei, University of Mississippi

A graph G is locally connected if for any vertex x of G the subgraph induced by $N(x)$ is connected. A graph G is said to be path extendable, if for each pair of vertices x, y and for each nonhamiltonian (x, y) -path P in G , there is an (x, y) -path P' in G such that $V(P) \cap V(P') = \{x, y\}$ and $|P'| = |P| + 1$. In this talk, we will present a result which implies that every connected, locally connected, claw-free graph is path extendable. This is a joint work with Y. Sheng, F. Tian, J. Wang and Y. Zhu.

Keywords: Claw-free graph, locally connected, path extendable

112 An Improvement on a Computer-Free Proof for the Cut Number of the 5-cube

L. Uribe and M.R. Emamy-K., University of Puerto Rico

The cut number $S(d)$ of the d -cube is the minimum number of the hyperplanes in $\mathbb{R}^d$ that slice, i. e. cutting the cube and avoiding vertices, all the edges of the d -cube. The cut number problem for the hypercube of dimensions $d = 4$ was posed by P. O. Neil more than thirty years ago. The identity $S(3) = 3$ is easy and that of $S(4) = 4$ is a well-known result obtained by the second author in 1988. Recently, Sohler and Ziegler have obtained a computational solution to the 5-cube problem. To find a short and computer-free proof for the 5-cube will remain a challenging open problem. On this theoretical approach, we improve the recent result of the authors and I. C. Tomasini.

113 A characterization of triangle-free tolerance graphs

Arthur H. Busch, University of Colorado at Denver

In the cycle-free case, the class of tolerance graphs coincides with the class of cycle-free interval bigraphs. We show that in general, the class of interval bigraphs contains tolerance graphs that are triangle-free (and hence bipartite). By extending this result, we obtain a characterization of triangle-free tolerance graphs. We also give separating examples to show that this containment relationship is proper.

Keywords: tolerance graphs, interval bigraphs, bipartite graphs, consecutive orderings, probe interval graphs

114 A Time Saving Region Restriction for Calculating the Minimum Crossing Number of K_n

Judith R. Fredrickson, Bei Yuan, and Frederick C. Harris, Jr., University of Nevada

In 1995 Harris and Harris presented an algorithm for calculating the minimum crossing number of a graph. This was implemented in parallel by Tadjiev and Harris in 1997. Since the algorithm presented builds huge search trees, the need to restrict our search space is very important as graph sizes grow. This paper presents a technique for search space reduction for an algorithm calculating the minimum crossing number of a complete graph. We introduce a restriction on the placement of a new edge in a "good" graph construction. Graph region examination plays a role in the restriction process. Preliminary results show execution time for calculating the Minimum Crossing Number of K_n has decreased by an order of magnitude when employing this restriction.

Keywords: minimum crossing number, complete graph, region restrictions

115 Extremal graphs for some problems on cycles in graphs

Klas Markstrom, UmeaUniversity

In this talk I will give a brief survey of a few problems concerning cycles in graphs. I will present a number of new extremal graphs for these problems, found by exhaustive computer search. I will list the extremal graphs and values for the maximum and minimum number of cycles in a graph, graphs without cycles of length 4 and 8, relating to a conjecture of Erdos and Gyarfás and the smallest 3-connected non-hamiltonian cubic graphs of class I.

Keywords: Cycles, Extremal graphs

116 Note on transformation of posets with the same upper bound graphs

K. Ogawa and M. Tsuchiya, Tokai University

The upper bound graph (UB-graph) of a poset $P = (X, \leq_P)$ is the graph $UB(P) = (X, E_{UB}(P))$, where $xy \in E_{UB}(P)$ if and only if $x \not\leq_P y$ and there exist $m \in X$ such that $x \leq_P m$ and $y \leq_P m$. In this talk, we consider transformations between posets P and Q , whose upper bound graphs are the same.

For a poset P , the canonical poset of P is the poset $Can(P) = (V(P), \leq_{Can(P)})$, where $x \leq_{Can(P)} y$ if and only if

- (1) y is a maximal element of P and $x \leq_P y$; or
- (2) $x = y$.

We obtain the following result. Two posets with the same canonical poset and the same upper bound graph can be transformed into each other by a finite sequence of two kinds of transformations, called additions and deletions on minimal elements of the canonical posets.

Keywords: upper bound graph, addition, deletion

117 Bar k -Visibility Graphs

Joan Hutchinson, Macalester College; and Josh Laison, Colorado College

Let S be a set of horizontal line segments, or bars, in the plane. We say that G is a bar visibility graph, and S its bar visibility representation, if there exists a one-to-one correspondence between vertices of G and bars in S , such that there is an edge between two vertices in G if and only if there exists an unobstructed vertical line of sight between their corresponding bars. If bars are allowed to see through each other, the graphs representable in this way are precisely the interval graphs. We consider representations in which bars are allowed to see through at most k other bars, and ask which graphs can be represented in this way.

Keywords: Visibility graphs, Interval graphs, Intersection graphs

118 Construction of Graphs with Maximum Graphical Structure and tenacity T

Dara Moazzami, University of Tehran

The tenacity of a graph G , $T(G)$, is defined by $T(G) = \min_{A \subseteq V} \frac{|A|}{r(G-A)}$, where the minimum is taken over all vertex cutset A of G . We define $G-A$ to be the graph induced by the vertices of $V-A$. $r(G-A)$ is the number of vertices in the largest component of the graph by $G-A$ and $w(G-A)$ is the number of components of $G-A$.

In this paper, the maximum graphical structure is obtained when the number of vertices p of a connected graph G and tenacity $T(G) = T$ are given. Finally the method of constructing the sort of graphs are also presented.

119 Nonseparable Hamilton decompositions

Sarah Holliday, Auburn University; and Matt Walsh, IPFW

A nonseparable Hamilton decomposition of $\geq K_{2n+1}$ is one in which no n of the Hamilton cycles can be used to construct a copy of K_{2n+1} . More generally, a decomposition is irreducible if no μn of its cycles can be used to form a copy of μK_{2n+1} for any positive integer $\mu < \infty$. We examine some constructions for nonseparable and irreducible decompositions of $\geq K_{2n+1}$ for various values of $\geq$ and n .

120 Jordan Surfaces in Discrete Topologies

Ralph D. Kopperman, CUNY; and John L. Pfaltz, University of Virginia

The characterization of Jordan surfaces, which separate a "connected" interior from a "connected" exterior, becomes rather tricky in discrete spaces. We illustrate some of the various difficulties and present (without proof) a valid "Jordan surface theorem" for n -dimensional discrete topologies that is based on degrees of connectivity.

Keywords: Discrete topology, connectivity, closure, pixel spaces, Jordan surface

121 Intersection graphs of convex sets: A brief survey

Jan Kratochvíl, Charles University

Intersection graphs of geometrical objects are studied both for their practical motivation and interesting theoretical properties. We will survey recent results on intersection graphs of straight-line segments, convex sets and curves in the plane. Special attention will be paid to the question of how many corners are needed (and suffice) for representations by convex polygons. Surprisingly tight asymptotic bounds will be presented for the case of polygon-circle graphs (i.e., when the polygons are required to be inscribed to a fixed circle). Part of the results are based on a joint work with M. Pergel.

122 Editing Distance of a Graph

Maria Axenovich, Iowa State University; Andre Kezdy, University of Louisville; and Ryan Martin, Iowa State University

Given a family of graphs $\mathcal{H}$, the *editing distance* of a graph G from $\mathcal{H}$ is the number of combined deletions and additions of edges performed on G so that the resulting graph is in $\mathcal{H}$. We fix a graph H and consider a family $\text{Forb}(H)$ of all graphs on n vertices with no induced subgraph isomorphic to H . We provide bounds for the maximum editing distance among all n -vertex graphs from $\text{Forb}(H)$. In doing so, we introduce a graph invariant which we call the *binary chromatic number*.

Keywords: edit, forbidden subgraphs, distance, binary chromatic number

123 An asymptotic Result for the Path Partition Conjecture

Marietjie Frick, University of South Africa; and Ingo Schiermeyer, Technische Universität Bergakademie Freiberg

The detour order of a graph G , denoted by $r(G)$, is the order of a longest path in G . A partition of the vertex set of G into two sets, A and B , such that $r(A) \leq a$ and $r(B) \leq b$ is called an (a, b) -partition of G . If G has an (a, b) -partition for every pair (a, b) of positive integers such that $a + b = r(G)$, then we say that G is r -partitionable. The Path Partition Conjecture (PPC), which was discussed by Lovasz and Mihok in 1981 in Szeged, is that every graph is r -partitionable. It is known that a graph G of order n and detour order $r = n - p$ is r -partitionable if $p = 0, 1$. We show that this is also true for $p = 2, 3$, and for all $p \geq 4$ provided that $n \geq p(10p - 3)$.

124 Ternary Strings that Avoid the Pattern 21

Ralph P. Grimaldi, Rose-Hulman Institute of Technology

For $n \geq 1$ let a_n count the ternary strings of length n that avoid the pattern 21. One finds that $a_n = F_{2n+2}$, where $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$, $n \geq 2$. In this talk we investigate further properties of these a_n ternary strings and count, for example,

- (1) the number of 0's, 1's, and 2's that appear;
- (2) the number of runs that occur;
- (3) the sum of these strings when considered as base 3 integers; and
- (4) the number of rises, levels, and falls that occur.

Keywords: Fibonacci Numbers, Alternate Fibonacci Numbers, Ternary Strings

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126 Path and cycles in path graphs

Daniela Ferrero, Texas State University

For a given graph G and a positive integer k the P_k -path graph, $P_k(G)$, has for vertices the set of all paths of length k in G . Two vertices are adjacent when the intersection of the corresponding paths forms a path of length $k - 1$ in G and their union forms either a cycle or a path of length $k + 1$ in G . Path graphs were proposed as an extension of line graphs. Indeed, $P_1(G) = L(G)$, the line graph of G . We shall present some properties of the number of disjoint paths of bounded length between any two different vertices of a path graph, and apply the results obtained to the study of the existence of cycles of a given length.

Keywords: path, cycle, path graph, container

127 Almost the largest possible Cycle on the subgraph, of two consecutive levels, of the Hypercube

Mahmoud El-Hashash, Bridgewater State College

Let Q_n be the n -dimensional hypercube and the weight of a node be the number of ones in its binary representation. If $n = 2k + 1$, then the subgraph $H_n = Q_n(k, k + 1)$ of Q_n induced by the nodes having exactly weights k or $k + 1$ is called the middle graph of the hypercube. In a previous paper, we presented a recursive construction for a long cycle in $Q_n(k, k + 1)$ and a recurrence relation for its length and solved it:

$$a_{n,k} = \sum_{i=0}^{n-k-3} \binom{k-2+i}{i} a_{n-k+1-i,l} + \sum_{i=0}^{k-2} \binom{n-k-3+i}{i} a_{k+2-i,l}.$$

In this paper, we show that, for a fixed k , the limit, when n goes to ∞ of the length of that cycle divided by the length of the largest possible cycle is 1.

128 Compositions into Powers of 2

Phyllis Chinn*, Humboldt State University; and Heinrich Niederhausen, Florida Atlantic University

A *composition* of n is an ordered collection of one or more positive integers whose sum is n . The number of summands is called the number of *parts* of the composition. A *palindromic composition* is a composition in which the summands are the same in the given or in reverse order. We count the number of compositions and the number of palindromic compositions where all summands are powers of 2. We also explore patterns involving the number of parts and the total number of occurrences of each positive integer among compositions where all summands are powers of 2.

Keywords: Compositions, palindromes

Thursday, March 11, 2004

8:00am	Registration - Grand Palm Room Available until 5:00pm			
	Live Oak Pavillion			
	Room A	RoomB	Roome	RoomD
8:20am	133:	134:	135:	136:
8:40am	137: E Moriya	138: E Moore	139: X Lin	140: R Craigen
9:00am	Invited Speaker: Brualdi - Grand Palm Room			
10:00am	Invited Speaker: Harary - Grand Palm Room			
10:30am	Coffee			
10:50am	141: B Hartnell	142:	143: A Soifer	144:
11:10am	145: A Dean	146: X Li	147: S De Agostino	148: N Finizio
11:30am	149: E Gethner	150: M Siggers	151: K Roblee	152: D Berman
11:50am	153: A Evans	154: L Eroh	155: N Vishnoi	156: B Bajnok
12:10pm	157: T. McKee	158: V. Levit	159: G. Bullington	160: C. Colbourn
12:30pm	Lunch			
2:00pm	161: S Klasa	162: M Bartha	163: D. McQuillan	164: K Humphreys
2:20pm	165: L Lurie	166: E Yazici	167: S Wnters	168: S Bonvicini
2:40pm	169: Z Arnavut	170: J Holliday	171: J. Urick	172: A Hilton
3:00pm	173: W. Myrvold	174: W.waller	175: P. Sun	176: J. Kuhl
3:20pm	Coffee			
3:40pm	177: A. Kooshesh	178: S. Alsardary	179: P. Azimi	180: A. Burgess
4:00pm	181: K Gopalakrishnan	182: A Sarkar	183: P. Kovar	184: W. Wei
4:20pm	185: J. Lucas	186: L Lima	187: R Low	188: K Ozawa
4:40pm	189: L Kazmierczak	190: M Ferrara	191: M. DeDeo	192: R Laue
5:00pm	193: G. Dufore	194: H Liu	195: L 'Nang	196: 8. Tankersley
5:20pm	197:	198:	199:	200:

Coffee and refreshments will be served in the registration area of the Grand Palm Room.

137 The Subgraph Connecting Problem - Its Variants and Their Computational Complexity

Taishi Nishida, and Etsuro Moriya, Waseda University

We consider the following problem: Given a graph G and its subgraphs $G_1, \dots, G_m$, is there a connected subgraph G' of G such that it intersects each G_i , $1 \leq i \leq m$? A variety of variants of this problem are introduced and it is shown that they are complete for five familiar classes of the complexity hierarchy, NL, P, NP, PSPACE, and NEXP. For example, if G' is required to share exactly one vertex with each subgraph, then the problem is NP-complete even if each G_i contains only two vertices. The idea for the proof of NP-completeness is extended to show P-, PSPACE-, and NEXP-completeness of the verification version, of the two-player game version, and of the succinct representation version, respectively.

Keywords: complexity class, NL-complete, P-complete, NP-complete, PSPACE-complete, NEXP-complete

138 Distance Constraints in Graph Color Extensions

Joan P. Hutchinson, Macalester College; and Emily H. Moore, Grinnell College

Graph color extension problems ask for circumstances under which we can precolor a set of vertices P in G and be guaranteed that the coloring can be extended to the entire graph. Let $r = \chi(G)$, s the chromatic number of the induced subgraph $G[P]$ on P , and t the total number of colors used. M. Albertson and J. Hutchinson have shown that graph color extension theorems exist in the cases (1) $t = r + s$ and (2) $t = r + s - 1$ (with topological constraints on G), provided the components of $G[P]$ are sufficiently far apart. In the current work, we give lower bounds for the distances required between precolored components. In the first case, we give tight bounds for this distance; in the second we give a small range for the distance. Examples come from r -chromatic, K_{r+2} -minor-free graphs that are not $(r+1)$ -list colorable with lists from colors $\{1, \dots, r+2\}$.

Keywords: graph color extensions, K_n -minor-free, list colorings

139 Cycles as Constraint Graphs in Multi-type Percolation

Xue Lin" and John C. Wierman, The Johns Hopkins University

In a multi-type percolation model we randomly color the vertices of an infinite underlying graph and the presence or absence of a bond between two neighboring vertices is determined by their colors through a constraint graph. A previous result is that infinite multi-type percolation is impossible on a class of bipartite underlying graphs if the constraint graph is an even cycle. We discuss the case of an odd cycle as the constraint graph and get a result applying to all cycles, odd or even. We prove the probability that an infinite multi-type percolation cluster exists equals zero when the constraint graph is a k -cycle ($k \geq 5$) and the degree of the underlying graph is less than or equal to 3. The proof involves nonlinear optimization, matrix analysis and Markov chains.

Keywords: multi-type percolation, constraint graph, cycles, Karush-Kuhn-Tucker(KKT) point, spectral radius

140 Some Classes of Circulant Orthogonal Matrices

R. Craigen, University of Manitoba

A paper tossed off by Sylvester in 1867, in the middle of the formative years of modern matrix theory, has led to over a century of investigations into difficult questions surrounding certain varieties of orthogonal complex matrices, which include Hadamard matrices, and their classification. From Sylvester's paper may be distilled a conjecture saying that there is a unique class of every prime order-the Vandermonde of the n th roots of unity. A year ago this conjecture fell, when it was discovered that there is more than one class of every prime order greater than 3.

Remarkably, the examples we have found have a very simple (i.e., circulant) structure. Further, all small examples, and many infinite classes of larger matrices of this type, appear to be circulant. This is quite unexpected, particularly in light of the longstanding conjecture that there are no circulant Hadamard matrices of order greater than 4.

We obtain new insight into larger classification problems including some that relate to Hadamard matrices and projective planes. We shall end with a series of new unsolved questions relating to the classification of orthogonal matrices.

Keywords: generalized Hadamard matrices, unit Hadamard matrices, orthogonal matrices, matrix classification, circulant matrices, Sylvester's conjecture

141 Falling Stars: A Game on a Graph

B. Hartnell, N. Hunter, V. LeBlanc, K. Patterson, G. Reid, and A. Spanik, Saint Mary's University

There are many results dealing with the problem of decomposing a fixed graph (often the complete graph) into isomorphic subgraphs (cliques or cycles, for instance). We are interested in considering a game based on that concept. In particular, two players alternate deleting a subgraph of a specified type from a given graph until the resulting graph no longer contains such a subgraph. The last player able to make a legal move wins. Thus, in general, the game will be over before all the vertices and edges have been removed. In the situation that the subgraph is a small star, a winning strategy for a very restricted collection of graphs will be outlined.

Keywords: subgraph deletion, game

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143 Axiom of Choice and Chromatic Number of Distance Graphs: Examples on the Plane and $\mathbb{R}^n$

Alexander Soifer, Princeton University; and Saharon Shelah, The Hebrew University

Define a **Unit Distance Plane** as a graph U^2 on the set of all points of the plane $\mathbb{R}^2$ as its vertex set, with two points adjacent iff they are distance 1 apart. The chromatic number χ of U^2 is called **the chromatic number of the plane**. It makes sense to talk about a **distance graph** when its set of vertices belongs to a metric space, and two points are adjacent iff the distance between them belongs to a given set of 'forbidden' distances.

In our 2003 report to this conference, we formulated a **Conditional Chromatic Number Theorem**, which described a setting in which the chromatic number of the plane takes on two different values depending upon the axioms for set theory. We also constructed an example of a distance graph on the real line $\mathbb{R}$ whose chromatic number depends upon the system of axioms we choose for set theory. Ideas developed there are extended here to construct distance graphs G_2 , G_3 , and G_4 on the plane $\mathbb{R}^2$, thus coming much closer to the setting of the chromatic number of the plane problem. The chromatic numbers of G_i , $i = 2, 3, 4$ in the Zermelo-Fraenkel-Choice system of axioms are 2, 3, and 4 respectively, and are not countable (if they exist) in a consistent system of axioms with limited choice, studied by Robert M. Solovay.

These ideas allow a generalization to n -dimensional Euclidean space $\mathbb{R}^n$, and thus a construction of distance graphs on $\mathbb{R}^n$, whose chromatic number depends upon the system of axioms we choose for set theory.

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145 Necessary Conditions for Unit Bar-visibility Layout of Triangulated Polygons

Alice M. Dean, Skidmore College; Ellen Gethner, University of Colorado at Denver; and Joan P. Hutchinson, Macalester College

A bar-visibility layout of a graph represents each vertex as a horizontal bar in the plane, with adjacencies corresponding to nondegenerate, unobstructed vertical bands of visibility between bars. Graphs possessing such a layout were completely characterized by several writers in the 1980s, and efficient algorithms were given for recognition and layout. We consider a restricted subclass, called unit bar-visibility graphs (UBVGs), in which all bars are required to have equal length. This class of graphs has not been completely characterized, but the subset of UBVGs that represent triangulated polygons has a characterization that is geometrically attractive and algorithmically efficient. This talk describes the necessary conditions for a triangulated polygon to be a UBVG.

Keywords: bar-visibility graph, planar, outerplanar, graph drawing, computational geometry, polygon, near-triangulation

146 Partitioning an Edge-Colored Graph into Multicolored Subgraphs

Zemin Jin, Xiaoyan Zhang, Nankai University; and Xueliang Li, University of Mississippi

Given an edge colored graph $G = (V, E)$, a subgraph H of G is said to be *multicolored* if no two edges of H have the same color. We show that finding the minimum number of multicolored trees (cycles) that partition $V(G)$ is NP-complete. Some inapproximability results are obtained. We also show that finding the minimum number of such multicolored paths is NP-complete for graphs with edge-colored by 2 colors, which implies that finding the minimum number of such multicolored trees remains NP-complete for graphs with edge-colored by 2 colors.

Keywords: multicolored, partition, NP-complete, inapproximability

147 A Conjecture on Biconnected Graphs and Regular 3-Cell Complexes

Sergio De Agostino, Armstrong Atlantic State University

The notions of spatial graph, spatial representation and spatiality degree of a graph were presented in (F. Luccio and L. Pagli, *Introducing Spatial Graphs*, *Congressus Numerantium* 110, 33-41, 1995). Later, the notion of BP-spatial representation of a biconnected graph $G = (V, E)$ was introduced in (P. Crescenzi, S. De Agostino and R. Silvestri, *A Note on the Spatiality Degree of Graphs*, *Ars Combinatoria* 63, 185-191, 2002) and it was shown that the spatiality degree of a BP-spatial representable graph is $2(|E| - |V|)$. While the spatiality degree of a graph is a fairly new concept in graph theory, it can be seen that the notions of spatial graph and its spatial representation relate to the concepts of two- and three-dimensional cell complexes. More precisely, the spatial representation induces a CW-complex on the three-dimensional euclidean space with its standard topology while its two dimensional skeleton is a regular CW-complex representing the spatial graph. The spatiality degree of a graph $G = (V, E)$ is the maximum number of 3-cells that a CW-complex with a 1-dimensional skeleton representing G can have on the three-dimensional euclidean space with its standard topology, assuming that two distinct 2-cells of the complex cannot share the same boundary. It follows from the Euler-Poincare' formula that $2(|E| - |V|)$ is the theoretical upper bound to the spatiality degree. Based on the evidence of previous results, we formulate the conjecture that for any biconnected graph $G = (V, E)$ with at least four simple cycles there exists a regular CW-complex with a 1-dimensional skeleton representing G on the three-dimensional euclidean space with its standard topology, which has $2(|E| - |V|)$ 3-cells and each cell has a distinct boundary.

Keywords: Biconnected graph, CW-complex, topological graph theory

148 Pseudo-Z-cyclic Whist Designs

Marco Buratti and Norman J. Finizio, University of Rhode Island

A new specialization of whist tournament design will be introduced.

149 A Unit Bar-Visibility Layout Algorithm for Outerplanar Near Triangulations

Ellen Gethner, University of Colorado at Denver

Bar-visibility graphs are those graphs whose vertices can be represented in the plane by closed, disjoint horizontal line segments (called bars); two bars are adjacent exactly when there is a nondegenerate, unobstructed rectangle of visibility between them. Those (necessarily planar) graphs that are bar-visibility graphs have been completely characterized and well-understood since the mid 1980's. Moreover, there are a variety of efficient algorithms that accomplish bar visibility layout.

The characterization of Unit Bar-Visibility Graphs (bar-visibility graphs in which each bar has unit length) is underway. We will talk about the algorithmic aspects of this problem and illuminate what is known about 2-connected outerplanar near triangulations; tools from classical geometry provide the foundation for an efficient and aesthetic layout algorithm.

Keywords: bar-visibility graph, planar, outerplanar, graph drawing, computational geometry

150 Color Critical Hypergraphs with Many Edges

V. Rodi and M. Siggers, Emory University

A k -coloring of a hypergraph H is a k -vertex-coloring with no monochromatic edges. H is k -critical if it is k -chromatic but becomes $(k - 1)$ -chromatic with the removal of any edge.

The only 3-critical 2-uniform hypergraphs (graphs) are odd-cycles. In 1952 Dirac showed that not all k -critical graphs are sparse by showing that for $k \geq 6$ and n large enough, there exists a k -critical graph with n vertices and more than cn^2 edges. In 1970 Toft proved the same for $k = 4$.

Generalizing to hypergraphs, Toft showed in 1973, that the maximum number of edges of a k -critical r -uniform hypergraph is of order n^r for $k > 3$. This is maximum possible order. In 1976 Lovasz showed that the maximum number of edges of a 3-critical r -uniform system is order n^{r-1} .

Several recent papers have investigated the chromatic numbers of (r, D) -systems (r -uniform hypergraphs in which no l -set of vertices occurs in more than one edge). We extend the above results to show that for all $k \geq 3$, $r > 1$, and large enough n , there exist k -critical (r, D) -systems on n vertices with order n^1 edges; this is maximum possible order.

Keywords: dense, color critical, uniform, hypergraph, partial Steiner systems

151 Extremal Families of Nearly Strongly Regular Graphs

Peter Johnson, Auburn University; and Ken Roblee, Troy State University

A graph is nearly strongly regular with parameters n, d, t (or, the graph is in $NSR(n, d, t)$) iff it is ad-regular graph on n vertices, and each pair of adjacent vertices have t common neighbors. In a quixotic quest for a description of some of these graphs, we fix our attention on $p = n - 2d + t$, the number of vertices outside the joint neighbor set of any two adjacent vertices in such a graph.

In earlier work we showed that if $t > 0$ and $NSR(n, d, t)$ is non-empty for some n and d , then n is no greater than $3t + 3p$. Such graphs for which $n = 3t + 3p$ have been completely characterized, for $p = 0$ and $p = 2$, and the second author has described all but a finite number of such graphs, for each even value of p . Here we show that if p is odd, and if $NSR(n, d, t)$ is non-empty for some n, d and t satisfying $p = n - 2d + t$, then n is no greater than $3t + 3p - 2$, and our aim is to find the graphs for which equality holds. When $p = 1$ there is only one, the complement of the Petersen graph; as of this writing no such graphs have been found with $p > 1$, odd.

Keywords: strongly regular, joint neighbor set

152 Some New Latin Triangles

David R. Berman, Nolan B. McMurray and Douglas D. Smith, University of North Carolina at Wilmington

Latin triangles are analogous to Latin squares but require lines in each of three directions to include each symbol exactly once. Each point of a Latin triangle is specified as a coordinate triple corresponding to the three directions. A median in a Latin triangle induces a permutation on the underlying coordinates. We show that this permutation induces a permutation of coordinate triples and transversals and can be used to construct Latin triangles of even order.

Keywords: Latin triangle, median, transversal

153 Representations of disjoint unions of complete graphs

Anthony B. Evans, Wright State University

A representation of a graph G modulo n is an assignment of distinct labels between 0 and $n - 1$ to the vertices of G so that the difference of two labels is relatively prime to n if and only if the corresponding vertices are adjacent. The representation number of G is the smallest positive integer n for which G is representable modulo n . In this paper we will present what is known about representations of disjoint unions of complete graphs and prove some new results. These representations are closely related to mutually orthogonal sets of latin squares.

154 List-Coloring Certain Complete n -Partite Graphs

Linda Eroh, University of Wisconsin Oshkosh

A graph is k -choosable if, for any assignment of lists of length k to each vertex, there is a proper coloring of the graph in which each vertex is assigned a color from its list. A graph is equitably k -choosable if, for any assignment of lists of length k to each vertex, there is a proper coloring of the graph in which every vertex is assigned a color from its list so that no color is used more than $\lceil n/k \rceil$ times, where n is the order of the graph. We show that if every graph in some hereditary set of graphs is equitably k -choosable, then every graph in that set is equitably $k + 1$ -choosable. However, we conjecture that there is a graph, $K_{3,3,2,2}$, that is equitably 4-choosable but not equitably 5-choosable. This conjecture depends on whether $K_{3,3,2,2}$ is 4-choosable or not. We show that $K_n(2)$ is n -choosable and $K_n(3)$ is not n -choosable, for any positive integer n . Furthermore, $K_{1(3),n-1(2)}$ is n -choosable for any positive integer n . However, the status of $K_{3,3,2,2}$ is still open.

Keywords: equitably n -choosable, n -choosable, list coloring, vertex coloring

155 A Generalization of the Characteristic Polynomial of a Graph

Richard J. Lipton, Nisheeth K. Vishnoi and Zeke Zalcstein, Georgia Institute of Technology

Given a graph G with its adjacency matrix A , consider the matrix $A(x, y)$ in which the 1s are replaced by the indeterminate x and 0s (other than the diagonals) are replaced by y . The $\mathcal{E}$ -polynomial of G is defined as:

$$\mathcal{E}(x, y) := \det(A(x, y) - yI)$$

This polynomial is a natural generalization of the standard *characteristic polynomial* of a graph.

In this note we characterize graphs which have the same $\mathcal{E}$ -polynomial. The answer is rather simple: Two graphs G and H have the same $\mathcal{E}$ -polynomial if and only if G and H are co-spectral and G_c and H_c are co-spectral. (Here G_c (resp. H_c) is the complement of G (resp. H)).

156 A generalization of sum-free sets in abelian groups and constructions of spherical designs

Bela Bajnok, Gettysburg College

A finite set X of points on the d -sphere S^d is a *spherical t -design*, if for every polynomial f in $d + 1$ variables and of total degree t or less, the average value of f over the whole sphere equals the arithmetic average of its values on X . For example, the $2d + 2$ vertices of a regular octahedron form a (minimum size) 3-design on S^d . This concept is the spherical analog of t - (v, k, λ) designs, and has been studied in various contexts, including representation theory and approximation theory. The main question in the field is to find all integers N for which a spherical t -design of size N exists on S^d , and to provide explicit constructions.

We call a subset S of the finite (additive) abelian group G *t -independent*, if no multi-subset of S with at most t elements can be divided into two disjoint parts so that the two parts have the same (multi-subset) sum. For example, the set $\{2, 3, 7\}$ is a (maximum size) 3-independent set in Z_{15} , but it is not 4-independent since $7 + 7 + 3 = 2$. This concept generalizes the well known concepts of sum-free sets and Sidon-sets. The main question here is to find the largest integer s for which a t -independent set of size s exists in G and to provide explicit constructions.

In this talk we find the maximum size of a 3-independent set in Z_n for all $n \in \mathbb{N}$ and discuss how 3-independent sets can be used to construct spherical 3-designs.

157 Clique Representations of Graphs

Terry McKee, Wright State University

Clique representations of arbitrary graphs generalize clique trees of chordal graphs by replacing trees with certain graphs that have distinguished families of special cycles, 'polyhedra,' and higher-dimension analogs.

This survey sketches the basic construction of clique representations, uses them in a new proof of Gavril's result that every graph is the intersection graph of subtrees of a triangle-free graph, and proves additional numerical relationships between a graph and its clique representation.

Keywords: Intersection graphs, chordal graphs, clique representations

158 Very well-covered graphs with log-concave independence polynomials

Vadim E. Levit and Eugen Mandrescu, Rolon Academic Institute of Technology

If $\mathfrak{K}$ denotes the number of stable sets of cardinality k in graph G , and $a(G)$ is the size of a maximum stable set, then $I(G; x) = \sum_{k=0}^{a(G)} \mathfrak{K} x^k$ is the independence polynomial of G (Gutman and Harary, 1983). A graph G is very well-covered if it is well covered (all its maximal stable sets have the same size (Plummer, 1970)), it has no isolated vertices, and its order equals $2a(G)$ (Favaron, 1982). For instance, the graph $c \bullet$, obtained from G by appending a leaf to each vertex, is very well-covered.

Alavi, Malde, Schwenk and Erdos (1987) conjectured that $I(G; x)$ is unimodal for any forest G . Our guess is that the independence polynomial of a forest is log-concave.

In this paper we propose a way to prove this strengthened conjecture step by step starting from very well-covered forests. For instance, we show that $I(G^*; x)$ is log-concave for any graph G having $a(G) \geq 3$. It is worth noticing that any very well-covered forest of order $2n$ can be represented as G^* for some forest G . We also prove that if $G \in \{K_{1,m}, P_n : n \geq 1\}$, then $I(G; x)$ is log-concave. Consequently, we conclude that for any positive integer a there is a very well-covered graph H with $a(H) = a$, whose $I(H; x)$ is log-concave.

Keywords: independence polynomial, log-concave sequence, well-covered graph

159 The Touring Number of a Graph

G. Bullington, L. Eroh, J. Koker, K. McDougal, H. Moghadam, S. Winters, University of Wisconsin-Oshkosh; and S. Stalder, University of Wisconsin-Waukesha

For a connected graph G let the touring distance $T(u, v)$ between vertices u and v be the length of the longest u - v trail. If a u - v trail has length $T(u, v)$, then we call it a u - v tour. For any subset S of the vertex set $V(G)$, let $Is(G)$ denote the set of all vertices that are on any tour to and from vertices within S . We define the (vertex-) tour number to be the minimum cardinality possible for S having the property that $Is(G) = V(G)$. In this talk, we will present some general results and bounds for this touring number, discuss some alternatives to these definitions to explore, and pose open questions.

Keywords: distance

160 Maximum and Average Access Cost in Double Erasure RAID Disk Arrays

Marge M. Coahran, University of Toronto; and Charles J. Colbourn, Arizona State University

In a systematic erasure code for the correction of two simultaneous erasures, each information symbol must have two associated parity symbols. When implemented in a redundant array of independent disks (RAID), performance requirements on the update penalty necessitate that each information symbol be associated with no more parity symbols than the two required. This leads to a simple graph model of the erasure codes, with parity symbols as vertices and information symbols as edges. Ordering the edges so that no more than f check disks (vertices) appear among any set of d consecutive edges is found to optimize access performance of the disk array when d is maximized. Computational experiments are reported to determine the optimal solutions for small values of n and d . Optimal solutions in the average case are determined when $d \geq 5$, and optimal solutions in the worst case are characterized when $d \geq 12$.

161 Chernoff bound on classification error for multivariate parametric and nonparametric classes

Adam Krzyzak and Stan Klasa, Concordia University

In the paper we first derive Chernoff and Bhattacharyya upper bounds on the probability of error for the problem of discriminating two multivariate Gaussian signals. We further generalize the results and obtain the upper bound estimate on the Bayes misclassification error derived by using a Parzen kernel estimate of probability density functions. We study the convergence of the estimate and the rate of convergence for a large class of densities.

162 The Berge Formula for maximum internal matchings in graphs

M. Bartha, Memorial University of Newfoundland

In matching theory, the deficiency of graph G is the number of vertices left uncovered by any maximum matching of G . Berge's formula provides an upper bound for the deficiency of graphs in terms of so called separator sets of vertices. A barrier is a separator set for which the upper bound is reached in Berge's formula. We provide a generalization of the Berge Formula for maximum internal matchings, i.e., matchings that are expected to cover only the vertices with degree at least two. Based on this formula, a new description of barriers is given for maximum internal matchings in graphs. Surprising consequences are obtained regarding the barriers of some important graphs having a perfect internal matching, which are in accordance with the author's previous results on the structure of such graphs. It becomes possible to enumerate all the maximal barriers of any graph having a perfect or perfect internal matching by an efficient algorithm.

Keywords: graph matchings, matching deficiency, barriers

163 A Spectrum Problem For Odd Complete Graphs

Dan McQuillan and Katy Smith, Norwich University

A total labeling of a graph is an assignment of consecutive integers $1, 2, \dots, v + e$ to the vertices and edges of the graph. The weight of each vertex is the sum of its label and the labels of its incident edges. A vertex magic total labeling (VMTL) is a total labeling in which the weight of each vertex is constant.

We will focus on the odd complete graphs. Previously, magic squares had been used to construct VMTL's with only a few different magic constants. We will introduce a new technique that is used to construct VMTL's for all possible magic constants. This solves an important conjecture for odd complete graphs. We provide explicit formulas and indicate how these methods can be used to find VMTL's for other graphs.

Keywords: Magic graphs, magic labelings, complete graphs, spectrum problem

164 Counting Infinite Step Set Lattice Paths using Umbral Calculus

Katherine Humphreys and Heinrich Niederhausen, Florida Atlantic University

A lattice path in the $\mathbb{N} \times \mathbb{N}$ plane with an infinite step set S can go to infinitely many lattice points within the boundary but can only come from finitely many points. In previous work, we have shown explicit solutions to counting lattice paths from $(0, 0)$ to (n, m) in the first quadrant, above a boundary line $y = ax - f$ ($a, f \in \mathbb{N}$), or also a path that has a special access to the boundary using an additional set of (privileged) step vectors P . We find closed form solutions via Sheffer sequences and functionals using results of the Umbral Calculus.

All of our examples of infinite step sets could be interpreted as step sets with a finite number of weighted steps. In this talk we demonstrate the possibility of this method with an infinite step set that does not reduce to a finite step set.

Keywords: Lattice path counting, infinite step set, Umbral Calculus, Sheffer polynomials, privileged access

165 Hybrid Model for Classification

Michael Gargano and Lorraine Lurie, Pace University

Self-Ordering Maps and Swarm Intelligence methods of classification will be discussed. A novel hybrid method will also be introduced.

Keywords: Neural Networks, Swarm Intelligence, Classification

166 The Triangle Intersection Problem for Kite Systems

Emine Sule Yazici, Auburn University

The graph  is called a kite and the decomposition of K_n into kites is called a kite system. Such systems exist precisely when $n \equiv 0$ or $1 \pmod{8}$. In 1975 C.C. Lindner and A. Rosa solved the intersection problem for the Steiner triple systems. The object of this paper is to give a complete solution to the triangle intersection problem for kite systems. (= How many triangles can two kite systems of order n have in common). We show that if $x \in \{0, 1, 2, \dots, n(n-1)/8\}$, then there exists a pair of kite systems of order n having exactly x triangles in common.

167 Some New Classes of Graceful Graphs

Brenda Clemens, Romain Coulibaly, Jennifer Garvens, Jennifer Gonnering, Jeff Lucas and Steven J. Winters, University of Wisconsin Oshkosh

A graph with m edges is called *graceful* if it is possible to label the vertices of the graph with distinct elements from the set $\{0, 1, 2, \dots, m\}$ in such a way that the induced edge labelling, which prescribes the integer $|i - j|$ to the edge joining vertices labelled i and j , assigns the labels $1, 2, \dots, m$ to the edges of the graph. We will test some new classes of graphs for gracefulness. We will consider the one-point union of some common graphs such as paths, complete graphs and complete bipartite graphs. In addition, we will consider adding or removing edges from complete bipartite and complete tripartite graphs.

Keywords: graceful

168 Starters: Doubling Constructions

Simona Bonvicini, University of Modena and Reggio Emilia

Let G be a finite group of even order $2n$. Can G be realized as an automorphism group of a one-factorization of the complete graph K_{2n} acting sharply transitively on its vertex set? Partial answers to this question were given by several authors. Here we consider the case in which n is even and G possesses a subgroup H of index 2 which can be realized as an automorphism group of the complete graph K_n acting sharply transitively on vertices. We present a construction of a one-factorization of K_{2n} starting by a one-factorization of K_n using the technique of starters.

169 A Fast and Efficient Algorithm for Transforming between Permutations and Lehmer-Inversion Tables

Ziya Arnavut, State University of New York

The notion of *inversion* for a given permutation was introduced quite early in an effort to provide concise representations of ordinary permutations. While several inversions techniques were introduced, to the best of our knowledge, not much is done in terms of developing efficient algorithms for permutation based inversion techniques. The fastest algorithm of which we are aware for transforming between permutations and Lehmer-inversion tables requires $O(n \log n)$ time. The method requires four integer values to be stored in each node's information field and utilizes a *VL tree* data structure, which requires rotations in order to retain a binary tree height-balanced.

In this work, we examine the *Lehmer-inversion* techniques, and introduce a fast and more memory efficient algorithm for transforming between permutations and Lehmer-inversion tables.

Keywords: Permutations, Inversions Tables, Lehmer-inversions, Algorithms

170 Counterexample to a Conjecture on Optimal Shields-Harary Weightings

John Holliday and Peter Johnson, Auburn University

The Shields-Harary number of a graph, with respect to a given (continuous) cost function, is the maximum, over all non-negative weightings of the vertices, of the minimum "dismantling cost" of the graph, with those weights. A weighting that achieves this maximum is *optimal*, and obviously finding optimal weightings of a graph, with respect to a given cost function, is a major objective in this area of study. It has long been conjectured that any vertex transitive graph **will** have a constant optimal weighting, for any cost function. A more recent, related conjecture, the "constant-weight-on-orbits" conjecture, is that any graph, with any cost function, will have an optimal weighting constant on each automorphism class of vertices. It is this conjecture that we dismiss with a counterexample. The conjecture about vertex transitive graphs is still open.

Keywords: connected component, Shields-Harary

171 On the Representation Number of a Split Graph

James Urick and Darren A. Narayan, Rochester Institute of Technology

A graph G has a representation modulo n if there exists an injective map $f : V(G) \rightarrow \{0, 1, \dots, n-1\}$ such that vertices u and v are adjacent if and only if $|f(u) - f(v)|$ is relatively prime to n . The representation number $rep(G)$ is the smallest n such that G has a representation modulo n . A split graph is one that can be partitioned into a complete subgraph and an independent set. We present new results involving the representation number of a split graph.

Keywords: vertex labeling, representation number, product dimension

172 Amalgamations of Partial Latin Squares

A.J.W. Hilton, University of Reading

Carlos and Glebsky have recently extended the theorem of the author that every outline latin square is an amalgamated latin square. The extension is to certain kinds of outline partial latin squares. Here we generalize the Carlos-Glebsky result to all outline partial latin squares.

Glebsky and Gordon recently announced that the original result of the author provides an essential tool in a recent characterization of theirs concerning approximations to compact groups, and that the Carlos-Glebsky result is needed for the same characterization concerning approximations to locally compact groups.

Keywords: Amalgamations, quotients, latin squares, quasigroups

173 On Filotti's Algorithm for Embedding 3-Regular Graphs on the Torus

John Chambers, William Kocay and Wendy Myrvold*, University of Victoria

In 1980, Filotti proposed a polynomial time algorithm for embedding 3-regular graphs on the torus. A topological obstruction G for the torus is a graph G with minimum degree at least three such that G is not toroidal but $G - e$ is for all edges e of G . In the process of studying this algorithm more closely in order to search for the 3-regular topological obstructions to the torus, we discovered a fatal flaw in his approach. We will discuss this and other pertinent issues regarding torus embedding algorithms and the search for torus obstructions.

Keywords: Embedding graphs on surfaces, torus embedding algorithms

174 On Some Conjectures of Graffiti.pc Concerning the Bipartite Number and Path Number of a Graph

Ermelinda DeLaVina and Bill Waller*, University of Houston-Downtown

The path number of a graph is the maximum number of vertices of an induced path in the graph. The bipartite number of a graph is the maximum number of vertices of an induced bipartite subgraph of the graph. We discuss several lower bounds for these invariants that were conjectured by Graffiti.pc, and the relationship between these lower bounds and known theorems or prior conjectures.

Keywords: average distance, bipartite number, diameter, Graffiti.pc, independence number, path number, radius

175 Constructions of A -magic graphs and some A -magic graphs with A of even order

W.C. Shiu, P.C.B. Lam and P.K. Sun*, Hong Kong Baptist University

Let A be an abelian group. An A -magic of a graph $G = (V, E)$ is a labeling $l : E(G) \rightarrow A \setminus \{0\}$ such that the sum of the labels of the edges incident with $u \in V$ are all the same, where 0 is the identity element of the group A . In this paper, we will show that some classes of graphs are A -magic, where A is an abelian group of even order greater than 2. Finally, we proved that product and composition of A -magic graphs are also A -magic.

176 Completing Partial Latin Squares

Jaromy Kuhl, University of Mississippi

A latin square is an $n \times n$ grid accompanied with n symbols and filled so that each symbol appears only once in each column and row. A partial latin square is one that is not filled completely. In 1980 Higgkvist presented the following conjecture: If R is an $n \times n$ partial latin square where at most $(n - 1)$ of the $n \times n$ squares are filled, then R can be completed. The historical roots of this conjecture will be presented in addition to some partial solutions.

177 Approximate Computation of Exponential Function and Related Problems

A.A. Kooshesh and B. Ravikumar, Sonoma State University

Computing 2^x (say input x is in binary, the output is in ternary) is computationally intractable in a trivial sense - the size of the output is exponential in the size of the input. But usually, we are only interested in a good approximation to 2^x . Recently, Hirvensalo and Karhumäki [Hir] provided a solution to this problem. Specifically, they showed how to compute the number of "trits" in 2^x and the leading trit of 2^x in time polynomial in $|x|$. We extend their work in the following directions:

(1) We provide an efficient implementation of the algorithm of [Hir] and extend it to compute the leading k -trits of 2^k for a fixed k .

(2) We propose an alternative *randomized* algorithm for this problem and compare it to [Hir].

(3) We also consider the problem of computing the leading trit of the $(1,1)$ -entry of A^n where A is an integer matrix. For this problem, the inputs are the entries of matrix A and (the binary representation of) n . This problem includes approximate computation of the n -th Fibonacci number as a special case.

Keywords: approximation algorithms, randomized algorithms

178 New Edge Neighborhood Graphs

Ali A. Ali, Mosul University; and Salar Alsardary, University of the Sciences in Philadelphia

Let G be an undirected simple connected graph, and $e = uv$ be an edge of G . Let $Na(e)$ be the subgraph of G induced by the set of all vertices of G which are not incident to e but are adjacent to u or v . Let N_e be the class of all graphs H such that, for some graph G , $Na(e) = H$ for every edge e of G . Zelinka studied edge neighborhood graphs and obtained some special graphs in N_e . Balasubramanian and Alsardary obtained other graphs in N_e . In this paper we give some new graphs in N_e .

179 An Integer Programming Model for the Graceful Labeling Problem

Kourosh Eshghi and Parham Azimi, Sharif University of Technology

A graceful labeling of a graph $G = (V, E)$ with n vertices and m edges is a one-to-one mapping f of the vertex set $V(G)$ into the set $\{0, 1, 2, \dots, m\}$ with this property: If we define, for any edge $e = (u, v) \in E(G)$, the value $n(e) = |f(u) - f(v)|$ then n is a one-to-one mapping of the set $E(G)$ onto the set $\{1, 2, \dots, m\}$. A graceful labeling problem is to find that whether a given graph is graceful and if it is graceful how to label the vertices. In this paper, first a new approach based on integer programming technique is presented to model the graceful labeling problem. Then a "Branching Method" is developed to solve the model for special classes of graphs. The proposed algorithm has been extensively tested on a set of different classes of randomly generated graphs. Computational results show the efficiency of the proposed algorithm for different classes of graphs.

Keywords: Graceful graphs, graph labeling, integer programming

180 Invariants for Even Cycle Systems

Andrea C. Burgess and David A. Pike, Memorial University of Newfoundland

An m -cycle system of order n is a partition of the edges of K_n , the complete graph on n vertices, into m -cycles. An m -cycle system of order n exists if and only if n is odd, m divides $n-1$ and either $n = 1$ or $n \geq m$.

Let $N_m(n)$ denote the number of pairwise nonisomorphic m -cycle systems of order n . The values $N_4(9) = 8$ and $N_6(9) = 640$ were determined by Dejter, Rivera-Vega and Rosa, who made use of several invariants for cycle systems to aid in distinguishing nonisomorphic systems. We employ a relaxation of one of these invariants in generating nonisomorphic even cycle systems (i.e. m -cycle systems for which m is even) and report on the utility of our invariant.

181 On Removing Randomness from Probabilistic Algorithms

K. Gopalakrishnan, East Carolina University

The amount of random bits used in a probabilistic algorithms can be considered as a complexity measure just like time and space. Hence, in recent years, many researchers have developed techniques to reduce the number of random bits needed in computation and to eliminate completely, if possible, the use of random bits. We propose a new technique in its abstract form for derandomization using sample spaces constructed from matroids.

Keywords: Derandomization, Probabilistic Algorithms, Binary Matroids

182 Connectivity of Random Geometric Graphs

Paul Balister, Bela Bollobas, Amites Sarkar, University of Memphis; and Mark Walters, Trinity College - Cambridge

Let P be a Poisson process of intensity one in a square S_n of area n . We construct a random geometric graph $G_{n,k}$ by joining each point of P to its k nearest neighbors. Recently, Xue and Kumar proved that if $k = 0.074 \cdot \log n$ then the probability that $G_{n,k}$ is connected tends to zero as $n \rightarrow \infty$, while if $k = 5.1774 \cdot \log n$ then the probability that $G_{n,k}$ is connected tends to one as $n \rightarrow \infty$. They conjectured that the threshold for connectivity is $k = \log n$. We improve these lower and upper bounds to $k = 0.3043 \cdot \log n$ and $k = 0.5139 \cdot \log n$ respectively, disproving this conjecture. We also establish lower and upper bounds of $k = 0.7209 \cdot \log n$ and $k = 0.9967 \cdot \log n$ for the directed version of the problem.

183 Vertex magic total labeling of products of regular VMT graphs and regular class 1 supermagic graphs

Petr Kovar, University of Minnesota Duluth

A vertex-magic total labeling of a graph $G(V, E)$ is defined as one-to-one mapping from $V \cup E$ to the set of integers $\{1, 2, \dots, |V| + |E|\}$ with the property that the sum of the label of a vertex and the labels of all edges incident to this vertex is the same constant for all vertices of the graph. A supermagic labeling of a graph $G(V, E)$ is defined as one-to-one mapping from E to the set integers $\{1, 2, \dots, |E|\}$ with the property that the sum of the labels of all edges incident to a vertex is the same constant for all vertices of the graph.

In the talk we present a technique for constructing vertex magic total labelings of Cartesian products of certain vertex magic total r -regular graphs G and certain s -regular supermagic graphs H with proper edges-coloring. The parameters r and s can be arbitrary with the restriction that if r is even then $|H|$ has to be odd and if s is odd then $|G|$ has to be odd.

If time permits we compare this technique to other methods based on decomposing G into two regular factors or/and H into two regular factors.

Keywords: vertex-magic total labeling, supermagic labeling, Cartesian product

184 On the Number of Latin Rectangles and Generalized Latin Squares

Spyros S. Magliveras, Wandu Wei, Florida Atlantic University; and Dale Mesner, University of Nebraska-Lincoln

A generalized Latin square of order n with index m ($m < n$) is a matrix of order n with entries in $([1, n] \cup \{b\})$, where $[1, n]$ denotes the set of integers between 1 and n and b is a symbol standing for blank, such that each row contains each $i \in [1, n]$ exactly once and so does each column. A $k \times n$ Latin rectangle with $k \leq n$ is a $k \times n$ matrix with entries in $[1, n]$ such that each row is a permutation of $[1, n]$ and each column is a k -permutation of $[1, n]$. In this paper, a formula for the number of generalized Latin squares is given as well as a formula of the number for Latin rectangles with some entries pre-determined.

Keywords: Latin Rectangles, generalized Latin Squares

185 Localized Rotation Distance in Binary Trees

Joan M. Lucas, State University of New York - Brockport

The binary tree is a fundamental data structure for storing totally ordered data. The most common operation for restructuring a binary tree is the rotation. Rotations are used in many binary search tree algorithms to achieve a balanced tree, which allows highly efficient access to the data in the tree. The combinatorial properties of rotations in binary trees have been extensively studied. Many authors have advanced definitions for restricted versions of the general rotation operation. In this paper we unify several previous results regarding restricted forms of rotations in binary trees under a general framework concerning the locality of consecutive rotations in a series of rotation operations. This framework utilizes the equivalence of n -node binary trees to triangulations of an $(n+2)$ -node convex polygon. Using this framework we introduce a new definition of restricted rotations and we show that any binary tree can be transformed into any other binary tree under this definition, and that the number of rotations required is bounded above by $n-3$ when the initial rotation can be freely chosen, and by $n-3+\lceil \log(n) \rceil$ when the initial rotation is specified by an adversary. This bound is very close to the $2n-6$ upper bound on general rotation distance, despite the severe locality restriction imposed. One localization restriction on consecutive rotations has been shown to be closely connected to other combinatoric structures, such as Thompson's group F .

Keywords: binary trees, rotation distance

186 Some properties of graphs in (a,b) -linear classes

Leonardo Silva de Lima, Nair Maria Maia de Abreu, Universidade Federal do Rio de Janeiro; Patricia Erthal de Moraes, Universidade Federal Fluminense; and Christine Sert, Pontificia Universidade Católica do Rio de Janeiro

We characterize the (a,b) -linear class of graphs as a function of the average degree of G . Besides, we prove, under certain conditions, that every graph with n vertices and m edges belongs to only one (a,b) -linear class. We present an algorithm which builds graphs with maximum vertex connectivity and maximum edge connectivity among all graphs in its class and show that the vertex connectivity and the edge connectivity for these graphs are equal to either $2a-1$ or $2a$. Finally, when G is a forest, we prove that $a=1$ and b is the number of its trees.

Keywords: graphs, vertex, edge and algebraic connectivities

187 On the Integer-magic Spectra of Maximal Outerplanar Graphs

Sin-Min Lee, Richard M. Low, San Jose State University; and Yong-Song Ho, Nan-Chiau High School

For $k \geq 2$, a graph $G = (V, E)$ is called Z_k -magic if there exists a labeling $f: E(G) \rightarrow Z_k$ such that the induced vertex set labeling $j^+: V(G) \rightarrow Z_k$, defined by $j^+(v) = \sum_{(u,v) \in E(G)} f(u,v)$ where $(u,v) \in E(G)$, is a constant map. In this paper, we investigate $\{k: G \text{ is } Z_k\text{-magic}\}$ for maximal outerplanar graphs G .

188 Constructions of Rectangular Designs

Kazuhiro Ozawa, Miwako Mishima, Gifu University; Shinji Kuriki, Osaka Prefecture University; and Masakazu Jimbo, Keio University

A *rectangular design* introduced by Vartak (1955) is a 3-associate partially balanced incomplete block (PBIB) design based on rectangular association scheme. In a rectangular design, $v = mn$ points are arranged in a rectangle of m rows and n columns, and the design consists of b blocks of size k ($k < v$). Each point is replicated r times, and with respect to each point, the *first associates* are the other $n-1$ points of the same row, the *second associates* are the other $m-1$ points of the same column and the remaining $(m-1)(n-1)$ points are the *third associates*, and any pair of points which are i th associates is repeated A_i ($i = 1, 2, 3$) times. Then $vr = bk$ and $(n-1)r_1 + (m-1)r_2 + (m-1)(n-1)r_3 = (k-1)r$ hold between the parameters.

In this talk, we will show some constructions of rectangular designs and a table of rectangular designs obtained from those constructions.

Keywords: balanced incomplete block design, rectangular design, symmetric balanced nested design

189 Realizability Results Involving Two Connectivity Parameters

L. William Kazmierczak¹ and Charles Suffel, Stevens Institute of Technology

There are networks that can be modeled by simple graphs, where edges are perfectly reliable but nodes are subject to failure, e.g. hardwired computer systems. One measure of the "vulnerability" of the network is the connectivity K of the graph. Another, somewhat related, vulnerability parameter is the component order connectivity K_k , i.e. the smallest number of nodes that must fail in order to ensure that all remaining components have order less than some value k . For this talk we present necessary and sufficient conditions on a 4-tuple (n, k, a, b) for a graph G to exist having n nodes, $K = a$, and $K_k = b$. Sufficiency of the conditions follows from a specific construction described in our work. Using this construction we obtain ranges of values for the number of edges in a graph having n nodes, $K = a$ and $K_k = b$ thereby obtaining sufficient conditions on the 5-tuple (n, e, k, a, b) for a graph to exist having n nodes, e edges, $K = a$, and $K_k = b$. In a limited number of special cases, we show the conditions on (n, e, k, a, b) to be necessary as well.

Keywords: connectivity, component order connectivity, realizability, vulnerability

190 On a Problem of Erdos, Jacobson and Lehel

Michael Ferrara¹, Ron Gould¹ and John Schmitt, Emory University

Let $1r$ be an n -element graphical sequence, and $a(1r)$ be the sum of the terms in $1r$. For any graph H and integer $n \geq |V(H)|$, we wish to determine the smallest m such that any n -term graphical sequence $1r$ having $a(1r) \leq m$ has a realization containing G as a subgraph. Denote this value m by $a(H, n)$. Here we determine the value m for the graph K_t - the complete multipartite graph with t partite sets each of sizes. Additionally, we give a purely graph theoretic proof of a conjecture of Erdos, Jacobson and Lehel that $a(K_t, n) = (t-2)(2n+1-t) + 2$, which was solved affirmatively by Li, Song and Luo in 1998 using linear algebraic methods.

191 Symplectic Maps and Generalizations of the Toda Lattice

Michelle DeDeo, University of North Florida

Toda originally designed his lattice and a type of canonical transformation, or symplectic mapping, known as the dual transformation to describe phenomena in solid-state lattice theory. Toda constructed a linear one-dimensional lattice of particles and strings that obeyed Hooke's laws and applied a canonical transformation to that system. That transformation, or mapping, was called the dual transformation. The term "dual" was typically used to emphasize the essential ingredient in the mapping: particles "became" springs and springs "became" particles in the new, or dual, system. The mapping was later applied to springs with non-linear potentials.

We hope to construct particular Hamiltonian systems associated to connected graphs embedded in Euclidean n -dimensional space with the standard metric. By applying the dual transformation to this system, we hope to extend the one-dimensional dual transformation associated to the Toda lattice. We also hope to discuss some applications to solid-state lattice theory.

192 Resolvable t -Designs

Reinhard Laue, University of Bayreuth

Orbits of some group of automorphisms of Platonian or Archimedean Solids on the sets of their vertices are selected to form a partition of the set of all vertices into blocks of equal size. If the full automorphism group of the solid is embedded into $PSL(2, q)$ then the orbit of one embedding under $PSL(2, q)$ corresponds to a family of partitions that form a resolvable design. There result several infinite families of resolvable 3-designs. These designs can be visualized by patterns on the solids which is done for some examples. Some of the known Steiner 5-designs are composed of such 3-designs and, thus, are easily seen to be resolvable.

Keywords: Resolvable Designs, Group Orbits, 3-Designs

193 When Bad Things Happen to Good Rankings; ILP Methods for Finding Minimum Feedback Arc Sets

Gregory Dufore^{*} and Darren A. Narayan, Rochester Institute of Technology

We consider the following question: Given a set of n players in a round robin tournament, what is the smallest sized tournament for which there exists an optimal ranking where all of the original n players ranked wrong with respect to each other? We investigate this problem using methods from graph theory and integer programming. Given an acyclic digraph D we seek a smallest sized tournament T that has D as a minimum feedback arc set. The reversing number of a digraph, $r(D)$ equals $|V(T)| - |V(D)|$. Isaak studied the case where D itself is the acyclic tournament T_n and used an integer programming formulation to establish lower bounds for $r(T_n)$. We investigate $r(T_n)$ when $n = 2^k + 2^{k-3}$ and prove that the known lower bounds for this class of tournaments are best possible. In addition we will examine methods for extending known results to obtain new reversing numbers.

194 Another Ramsey type problem

Henry Liu^{*} and Robert Morris, The University of Memphis

We prove the following Ramsey type result. *For any 2-colouring of the edges of K_n , where $n \geq 9$, we can find a monochromatic, 3-connected subgraph on at least $n - 4$ vertices.* We then mention some open questions and known results which generalize to k -connectedness.

Keywords: k -connected

195 Complement of Fibonacci Cube in Hypercube

Lan Wang, University of Puerto Rico

Let $CF(n)$ be the induced subgraph of the hypercube consisted by those vertices whose coordinate contains at least one pair of consecutive 1s. This graph is the complement of the Fibonacci cube in the hypercube. We present some properties of $CF(n)$.

196 A Combinatorial Interpretation of the Motzkin Triangle

Barbara Tankersley, Howard University

The Motzkin sequence 1, 1, 2, 4, 9, 21, ... has several combinatorial interpretations. For instance, the Motzkin sequence counts the number of ordered trees with the out degree of every vertex at most 2. We consider the lower triangular matrix obtained via the Hankel matrix. The Hankel matrix of a sequence $a_0, a_1, a_2, a_3, \dots$ is an infinite matrix $H = (h_{n,k})_{n,k=0}^{\infty}$ such that $h_{n,k} = a_{n+k}$. If H is positive definite then $H = LDU$ where L is the lower triangular matrix with all ones on the main diagonal, $U = L^T$, and D is the diagonal matrix. We use a similar tree representation to give a combinatorial interpretation for the Motzkin triangle.

Friday, March 12, 2004

8:00am	Registration - Grand Palm Room			
	Live Oak Pavillion			
	Room A	RoomB	Roome	RoomD
8:20am	201: P. Lisonek	202: A Hobbs	203: J. Limbupasiriporn	204: F. Hadlock
8:40am	205: M. Reid	206: A. Hubenko	207: F. Ruskey	208: W. Wallis
9:00am	209: G. Lam	210: E. Morgan	211: Y. Kaneko	212: F. Sagols
9:30am	Invited Speaker: ??? - Grand Palm Room			
10:30am	Coffee			
10:50am	213: T. Seidel	214: J. Bode	215: M. Buratti	216: D. Naor
11:10am	217: N. Carnes	218: J. Hattingh	219: L. Cummings	220: S. Moradi
11:30am	221: N. Cameron	222: R. Rubalcaba	223: J. Fukuyama	224: J. Werman
11:50am	225: S. Heubach	226: N. Clarke	227: R. Morris	228: W. Grundlingh
12:10pm	229: P. Guan	230: L. Beasley	231: H. Lefmann	232: S. Nanda
12:30pm	Lunch (on your own)			

Coffee and refreshments will be served in the registration area of the Grand Palm Room.

201 Binary caps with many free pairs of points

Petr Lisonek, Simon Fraser University

A cap in $PG(n, q)$ is a set S of points such that no three points of S are collinear. We say that two points $x, y \in S$ form a "free pair" if any plane containing x and y intersects S in at most three points. In the language of coding theory (viewing S as columns of a parity check matrix of a q -ary linear code of distance at least 4) this condition means that $\{x, y\}$ is not a subset of the support of any codeword of weight 4.

Given m, n and q we ask what is the maximum number of free pairs of points in a cap of cardinality m in $PG(n, q)$. In this talk we are concerned with the case $q = 2$. We review the known lower bounds and we improve the known lower bound in cases when $m \sim 2^{nt^2}$ using a construction based on BCH codes. The motivation for studying this problem comes from statistical experiment design where free pairs of points are called "clear 2-factor interactions."

Keywords: caps, finite projective spaces, BCH codes, statistical experiment design

202 William T. Tutte, 1917-2002

Arthur M. Hobbs", Texas A&M University; and James G. Oxley, Louisiana State University

During the past year, we gathered information and wrote a paper for the American Mathematical Society Notices (March 2004 issue) reviewing some of the many accomplishments of William T. Tutte. We made some surprising discoveries about his life and work; in this talk we will briefly describe his work and describe some of our discoveries.

203 Codes of Paley Graphs

Jirapha Limbupasiriporn, Clemson University

For any prime power n with $n \equiv 1 \pmod{4}$, the Paley graph $P(n)$ of order n has the finite field $GF(n)$ as vertex set and two vertices are adjacent if the difference is a non-zero square in $GF(n)$. The Paley graph is a strongly regular graph of type $(n, \frac{n-1}{2}, \frac{n-5}{4}, \frac{n-1}{4})$ and is isomorphic to its complement.

If A is the adjacency matrix of $P(n)$, the row span of A over the finite field $GF(p)$ forms a linear code of length n and of dimension $\frac{n-1}{2}$ provided that $p \nmid n-1$.

In this talk, we will examine the minimum weight of the binary code of the Paley graph and its dual. If n is a prime and $n \equiv 1 \pmod{8}$, the binary code of $P(n)$ is a quadratic residue code with parameter $[\frac{n-1}{2}, \frac{n-5}{4}, \frac{n-1}{4}]_2$ where $n \equiv 1 \pmod{8}$. We will show that if q is a prime power and $q \equiv 1 \pmod{8}$, the binary code of $P(q^2)$ is a $[\frac{q^2-1}{2}, \frac{q^2-5}{4}, \frac{q^2-1}{4}]_2$ code and its dual is a $[\frac{q^2-1}{2}, \frac{q^2-1}{4}, \frac{q^2-3}{4}]_2$ code. We will also briefly discuss permutation decoding for these codes.

This is joint work with J.D. Key of Clemson University.

Keywords: Paley graphs, binary codes

204 Manhattan Graphs

Frank Hadlock", Tennessee Technological University and Florida Atlantic University; and Frederick Hoffman, Florida Atlantic University

Manhattan distance is the rectilinear distance induced in a grid graph as the sum of the absolute differences in the x and y coordinates. This notion has been extended to unweighted trees by coordinatizing the vertices. For a tree with n terminal vertices, $m = \text{int}((n+1)/2)$ integer coordinates suffice so that the tree distance is the sum of the absolute differences in the m coordinates. This article presents a new way of coordinatizing trees to realize a Manhattan distance on the tree, and extends the notion to unweighted graphs by constructing a forest of subtrees. Each tree factor is coordinatized to obtain a Manhattan tree and the product of the tree coordinates over the forest is assigned to the graph vertices. Graph distance can then be calculated as the minimum Manhattan distance over any tree factor. Given a tree factor, a current vertex and a destination vertex, the next vertex on the shortest path to the destination is available from coordinate calculations, as well as the information as to whether a given vertex lies on the shortest path.

205 Tiling with Polyominoes and Boundary Words

Michael Reid, University of Central Florida

In 1990, Conway and Lagarias gave a technique for analyzing some types of tiling problems by using boundary words. This method transforms these tiling questions into questions involving finitely presented groups. Many basic problems involving finitely presented groups are known to be algorithmically unsolvable. We give a strategy for approaching the resulting questions in finitely presented groups and give examples where the boundary word method gives interesting results.

Keywords: polyomino, tiling, boundary words

206 On a cyclic property of directed graphs

A. Hubenko, University of Memphis

In his 1999 paper Adam formulated 10 cyclic properties of directed graphs and established the connection between most of these properties. Heteyi in his 2001 paper answered all but two of the open questions of Adam. One of these open questions can be formulated as follows. Let D be a digraph, such that for every two vertices x and y there is a cycle C containing both x and y . Is it true that there is an arc e in D such that for every vertex x there is a cycle C containing both x and e ? We answer this question for several special digraphs D . Our main result shows that if D is a bitournament (oriented complete bipartite graph) then such arc e exists.

207 Counting strings over Z_4 with given symmetric function evaluations

Frank Ruskey and C. Robert Miers, University of Victoria

Let a be a string over Z_4 . The j -th elementary symmetric function evaluated at a is denoted $T_j(a)$. We study the cardinalities $S_4(n; r_1, r_2, \dots, r_4)$ of the set of length n strings for which $T_i(a) = r_i$. The profile (k_1, k_2, k_3) of a string a is the sequence of frequencies with which each letter occurs. The profile of a determines $T_j(a)$, and hence S_4 . Let $f_n : Z \setminus \{1\} \rightarrow Z_4[Z] \text{ mod } z^{2^n}$ be the map that takes $(k_1, k_2, k_3) \text{ mod } 2^n + 1$ to the polynomial $1 + T_1 z + T_2 z^2 + \dots + T_{2^n-1} z^{2^n-1}$. We show that f_n is a group homomorphism and determine its kernel. The range of f_n is described. These results are used to "efficiently" compute $S_4(n; T_1, T_2, \dots, T_t)$. Some extensions to strings over Z_{2^d} are presented.

Keywords: Symmetric function, generating function, enumeration, polynomial, multinomial coefficient, homomorphism, kernel

208 Overlarge sets of one-factors

W.D. Wallis, Southern Illinois University

We address the following problem: What is the spectrum of k , such that there is a set of k one-factors of the complete graph K_{2n} which between them cover all edges, such that no $2n - 1$ of them form a one-factorization of K_{2n} .

209 Non-inclusions of Archimedean, Laves, and 2-Uniform Infinite Lattices

George Lam, Matthew Lad, Sajod Moradi, Brian Yagoda, and John Wierman, The Johns Hopkins University

Our study of Archimedean, Laves, and 2-Uniform lattices is motivated by applications to models from probability, combinatorics and mathematical physics - bond and site percolation, first-passage percolation and self-avoiding walks. An Archimedean lattice is a graph of a regular tiling that is vertex-transitive. A regular tiling is a tiling of the plane that consists entirely of regular polygons. Since the Archimedean lattices are planar graphs, each has a planar dual graph called a Laves lattice. 2-Uniform lattices are regular tilings with two classes of vertices. In obtaining bounds for the critical probability of bond percolation for these lattices, we have found inclusions among and between the Archimedean, Laves and 2-Uniform lattices. To prove that all possible inclusions have been found, we utilized previously existing criteria and developed new techniques to prove non-inclusions in all other cases. A number of more difficult cases required individual reasoning. We will discuss these methods and provide a few illustrative examples.

Keywords: Archimedean lattice, vertex-transitive, subgraph, percolation, non-inclusion

210 Strong (t, r) -regularity

Robert Jamison, Clemson University; Peter Johnson, Auburn University; Lisa Markus, De Anza College, and Evan Morgan, Louisiana State University

A graph on $n > t - 1$ vertices is strongly (t, r) -regular iff for every set of t vertices of the graph, the union of their open neighborhoods has cardinality r . ("Strongly" distinguishes this property from the similarly defined property of (t, r) -regularity in which the requirement is on independent sets of vertices, only.) Thus the clique $K(r)$ on r vertices is strongly (t, r) -regular for every $t = 2, \dots, r$.

Although we will make some observations about the cases $t > 2$, the case $t = 2$ is our main focus. A regular strongly $(2, r)$ -regular graph which is not $K(r)$ is a certain kind of strongly regular graph, and a good deal is known about these. The outstanding question at present is: Is there, for some r , a non-regular strongly $(2, r)$ -regular graph? Building on earlier work by Haynes and Markus, we show that if there is, then $r > 10$.

Keywords: strongly regular, open neighborhood, probe interval graphs

211 On Locating Sources of Optimal File Transfer Problem

Yoshihiro Kaneko, GIFU University; and Tsuyoshi Ogata, Dai Nippon Printing Co., Ltd.

An optimal file transfer here is the way that we transfer copies of some necessary information files with the minimum total cost on a given network model. Our network model is a weighted directed graph with vertex costs, vertex demands and arc costs. On such model, we can duplicate the file at any vertex from sources where original files are given from the outside. Under such condition we consider how to transfer files, that is, how many copies are made at vertices and are transmitted at arcs so that the sum of making and transmitting those copies are the minimum. This problem is generally NP-hard. However under some condition it is polynomially solvable in the time complexity of $O(n^3)$ for the vertex number n since it is a generalization of a single shortest path problem and a minimum spanning tree problem. Every optimal file transfers depend on sources, that is, the cost of optimal file transfers may differ if we change sources. In this talk, we show that we can solve our file transfer problem with the same time complexity even if we can choose sources with a fixed number. The degree constraint minimum spanning tree plays a key role to solve such problem.

Keywords: Network theory, vertex cost, vertex demand, arc cost, degree constrained minimum tree, polynomial time algorithm

212 A new proof of $D_{\Delta} \rightarrow Y$ reducibility of three terminal planar graphs

Isidoro Gitler and Feliu Sagols, Cinvestav Mexico

In 1966, G. Epifanov proved the Akers-Lehman conjecture, that any planar graph with two terminals can be reduced by means of $D_{\Delta} \rightarrow Y$ transformations to a single edge, the last two nodes being the original two terminals. In 1991 I. Gitler proved the three terminal planar conjecture. We give a new proof of the 3-terminal case by proving that any planar graph with three terminals can be reduced to a path of length three with vertex set being the original three terminals, using these operations on the medial graph.

Keywords: Delta-Wye reductions, medial graphs, planar embeddings and topological graph theory

213 Ovals in the Projective Plane of Order 25

Spyros S. Magliveras, Tanya E. Seidel, and Michal Sramka, Florida Atlantic University

We provide a breakdown of the collineation group of the Hall Projective Plane of order 25 in order to construct ovals. We construct all ovals preserved by permutations in the group of collineations by means of fusing cycles of prime-order elements.

Keywords: arcs, automorphism group, collineation, ovals, projective plane

214 Minimum regular rectilinear plane graph drawings with fixed numbers of edge lengths

Jens-P. Bode, Technische Universitat Braunschweig

Every planar graph can be drawn in the plane with nonintersecting edges (plane drawing). Moreover, this is possible using straight line segments for all edges (rectilinear plane drawings). As further restrictions one may ask the edges to be of equal length or to use integral lengths only. Here we will consider r -regular graphs and ask for the smallest examples using at most a given number s of edge lengths in its rectilinear plane drawing. Common work with Heiko Harborth.

215 Optimal optical orthogonal codes and difference families of block size 4

Julian Abel and Marco Buratti, University of Rome

A $(v, k, 1)$ difference family (DF) is a set F of k -subsets (blocks) of Z_v with the property that $\cup_{B \in F} B$ (the list of differences from F) covers each nonzero element of Z_v exactly once. The obvious necessary condition for its existence is that $v = 1 \pmod{k^2 - k}$. If this congruence is not satisfied one may ask for a set F of k -subsets of Z_v such that $\cup_{B \in F} B$ has no repeated elements and covers the maximum possible number of elements of Z_v . Such an F is called a $(v, k, 1)$ optimal optical orthogonal code (OOOC).

A $(v, k, 1)$ -DF generates a cyclic 2-design with the same parameters. This is the reason for which we are mainly interested in DFs rather than OOOCs. In spite of this, we have proved that certain $(p, 4, 1)$ -OOOCs with p a prime give rise to $(rp, 4, 1)$ -DFs where r is the remainder of the Euclidean division of p by 12. This allowed us to obtain some progress on $(v, 4, 1)$ -DFs.

Keywords: Cyclic 2-design, difference family, optimal optical orthogonal code

216 Improving Universal Formulas for Percolation Thresholds

Dora P. Naor- and John C. Wierman, The Johns Hopkins University

Universal formulas to predict values for percolation thresholds of periodic graphs make use of certain features of lattice graphs such as dimension and average degree. A relationship exists between the average and second-moment of the degree of a graph and the average degree of its line graph, as well as between the bond model percolation threshold of a graph and the site model percolation threshold of its line graph. Using these relationships and incorporating them into existing universal formulas we can create new formulas that improve the accuracy of percolation threshold predictions.

Keywords: percolation threshold, line graph, average degree

217 Some Antiautomorphisms of Mendelsohn Triple Systems

Neil P. Carnes, McNeese State University

A cyclic triple, (a, b, c) , is defined to be the set $\{(a, b), (b, c), (c, a)\}$ of ordered pairs. A Mendelsohn triple system of order v , $MTS(v)$, is a pair $(M, \mathcal{b})$, where M is a set of v points and $\mathcal{b}$ is a collection of cyclic triples of pairwise distinct points of M such that any ordered pair of distinct points of M is contained in precisely one triple of $\mathcal{b}$. An antiautomorphism of a Mendelsohn triple system is a permutation of M which maps b to b^{-1} , where $b^{-1} = \{(c, b, a) | (a, b, c) \in \mathcal{b}\}$. We give conditions for the existence of Mendelsohn triple systems of order v admitting certain types of antiautomorphisms.

Keywords: Mendelsohn triple system, antiautomorphism

218 On weakly connected domination in graphs I I

G.S. Domke, J. H. Hattingh*, Georgia State University; and L.R. Markus, De Anza College

A weakly connected dominating set for a connected graph is a dominating set D of vertices of the graph such that the edges not incident to any vertex in D do not separate the graph. The weakly connected domination number of G , denoted $\gamma_{wc}(G)$, is $\min\{|S| \mid S \text{ is a weakly connected dominating set of } G\}$. We characterize graphs G for which $\gamma(H) = \gamma_{wc}(H)$ for every connected induced subgraph H of G , where γ is the domination number of a graph. We provide a constructive characterization of trees T for which $\gamma(T) = \gamma_{wc}(T)$. Lastly, we constructively characterize the trees T in which every vertex belongs to some weakly connected dominating set of cardinality $\gamma_{wc}(T)$.

Keywords: weakly, connected, domination

219 Systematic Comma-Free Codes

Larry Cummings, University of Waterloo

Comma-free codes are overlap-free codes and so are one solution to the problem of correcting framing errors. Codewords of a systematic comma-free code have fixed positions which establish synchronization and arbitrary positions to carry information. The first systematic codes for synchronization were proposed by E.N. Gilbert.

A test for determining whether a systematic block code has overlaps is given. This allows testing systematic codes for comma-freeness. A easily constructed systematic comma-free code of every positive word length n and redundancy $\frac{1}{2} + \frac{1}{n}$ is given.

Keywords: systematic codes, comma-free codes

220 Inclusions of Archimedean, Laves, and 2-Uniform Infinite Lattices

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Our study of Archimedean, Laves, and 2-uniform lattices is motivated by applications to models from probability, combinatorics and mathematical physics - bond and site percolation, first-passage percolation and self-avoiding walks. An Archimedean lattice is a graph of a regular tiling that is vertex-transitive. A regular tiling is a tiling of the plane that consists entirely of regular polygons. Since the Archimedean lattices are planar graphs, each has a planar dual graph called a Laves lattice. 2-Uniform lattices are regular tilings with two classes of vertices. In percolation theory, if one lattice is included in another, the percolation threshold of the first is larger than that of the second. This "containment principle" can be used to derive bounds for percolation thresholds, but until recently has not been systematically exploited. We find numerous inclusions among and between Archimedean, Laves, and 2-uniform tilings, and use the inclusions to derive percolation threshold bounds. We illustrate many inclusions and discuss the resulting percolation threshold bounds.

Keywords: Archimedean lattice, vertex-transitive, subgraph, percolation, inclusion, percolation threshold

221 Elements of (Pseudo) Order Two in the Riordan Group

Naomi T. Cameron, Occidental College; and Asomoah Nkwanta, Morgan State University

A Riordan matrix is an infinite, lower triangular matrix whose columns are generating functions. The study of the Riordan group structure (induced by matrix multiplication) is of interest in its own right, but it can also lead to the discovery and proof of nice combinatorial identities. In this talk, I will investigate the combinatorial aspects of involutions in the Riordan group. In particular, I will describe a class of Riordan matrices having *pseudo-order* two (which includes Pascal's triangle), and illustrate how to obtain proofs of combinatorial identities, such as

$$4n-m \binom{n}{m} = \sum_{k=0}^n \frac{(k+m+1)}{n+1} \frac{(2n+2)}{2m+1} \binom{n-k}{n-k}$$

Keywords: lattice paths, Catalan numbers, generating functions, groups

222 Some New Facts about The Hamming Space

Junichiro Fukuyama, Indiana State University

Let U, U_1, U_2 be non-empty subsets of $\{1, \dots, n\}$, where $n, m \in \mathbb{Z}^+, m \leq n$. Denote by $\oplus$ the symmetric difference between $t_i \in (f; j)$. The *sparsity* of U is defined by $\sigma(U) = \ln(|U|) - \ln(|U|)$. It has been shown recently that the minimum Hamming distance $\min_{t_1 \oplus t_2 \in U} |t_1 - t_2|$ between u_i is $\geq \frac{1}{2} \sigma(U)$ whenever $\sigma(U) \geq m$ for some constant $\epsilon \in (0, 1)$. It is a solution to the conjecture called *The Isoperimetric Problem for m-sets*.

It seems to take deeper understanding on the topological structure of The Hamming Space, in order to obtain a better bound than the above. In this regard, it is significant to investigate

$$\text{Ext}(U, d) = \{s \in \{1, \dots, n\} \mid \exists t \in U, t \oplus s \in U\}, \text{ and}$$

$$\sigma(U, d) = \{s \in \{1, \dots, n\} \mid \exists t \in U, t \oplus s \in U, |s| \leq d\},$$

for a positive real number d less than m . They are called the *d-extension* and *d-cover* of U , respectively. The minimum Hamming distance between U_i is at most d , if and only if $\text{Ext}(U_1, d) \cap \text{Ext}(U_2, d) \neq \emptyset$, if and only if $\sigma(U_1, d) \cap \sigma(U_2, d) \neq \emptyset$.

In this paper, we will show that a large d -extension means a large d -cover. More precisely, the results include

1. $\sigma(U, d) \leq \sigma(\text{Ext}(U, d))$, and
2. $\sigma(U, d) \ll 1$ when $(\sigma(\text{Ext}(U, d)) \leq m) \wedge (d \gg 1)$.

The second result means that when the d -extension of U is reasonably dense and d is asymptotically larger than m , the d -cover of U forms a majority in $(\text{Ext}(U, d))$.

222 Domination null and packing null vertices of a graph

Peter D. Johnson Jr., Robert Rubalcaba, Auburn University; and Matt Walsh, Indiana-Perdue University at Fort Wayne

A vertex v of a finite simple graph G is domination null provided $f(v) = 0$ for all minimum fractional dominating functions f . A vertex v is packing null provided $g(v) = 0$ for all maximum fractional closed neighborhood packing functions g . We raise a number of questions about such vertices and provide very few answers.

Keywords: fractional domination, complementary slackness

224 Estimating Percolation Thresholds Using the Substitution Method

John C. Wierman and Rulian Cheng, The Johns Hopkins University

Bond percolation models are infinite random graph models, in which each edge is retained with probability p , $0 \leq p \leq 1$, independently of all other edges. The percolation threshold P_c is the value of p such that for $p > P_c$ there exists an infinite component with positive probability and for $p < P_c$ all components are finite almost surely. Percolation models are widely studied and applied in the physical sciences and engineering, with the percolation threshold representing a phase transition point.

The exact value of the percolation threshold is exactly known for very few infinite graphs. There are several methods of estimating percolation thresholds in the physics literature, including Monte Carlo simulation, series expansions, and renormalization methods. This talk describes preliminary investigations of a new estimation approach which shows some promise. It is based on the substitution method, which computes bounds for percolation threshold bounds by solving equations of probabilities of up-sets in a partition lattice derived from the percolation model.

Keywords: percolation, threshold, partition lattice, up-set

225 Counting rises, levels and drops in compositions with parts in a set A

Silvia Heubach, California State University Los Angeles, and Toufik Mansour, University of Haifa

A composition of $n \in \mathbb{N}$ is an ordered collection of one or more positive integers whose sum is n . A palindromic composition of n is a composition in which the summands are the same in the given and in reverse order. The number of summands is called the number of parts. We derive the generating function for the number of parts, rises (a summand followed by a larger summand), levels (a summand followed by itself) and drops (a summand followed by a smaller summand) for a general set A , and are able to derive all previously known results as special cases. We also derive new results for Carlitz compositions (no adjacent summands can be the same) and for partitions.

Keywords: Composition, Palindromic compositions, Carlitz compositions, partitions, generating functions

226 A Game of Cops and Robber Played with Partial Information

Nancy E. Clarke, Acadia University

Two versions of the Cops and Robber game are introduced in which the cops play with partial information provided via selected vertices of a graph. The robber has perfect information. When the partial information includes only the robber's position, we give bounds on the amount of information required by a single cop to guarantee the capture of a robber on a tree. When the partial information also includes information about the robber's direction, we give bounds on the amount of information required by a cop to capture a robber on a copwin graph.

Keywords: game, cop, partial information, pursuit, graph

227 Results on Frankl's Conjecture

Robert Morris, University of Memphis

Frankl's famous conjecture states that given any finite union-closed family of sets, not all empty, there exists an element which is contained in at least half the sets. The conjecture has been open for 25 years, and very little is known. We will present some partial progress, extending work of Poonen, Vaughan, Gao and Yu, and will pose several questions and conjectures.

Keywords: Frankl conjecture, FC-families, Union-Closed families

228 The lottery problem: A graph theoretic and computational approach

Werner Gröndlingh, University of Victoria

A variety of lotteries are operational across the world today. Suppose a lottery scheme consists of randomly selecting a winning n -set from a universal m -set, while a player participates in the scheme by purchasing a playing set of any number of n -sets from the universal set prior to the draw, and is awarded a prize if k (or more) numbers in the winning n -set match those of at least one of the player's n -sets in his/her playing set ($k \leq n \leq m$). In this talk, we study the well-known combinatorial *lottery problem* (introduced in 1960), which inquires the following: "How many n -sets is needed in a playing set in order to be guaranteed a certain prize?" Such a set is called a lottery set. We also discuss the notion of a *lottery graph* which transcribes the problem to the well-known field of graph domination theory. Some results on lottery numbers are presented using both analytical arguments and computer searches.

Keywords: Lottery problem, lottery, combinatorial designs

229 Minimal Critical Squarefree Subgraph OF Hypercube

Puhua Guan, University of Puerto Rico

Let $F(n)$ be the class of subgraphs of the hypercube $H(n)$ such that for any graph G belongs to $F(n)$, G does not contain a square, adding any edge of $H(n)$ to G that is not in G , the result graph will contain at least one square. What is the minimum number of edges of the graphs in $F(n)$? We present a construction to make such graphs with $\frac{n}{2}$ (the number of edges of $H(n)$).

230 Tournaments, Primitivity and Preservers

LeRoy B. Beasley, Utah State University

We investigate additive mappings on the set of digraphs which map tournaments to tournaments, primitive digraphs to primitive digraphs, 2-primitive digraphs/tournaments to 2-primitive digraphs/tournaments, etc.

Keywords: Tournament, primitive, preserver

231 Triangles of Large Minimum Area in the d-Dimensional Unit-Cube

Hanno Lefmann, TU-Chemnitz

We consider a variant of Heilbronn's triangle problem by asking for fixed dimension $d \geq 2$ for a distribution of n points in the d -dimensional unit-cube $[0, 1]^d$ such that the minimum (2-dimensional) area of a triangle among these n points is maximal. Denoting this maximum value by $ti_{d\text{-line}}(n)$ and $ti_{d\text{-line}}^{off}(n)$ for the off-line and the on-line situation, respectively, we will show that $d \cdot (\log n) 1/(d-1) / n^{2/(d-1)} \leq ti_{d\text{-line}}(n) \leq C_1 / n^{2/d}$ and $c_2 / n^{2/(d-1)} \leq ti_{d\text{-line}}^{off}(n) \leq C_2 / n^{2/d}$ for constants $c_1, c_2, C_1, C_2 > 0$ which depend on d only.

232 An Algorithm for a 2-Disk Fault-Tolerant Array with (Prime -1) Disks

Sanjeeb Nanda" and Narsingh Deo, University of Central Florida

In recent years commercial RAID (Redundant Arrays of Inexpensive Disks) systems have gained considerable appeal due to their enhanced I/O bandwidths, increased capacities and a drastic reduction in cost. However, the continued demand for larger capacities, while keeping the cost low, has led to the use of larger arrays with compromised disk quality. This has challenged RAID systems to offer better fault-tolerance without sacrificing performance or space. In this paper, we present an algorithm for recovery from two random disk failures in an array of $(P - 1)$ disks, where P is a prime number. The algorithm relies on exclusive OR parity bit computed uniformly across all the disks. We give a constructive proof of the correctness of the algorithm and demonstrate the key steps with examples.

Keywords: Arrays, Disks, Exclusive OR, Fault-tolerance, Parity