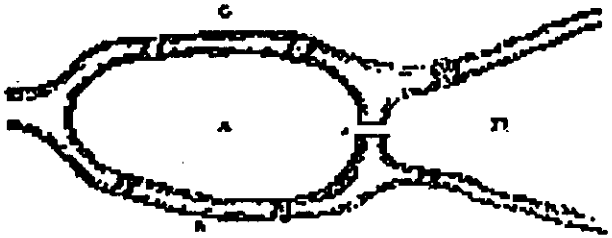
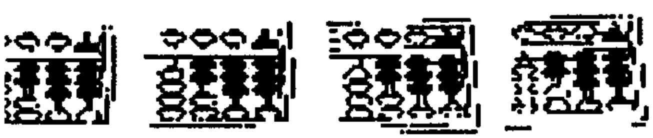
$$\frac{1}{n}\sum_{d|(b_1,\ldots,b_r)} \mu(d)\,\frac{(\frac{n}{d})!}{(\frac{b_1}{d})!\cdots(\frac{b_r}{d})!}$$





$$\frac{1}{n}\sum_{d|(b_1,\ldots,b_r)} \mu(d)\,\frac{(\frac{n}{d})!}{(\frac{b_1}{d})!\cdots(\frac{b_r}{d})!}$$

Thirtieth Southeastern International Conference on Combinatorics, Graph Theory, and Computing

March 8-12, 1999

Program and Abstracts

Monday, March 8, 1999

	Live Oak Pavillion			Senate Chambers
	Room A	Room BC	Room D	Room 230 UC
8:00 am	Registration until 5:00, downstairs lobby of the University Center (UC), where coffee will be served.			
9:00 am	Opening Ceremonies: FAU Provost Richard Osburn FAU C.E. Schmidt College of Science Dean John Wiesenfeld			
9:30 am	Bressoud			
10:30 am	Coffee			
10:50 am	001 Bagga	002 Hedetniemi	003 Jayawardene	004 Sheng
11:10 am	005 Lawrencenko	006 McRae	007 Myers	008 Wu
11:30 am	009 Bajnok	010 Piazza	011	012 H. Li
11:50 am	013 Batten	014 McDougal	015	016 Lipman
12:10 pm	017 Miyamoto	018 Shreve	019	020 E. Cheng
12:30 pm	Lunch			
2:00 pm	Bressoud			
3:00 pm	Coffee			
3:20 pm	021 H.L. Cheng	022 Sugita	023 Tapia-Recilas	024 Narayan
3:40 pm	025 Bode	026 Moriya	027 Bierbrauer	028 Altman
4:00 pm	029 Harborth	030 Bartha	031 McIntyre	032 Godbole
4:20 pm	033 Szekely	034 Rall	035	036 Y-L Lai
4:40 pm	037 Wallach	038 Cherry	039 D.G. Hoffman	040 LaFollette
5:00 pm	041 Turgeon	042 Shen	043 W.E. Clark	044 Vernet
5:20 pm	045	046 Szwarcfiter	047	048 Williams
5:45 pm	Transportation back to motels (returns for reception about 6:15)			
6:15 pm	Reception, Board of Regents Room, 3rd floor, Administration Building			
	Transportation back to motels (TBA)			

Tuesday, March 9, 1999

	Room A	Room BC	Room D	Room SC
8:15 am	Registration 8:15-noon and 1:30-4:00, UC, 2nd floor lobby			
8:40 am	049 Zeleke	050 Grimaldi	051 Albertson	052 Fischer
9:00 am	Book Exhibits 9:00-5:00, Room 229			
9:00 am	053 Chen	054 Heubach	055 Moore	056 Golumbic
9:30 am	Trenk			
10:30 am	Coffee			
10:50 am	057 W.N. Li	058 Shull	059 Hollingsworth	060 Isaak
11:10 am	061 Ali	062 Mulder	063 Walls	064 Grieg
11:30 am	065 Havas	066 Jamison	067 Ginn	068 McNulty
11:50 am	069 Guan	070 Bogart	071 Hoang	072 Chappell
12:10 pm	073 Hernandez	074 Golumbic & Trenk	075 West	076 Karro
12:30 pm	Lunch			
2:00 pm	Aigner			
3:00 pm	Coffee			
3:20 pm	077 Chartrand	078 Barth	079 Mishima	080 Dunbar
3:40 pm	081 Raines	082 Boukaabar	083 Hishida	084 Phillips
4:00 pm	085 VanderJagt	086 Voss	087 Mackenzie	088 van der Merwe
4:20 pm	089 O'Donnell	090 Valdes	091 B. Li	092 Haynes
4:40 pm	093 Fink	094 McKenna	095 Zuanni	096 Cockayne
5:00 pm	097 Erwin	098 Ellis-Monaghan	099 Francel	100 Henning
5:20 pm	101 P. Zhang	102 Dean	103 Dobson	104 Mynhardt
5:40 pm				
5:45 pm	Transportation leaves for motels			
5:50 pm	Transportation leaves UC for Conference activities			
6:00 pm	TBA			

Wednesday, March 10, 1999

	Room A	Room BC	Room D	Room SC
8:15 am	Registration 8:15-noon and 1:30-4:00, UC, 2nd floor lobby			
8:30 am	105 Heckman	106 Fitzpatrick	107 Weinreich	108 Liu
8:50 am	109 Shields	110 Pfender	111 Balogh	112 Hilton
9:00 am	Book Exhibits 9:00-5:00, Room 229			
9:10 am	TICA Session			
9:30 am	Dinitz			
9:30 am	Coffee			
10:50 am	113 Gasarch	114 Hurlbert	115 Thomson	116 P.D. Johnson
11:10 am	117 Myrvold	118 Oellermann	119 Bohme	120 H-J Lai
11:30 am	121 Ostergard	122 Q. Yu	123 Hlineny	124 Schiermeyer
11:50 am	125 Rogers	126 Gyori	127 Lundgren	128 Brigham
12:10 pm	Conference Photograph			
12:30 pm	Lunch			
2:00 pm	McKay			
3:00 pm	Coffee			
3:20 pm	129 Hinz	130 Winters	131 Nagle	132 Elmallah
3:40 pm	133 Davis	134 Kelmans	135 Kirkpatrick	136 Slater
4:00 pm	137 George	138 Huff	139 Abueida	140 Seyffarth
4:20 pm	141 Sritharan	142 Rodger	143 Buratti	144 Carrington
4:40 pm	145 Colbourn	146 Pike	147 Chateauneuf	148 J Liu
5:00 pm	149 Calkin	150 Heuss	151 U. Leck	152 Trees
5:20 pm	153 Gargano	154 Bennett	155 V. Leck	156 Doherty
5:45 pm	Transportation leaves for motels			
5:45 pm	Transportation leaves from motels and UC to Sheraton for banquet			
7:00 pm	Conference Banquet at Sheraton (cash bar opens at 6:00, seating at 6:45)			
	Transportation back to motels			

Thursday, March 11, 1999

	Room A	Room BC	Room D	Room SC
8:15 am	Registration 8:15-noon and 1:30-4:00, UC, 2nd floor lobby			
8:40 am	161 Roblee	162 Saito	163 Tolman	164 Soifer
9:00 am	Book Exhibits 9:00-5:00, Room 229			
9:00 am	165 Pfaltz	166 Raychaudhuri	167 Montagh	168 Eggleton
9:30 am	Babai			
10:30 am	Coffee			
10:50 am	169 Montague	170 Horak	171 Shapiro	172 Hartnell
11:10 am	173 J.T. Lewis	174 Daven	175 Peart	176 Barbosa
11:30 am	177 D.D. Smith	178 Barovich	179 Egecioglu	180 Levit
11:50 am	181 Finizio	182 Cameron	183 Starling	184 Alsardary
12:10 pm	185 S.J. Lewis	186 Michael	187 Atici	188 Parker
12:30 pm	Lunch			
2:00 pm	Odlyzko			
3:30 pm	Coffee			
3:40 pm	229	230 N. Graham	231 Ding	232
4:00 pm	233	234 Fleming	235	236
4:30 pm	McKay			
6:00 pm	Transportation leaves for motels, returns 6:30 from motels to informal party			
6:15 pm	Informal conference party until 7:45 in Cafeteria patio area			
	Transportation back to motels			

Friday, March 12, 1999

	Room A	Room BC	Room D	Room SC
8:15 am	Registration 8:15-12:45, UC, 2nd floor lobby			
8:40 am	189 Chopra	190 Meyerowitz	191 Chinn	192 Taksa
9:00 am	Book Exhibits 9:00-11:00, room 229			
9:00 am	193 Carnes	194 K.L. Clark	195 Molina	196 Krzyzak
9:30 am	Vanstone			
10:30 am	Coffee			
10:50 am	197 Preece	198 Lam	199 Hochberg	200 Micikevicius
11:10 am	201 Miao	202 Shiu	203 Bargteil	204 Abdalla
11:30 am	205 Georges	206 Sun	207 Traldi	208 Kaneko
11:50 am	209 Motsumoto	210 S-M Lee	211 Rodriguez	212 Ghriga
12:10 pm	213 Carella	214 Moghadam	215 Markenzon	216 Amin
12:30 pm	217 Jones	218	219 Eroh	220 Kruskal
12:50 pm	221 Ealy	222	223	224 Wood
1:10 pm	225	226	227	228
	Transportation TBA			
7:00 pm	Informal after-dinner Survivor's Party at the home of Aaron Meyerowitz and Andra Schuver, 454 NE Third Street. Tell us if you need transportation.			
	Transportation back to motels			

Invited talks

Martin Aigner. University of Berlin. *Penrose Polynomials and the 4-Color Theorem.*
Tuesday 2:00 pm.

Lazlo Babai, University of Chicago. *Constructing Ramsey Graphs: an Erdos Challenge.*
Thursday, 9:30 am.

David Bressoud , Dewitt Wallace Professor, Macalester College, *The Story of the Alternating Sign Matrix Conjecture 1: determinants, plane partitions and symmetric functions,* and 2*: symmetric designs, Pfafffians, and the Yang-Baxter Equation..*
Monday, 9:30 am and 2:00 pm.

Jeff Dinitz, University of Vermont, *Coloring Block Designs.*
Wednesday, 9:30 am.

Brendan McKay, Australian National University. *Constructing Combinatorial Objects on the Computer.*
Wednesday, 2:00 pm.

Brendan McKay, Australian National University. **Special Public Lecture:** *Bible codes: Fact or Fallacy?*
Thursday, 4:30 pm.

Andrew Odlyzko, A.T.&T. Laboratories-Research. *Random Permutations, Computers and Quantum Gravity.*
Thursday, 2:00 pm.

Ann Trenk, Wellesley College. *Seventeen Years of Tolerance Graphs.*
Tuesday, 9:30 am.

Scott Vanstone, University of Waterloo and CERTICOM Corporation. *Recent Advances in Elliptic Curve Cryptography.*
Friday, 9:30 am.

1

Some Properties of Directed Polygon Visibility Graphs

Jay S. Bagga*, John W. Emert, J. Michael McGrew Ball State University

Given a simple closed polygon in the plane, its polygon visibility graph has the same vertices as those of the polygon, while the edges are those on the polygon, together with the internal chords (that is, chords which do not intersect with the exterior of the polygon). In an earlier paper the authors extended this notion to directed polygon visibility graphs, and studied some properties including bounds on the size of such digraphs. In this paper we consider a more general situation. We obtain bounds on the size of those directed polygon visibility graphs which can be formed from an arbitrarily oriented polygon. These bounds are functions of the number of polygon vertices, and the number of changes in orientation along the polygon. KEYWORDS: Visibility graphs, computational geometry.

2

Getting a Charge Out of a Graph

Stephen Hedetniemi*, Clemson University; Sandra Hedetniemi, Clemson University;
Peter Slater, Univ. of Alabama in Huntsville; Teresa Haynes, East Tennessee State University

Let S be a set of vertices of a graph $G = (V, E)$. The *charge* of S is defined to be $chg(S) = |N(S) \cap V - S| - |N(S) \cap S|$, where $N(S)$ denotes the open neighborhood of S. The *charge* of a graph G is defined as $chg(G) = max\{chg(S) : S \subseteq V\}$. In this preliminary paper we explore the problems of getting a maximum charge out of a graph G, including the determination of the computational complexity and the existence of linear algorithms for trees and series-parallel graphs.

3

Ramsey Numbers $r(C_6, G)$ for all Graphs G of Order less than six

Chula J. Jayawardene and Cecil C. Rousseau
The University of Memphis

Let $r(C_6, K_n)$ be the smallest integer N such that if a graph on N vertices contains no C_6, then its independence number is at least n. First we will find $r(C_6, K_5)$ and the Ramsey numbers $r(C_6, G)$ for all disconnected graphs on 5 vertices. Next using these we will find $r(C_6, G)$ for all graphs G of order less than six.

Key Words: Graph theory, Ramsey numbers

4

Cycle Free Probe Interval Graphs

Li Sheng*, Drexel University, PA

Interval graphs arose 40 years ago in connection with problems of genetics and scheduling and have given rise to a vast amount of mathematical work. A graph is an interval graph if an interval can be assigned to each vertex such that two vertices are adjacent if and only if their corresponding intervals have a nonempty intersection. A graph $G = (V, E)$ is a split interval graph if there is partition $V = \{V_1, V_2\}$ so that $G(V_1)$ is an interval graph and $G(V_2)$ is an independent set. We introduce a special case of split interval graphs called probe interval graphs, and present their modern applications in molecular biology. Finally, we give a characterization for a cycle free graph to be a probe interval graph.

Keywords: Interval graphs, Split graphs, Split interval graphs, probe interval graphs.

Monday, March 8, 1999
11:10 AM

5

On the Construction of a Polynomial for the Volume of a Polyhedron

Nikolaos Galatos, Serge Lawrencenko*, Vanderbilt University
Idzhad Kh. Sabitov, Moscow State University

It is a theorem of the third author that the volume of a polyhedron in Euclidean 3-space is a root of a some polynomial whose coefficients only depend on the combinatorial structure and the metric of the polyhedron. In this note we construct a polynomial for the volume of a given polyhedron homeomorphic to the 2-sphere without applying the operation of cutting of the polyhedron. For this, we prove the following topological theorem: any triangulation T of the 2-sphere with at least five vertices has at least two nonadjacent vertices both not incident with an empty 3-cycle of T (if any)—that is, a cycle of three edges of T not bounding a face of T. *Keywords*: volume of a polyhedron; triangulation of a 2-manifold

6

Non-negatively Charged Graphs

Alice A. McRae* and Dolores A. Parks, Appalachian State University

Let S be any set of vertices of a graph G. We are interested in the relationship between the boundary of S and the interior of S, where $boundary(S) = N(S) - S$, and $interior(S) = N(S) \cap S$. The charge of a set S, denoted $chg(S) = |boundary(S)| - |interior(S)|$. We look at properties of sets that have a non-negative charge.

Keywords: boundary, interior, charge of a graph, independent sets

7

Basic Interval Orders

Amy N. Myers, Dartmouth

A finite interval order of length n has a unique representation as a collection of intervals of an n element linearly ordered set. A basic interval order of length n has the propertly that removal of any element yields an order of length less than n. One can construct and ennumerate the set of basic length n interval orders using a recurrence relation.

8

On Finding a Hamiltonian Path in a Tournament Using Semi-Heap

Jie Wu
Department of Computer Science
Florida Atlantic University
Boca Raton, FL 33431
jie@cse.fau.edu, http://www.cse.fau.edu/~jie

We consider the problem of finding a Hamiltonian path in a tournament. A tournament is a directed graph with a complete underlying graph. We first introduce a new data structure called a semi-heap and then use it to construct an optimal $\Theta(n \log n)$ algorithm to find a Hamiltonian path. Finally, we propose a cost-optimal parallel algorithm using semi-heap. The run time of this algorithm is $O(n)$ with $O(\log n)$ processors in the EREW PRAM model.

Keywords: Directed graph, heap, Hamiltonian path, sorting, total order, tournament

Monday, March 8, 1999
11:30 AM

9

Sidon-type Sequences and Spherical 3-Designs

Béla Bajnok, Gettysburg College

A finite set X of points on the d-sphere S^d is a *spherical t-design*, if for every polynomial f of total degree t or less, the average value of f over the whole sphere is equal to the arithmetic average of its values on X. The concept is the spherical analogue of $t-(v,k,\lambda)$ designs, and has been studied in various contexts, including representation theory and approximation theory. A central question in the field is to find all integer triples (t,d,N) for which a spherical t-design on S^d exists consisting of N points. This question is largely unsolved for $t \geq 3$.

We call a subset S of $\mathbf{Z}_n$ a *Sidon-type set of strength* t in $\mathbf{Z}_n$, if no multi-subset of S with at most t elements can be divided into two disjoint parts so that the two parts have the same (multi-subset) sum. We do not know the maximum cardinality $s(n,t)$ of a Sidon-type set of strength t in $\mathbf{Z}_n$ for $t \geq 3$.

In this talk we find a lower bound for $s(n,3)$ (which we believe is exact), and discuss how Sidon-type sets of strength 3 can be used to construct spherical 3-designs.

A Lower Bound for Edge-tenacity

10

Barry Piazza*, U. of Southern Mississippi
Sam Stueckle, Trevecca Nazarene University

The edge tenacity of a graph G is given by the minimum of $\left(|S| + m(G-S)\right)/\omega(G-S)$ taken over all edge disconnecting sets S, where $m(G-S)$ denotes the maximum order of a component of $G-S$ and $\omega(G-S)$ denotes the number of components of $G-S$.. A lower bound for the edge-tenacity of a graph in terms of the its eigenvalues is established. The technique is the one Beineke, Goddard, and Lipman used in the paper "Graphs with Maximum Edge-Integrity," **ARS Combinatoria 46(1997)**, pp.119-127, to obtain a lower bound for edge-integrity.

KEYWORDS: edge-tenacity, edge-integrity, edge boundary

On Calculating Connected Dominating Set for Efficient Routing in Ad Hoc Wireless Networks

12

Jie Wu and Hailan Li *
Department of Computer Science
Florida Atlantic University
Boca Raton, FL 33431
{jie, hli}@cse.fau.edu

Efficient routing among a set of mobile hosts (also called nodes) is one of the most important functions in ad-hoc wireless networks. Routing based on connected dominating set is a frequently used approach, where the searching space for a route is reduced to nodes in the set. A set is dominating if all the nodes in the system are either in the set or neighbors of nodes in the set. In this paper, we propose a simple and efficient distributed algorithm for calculating connected dominating set in ad-hoc wireless networks, where connections of nodes are determined by their geographical distances. Our simulation results show that the proposed approach outperforms a classical algorithm. Our approach can be potentially used in designing efficient routing algorithms based on connected dominating set.

Keywords: ad hoc wireless networks, dominating sets, routing, simulation

13

DECOMPOSITIONS OF FINITE PROJECTIVE PLANES

Lynn Margaret Batten, University of Manitoba

It is well known that square order Desarguesian projective planes can be partitioned into Baer subplanes. We consider the case of arbitrary finite projective planes and ask what decompositions into isomorphic copies of a particular subset can exist. In the square order, n^2, situation, our results indicate a fascinating connection between the existence of Baer subplanes, $(n-\sqrt{n}+1)$-arcs, and decompositions into collinear sets of size $n-\sqrt{n}+1$. Key Words: projective plane, partition.

14

Edge-added Eccentricities of Vertices in a Graph

Kevin McDougal, University of Wisconsin

The eccentricity of a vertex v in a connected graph G is the distance between v and a vertex farthest from v. For a vertex v, we define the edge-added eccentricity of v has the minimum eccentricity of v in the graphs G + e over all edges e in the complement of G. A graph is said to be edge-added stable (or just stable) if the eccentricity and the edge-added eccentricity are the same for all vertices in the graph. This paper defines the edge-added center of a graph and describes properties of edge-added stable graphs.

keywords: eccentricity, center

16

Fault Tolerant Routing in Split-Stars and Alternating Group Graphs

Eddie Cheng and Marc J. Lipman*, Oakland University, Rochester, MI

Akers, Harel and Kirshnamurthy proposed an interconnection topology, the star graph, as an alternative to the popular n-cube. Cheng, Lipman and Park proposed the split-star as an alternative to the star graph and a companion graph to the alternating group graph proposed by Jwo, Lakshmivarahan and Dhall. Star graphs, alternating group graphs and split-stars are advantageous over n-cubes in many aspects. In this paper, we give a routing algorithm when faults are present in alternating group graphs and split-stars.

Keywords:Interconnection networks, split-stars, alternating group graphs, faults, routing

Monday, March 8, 1999
12:10 PM

17

On incidence structures between points and planes of $PG(3, q^2)$

Nobuko Miyamoto, University of Tsukuba

Let $\Sigma = PG(4,q)$ be embedded in $\Sigma^* = PG(4,q^2)$. Then the set B of points on a hyperplane H of Σ^* not contained in Σ can be partitioned into $v_1 = (q^2+1)(q^2-q)$ orbits $G_1 \ldots G_{v_1}$ of size q^2+q+1 each by a Singer Cycle of Σ. We consider two sets V_1 and V_2 of planes each of which contains G_i, $1 \le i \le v_i$, and intersects in a plane of H, respectively. It will be shown that an incidence structure between V_1 and B gives an (r_1, λ_1)-design with $r_1 = q^4+q^2+1$ and $\lambda_1 = q^2+1$ and an incidence structure between V_2 and B also gives an (r_2,λ_2)-design with $r_2 = q^4+q^2+1$ and $\lambda_2 = q^2+q+1$. Moreover we show that (V_1, V_2, B) forms a regular pairwise balanced bipartite design.

Key Words: (r,λ)-design, regular pairwise balanced bipartite design, projective spaces

18

Clique-Dominating Cycles

G. Chen, Georgia State Univ.; R. Faudree, Univ. of Memphis;
W. Shreve*, North Dakota State Univ.

In this paper, we investigate the sufficient conditions for a graph G to contain a cycle (path) C such that G-V(C) is a disjoint union of cliques. In particular, suffi- cient conditions involving degree sum and neighborhood union are obtained.

Keywords: DOMINATING CYCLE, DEGREE SUM, NEIGHBORHOOD UNION.

20

Disjoint Paths in Split-Stars

Eddie Cheng* and Marc J. Lipman, Oakland University, Rochester, MI

A popular interconnection network is the star graph. It has many attractive properties including the existence of a large number of vertex disjoint paths. A graph has the k-disjoint path property if for all selection of k disjoint vertex pairs $\{s_1, t_1\}, \ldots, \{s_k, t_k\}$ there exists k vertex disjoint paths, one connecting each pair. A graph with the k-disjoint path property must be $(2k-1)$-connected. It is known that for star graphs this condition is sufficient. We prove the same for split stars.

Keywords: Interconnection networks, split-stars, disjoint paths

Monday, March 8, 1999
3:20 PM

21

Knight's Tour on a Hexagonal Honeycomb

by

Peter Che Bor Lam, Wai Chee Shiu and Hee Lin Cheng[1]
Hong Kong Baptist University.

Key words and phrases: Knight's tour, Triangular honeycomb, Hexagonal honeycomb

John J. Watkins showed that there exists a knight's tour on triangular honeycombs. In the paper we extend his results to hexagonal honeycombs.

22

Induced Permutation Automata and Coverings of Strongly Connected Automata

Kimio SUGITA*, Tokai University
Kenji UEMURA, Tsuru University
Takeo YAKU, Nihon University

Decomposition theory of finite automata is one of the major problem in algebraic automata thoery. The first leading result in this field was due to Krohn and Rhodes. One version stated by Krohn, Langer and Rhodes is that a strongly connected automaton A is decomposed into a cascade product of a group automaton and the factor automaton, if its automorphism group $\mathcal{A}(A)$ is not $\{\text{id}\}$. Generally, the class of strongly connected automata of this kind is small. In this article, we introduce the notion of the induced permutation automaton and consider a decomposition theory of them for larger class of automata, where $\mathcal{A}(A)$ is not $\{\text{id}\}$. Let $A = (S, \Sigma, N)$ be an automaton, the equivalent relation $x \sim y$ in Σ^* iff $N(s, x) = N(s, y)$ for any $s \in S$ induces a mapping semigroup on A. This semigroup $\mathcal{C}(A)$ is called the *characteristic semigroup* of A. Let $\{e_1, e_2, \ldots, e_n\}$ be a set of minimal idempotent of $\mathcal{C}(A)$, then the set $\pi = \{Se_1, Se_2, \ldots, Se_n\}$ is a disjoint partition of S satisfying $N(Se_i, \sigma) \subset Se_j$. This partition, *admissible partition*, induces an automaton $A/\pi = (\pi, \Sigma, \overline{N})$, where $\overline{N}(Se_i, \sigma) = Se_j$, and called *factor automaton*. The restriction of $e\mathcal{C}(A)e$ on Se is a permutation group, so we can define an automaton $B = (Se, eCe, M)$, where $M(se, exe) = sexe$, and called the *induced permutation automaton*. Using these automata, we make a *covering* of automata. A covering is a surjective automaton isomorphism. This covering is available for the class of strongly connected automata where $e\mathcal{C}(A)e$ is not $\{\text{id}\}$.

Theorem *Let $A = (S, \Sigma, N)$ be a strongly connected automaton. Then A is covered by a cascade product $B\omega(A/\pi) = (Se \times \pi, \Sigma, L)$ of the induced permutation automaton $B = (Se, e\mathcal{C}(A)e, M)$, where e is a minimal idempotent of $\mathcal{C}(A)$, and the generalized factor automaton for the admissible partition system $A/\pi = (\pi, \Sigma, \overline{N})$. The automorphism group of the generalized automaton is $\{\text{id}\}$.*

23

Some linear codes over the Veronese Variety

C. Rentería; ESFM, IPN, México
H. Tapia-Recillas(*); Dpto. Mat. UAM-I, México

Let K =GF(q) be a finite field with q elements, let $P_m(K)$ be the m-projective space over K and let $S = \{P_1, \ldots, P_s\}$ be a subset of $P_m(K)$. Let $\mathcal{L}$ be a finite dimensional K-linear space of functions which are defined on the set S and take values on K. Then the evaluation map:

$$ev : \mathcal{L} \to K^s, \quad ev(f) = (f(P_1)), \ldots, f(P_s))$$

defines a K-linear code: $C_S = ev(\mathcal{L})$. Describing the parameters of this code for a general set S is rather difficult.

Let $A = K[X_0, \ldots, X_m] = \oplus_{j \geq 0} A_j$ be the polynomial ring over the finite field K with the natural graduation. If $S \subseteq P_m(K)$ is as above and $\mathcal{L} = A_d$ is the d-graded homogeneous component of the polynomial ring A, the corresponding linear code $C_S(d) := ev(A_d)$, will be called the Reed-Muller linear code over the set S, which is isomorphic to $A_d/I_S(d)$, where $I_S = \oplus_{j \geq 0} I_S(j)$ is the (graded) vanishing ideal of S. The dimension of this code, a description and some other invariants of the vanishing ideal of S are given when the set S is the image of the m-projective space over a finite field under the Veronese mapping. Some examples are provided to illustrate the ideas.

24

The Reversing Number of a Digraph: A Disjoint Union of Paths

Darren Narayan* and Garth Isaak, Lehigh University

A minimum feedback arc set of a digraph is a smallest sized set of arcs that when reversed makes the digraph acyclic. Given an acyclic digraph D, we seek a smallest tournament T which has D as a minimum feedback arc set. The reversing number of D is then defined to be $r(D) = |V(T)| - |V(D)|$. We determine the reversing number for a disjoint union of directed paths and examine the reversing number of other arborescences. If $D = \sum_{i=1}^{m} P_{k_i}$ where $k_i \geq k_{i+1}$ for all $1 \leq i \leq m-1$, then $r\left(\sum_{i=1}^{m} P_{k_i}\right) = \max\left\{0, k_1 - 1 - \sum_{i=2}^{m} k_i, \left\lfloor \frac{k_1 + k_2 - 1}{3} \right\rfloor - \sum_{i=3}^{m} k_i\right\}$.

Keywords: acyclic digraph, feedback arc set, tournament

Monday, March 8, 1999
3:40 PM

25

Independent Knights on Hexagon Boards

Jens-P. Bode, Technische Universität Braunschweig, Germany

For knight graphs on different parts of the regular hexagon tessellation the independence number is discussed. Exact values are determined for the two classes of boards where the shapes are equilateral triangles or (nearly) regular hexagons. (Common work with Heiko Harborth and Hartmut Weiss.)

26

Turn-Bounded Pushdown Automata Revisited

Etsuro MORIYA, Waseda Univeristy, Japan

A pushdown automaton is said to make a *turn* at a moment if it changes at that moment from a stack increasing (decreasing) mode to a stack decreasing (increasing) mode. Let DPDA-TURN($f(n)$) (NPDA-TURN($f(n)$), resp.) be the class of languages accepted by deterministic (nondeterministic, resp.) pushdown automata that make at most $f(n)$ turns for any input of length n. A well-known classical theorem states that NPDA-TURN($O(1)$) equals the class of metalinear languages; in particular NPDA-TURN(1) equals the class of linear languages.

We show that for any function $f(n)$, NPDA-TURN($f(n)$) $\subseteq$ **NSPACE**($f(n)\log n$) and that NPDA-TURN($O(1)$) $\subseteq$ **NLOG**. We also show that NPDA-TURN($O(1)$) $\subseteq$ LOGLCFL, where LOGLCFL is the class of languages log-space reducible to linear context-free languages. For deterministic case, the corresponding results that DPDA-TURN($f(n)$) $\subseteq$ **DSPACE**($f(n)\log n$)) and that DPDA-TURN($O(1)$) $\subseteq$ **DLOG** hold.

Keyword: pushdown automaton, context-free language, complexity class, LOGCFL

27

From algebraic-geometric codes to limited bias and dependence

Jürgen Bierbrauer, Michigan Technological University

ϵ−biased and ϵ−dependent arrays (functions, random variables) are basic structures for certain areas of theoretical computer science (algorithms, universal hashing, pseudo-randomness, authentication, ...). Two landmark papers on the subject are Naor and Naor (1990,1993) and Alon, Goldreich, Hstad and Peralta (1990,1992). We will describe the fundamental construction of ϵ− biased arrays from the latter paper in a coding-theoretic setting. This leads to far-reaching generalizations and improvements. As ingredients we use algebraic-geometric codes and Ramanujan graphs. Hermitian codes provide efficiently constructed examples. The relation between the different types of structures is clarified by Fourier analysis. In the way of applications we focus on covering arrays (also called t-independent sets or universal sets) and authentication codes.

28

Connectivity and Reliable Communication for Replicating Graphs

Tom Altman, University of Colorado at Denver

Let $G_1 = (V_1, E_1)$ be a connected graph, called the base-graph, and let $G_2 = (V_2, E_2)$ be an identical copy of G_1. Let $E_3 = \{(v_{i_1}, v_{i_2})$ where $v_{i_1} \in V_1$ and $v_{i_2} \in V_2$ $\}$ be a set of edges. The graph $G_3 = (V_1 \bigcup V_2, E_1 \bigcup E_2 \bigcup E_3)$ is said to be a replication of G_1.

In this talk we discuss the connectivity and reliable communication protocols for architectures that can be represented via replicated graphs. We show that both the edge as well as the node-connectivity of a replicating graph is a function of the base-graph's connectivity and the number of replications. We then present a reliable communication protocol that maximizes the number of independent paths from any source node to any destination node.

Keywords: replicating graphs, connectivity, distributed architecures.

Monday, March 8, 1999
4:00 PM

29 Bishop and rook independence on triangle boards

Heiko Harborth, Technische University at Braunschweig, Germany

As game boards T_n we consider simple connected parts of the Euclidean tesselation where $n/2$ or $(n-1)/2$ rings of neighbor triangles surround a vertex or triangle, respectively. For rook graphs and bishop graphs on T_n the independence numbers are determined. (Common work with Martin Harborth.)

30 Characterizing deterministic soliton graphs

M. Bartha*, József Attila University, Hungary
M. Krész, Juhász Gyula Teacher Training College, Hungary

The underlying object of a soliton automaton is an undirected graph having a perfect internal matching, called a soliton graph. The states of the automaton are identified with the perfect internal matchings of the graph, and transitions are defined by making alternating walks from one external vertex to another. A soliton graph is deterministic if the automaton associated with it is deterministic in the usual sense.

First, a reduction procedure is given for soliton graphs, by which every graph has a minimal representation. It is then proved that a 1-extendible and minimal soliton graph with external vertices has an alternating cycle with respect to some of its states iff the graph contains an even-length cycle. This result leads to a characterization of deterministic soliton graphs, saying that every internal elementary component of such a graph is a single mandatory edge, and the minimal representation of each external component is a graph not containing even-length cycles. Using this characterization, the deterministic property becomes straightforward to check for soliton graphs.

Key words: perfect internal matchings in graphs, elementary decomposition, deterministic finite automata

Using Canonical Trees to Efficiently Implement Huffman Decode Information

31

D. R. McIntyre* and F. G. Wolff
Cleveland State University & Case Western Reserve University

Given an information source defined by a pair (S, F) of source symbols $S = \{s_1, s_2, \ldots, s_n\}$ and a set of corresponding nonnegative frequencies $F = \{f_1, f_2, \ldots, f_n\}$, the well-known static Huffman compression algorithm can be used to construct a binary tree with n leaf nodes and n-1 internal nodes. Huffman's tree has the minimum value of $\sum_{i=1}^{n} f_i l_i$ over all such binary trees, where l_i is the level at which symbol s_i with frequency f_i occurs in the tree. Binary trees with n leaves are in a one-to-one correspondence with sets of n strings on $\{0, 1\}$ that form a "minimal prefix code" (strings in which no string is a proper prefix of another). The correspondence between trees and codes is simply to represent the path from the root to each leaf node as a string of 0's and 1's, where 0 corresponds to a left branch and 1 to a right branch. Huffman codes are clearly information source dependent and hence decode information must be stored along with the compressed source. A representation of canonical trees is introduced to more efficiently store the decode information contained in the Huffman tree.

Key words: canonical trees, compression, data, Huffman, prefix codes.

32 A closer look at strict consecutive k-out-of-$n : F$ systems

Anant P. Godbole* and Papa A. Sissokho, MICHIGAN TECH UNIVERSITY

A strict consecutive k-out-of-$n : F$ system is defined as one consisting of n linearly arranged components, each failing with probability q, and with system failure occurring iff there is a failure "run" of length k, but no failure run of length $j \leq k-1$. Heuristic arguments have been given to claim that these are unrealistic systems since their reliability is almost unity. We prove this fact using the multivariate Stein-Chen method, and propose a new model, far more realistic than the old, called the *conditional* strict consecutive k-out-of-$n : F$ system. An exact formula is given for its reliability. Since this is complicated (involving sums of multinomial coefficients over an index set determined by the solutions of a diophantine equation), two approximations are provided for the reliability and error bounds are given.

33

KATONA TYPE PROOF FOR THE 2-INTERSECTING ERDOS-KO-RADO THEOREM

Laszlo A Szekely, University of South Carolina

The simplest proof of the Erdos-Ko-Rado theorem is Katona's proof which counts cyclic permutations. A paper of Erdos, Faigle and Kern emphasized that Katona's proof is group theoretic. This paper gives a Katona type proof for the 2-intersecting Erdos-Ko-Rado theorem. Now the affine linear group plays the role of the cyclic group in Katona's proof. The proof is based on a number theoretic result of Kevin Ford on 2-intersecting integer arithmetic progressions and rectification techniques. This is a joint work with Ralph Howard and Gyula Karolyi.

34

The Equal Union Property on Vertex Sets in a Graph

Steve Hedetniemi, Sandee Hedetniemi, David Jacobs, Robert Jamison, Renu Laskar, and Ted Doyle; Clemson University,
Jean Dunbar; Converse College, Alice McRae; Appalachian State University,
Douglas Rall*;Furman University

A family $\mathcal{F} = \{S_1, S_2, \ldots, S_m\}$ of distinct, nonempty subsets of a set X is said to have the **equal union property** if $\mathcal{F}$ has two nonempty subfamilies $\{S_i | i \in A\}$ and $\{S_j | j \in B\}$ such that $A \cap B = \emptyset$ and $\cup_{i \in A} S_i = \cup_{j \in B} S_j$. We are interested in the following general question for a graph $G = (V, E)$: If $\mathcal{F}$ is the family of all subsets of V of a certain type, does $\mathcal{F}$ have the equal union property? For example, $\mathcal{F}$ could be the set of all closed neighborhoods, the set of all maximal independent sets or the set of all cliques.

key words: neighborhoods, maximal independent set, equal union property

36

On the Profile of Corona of Graphs

Yung-Ling Lai, National Chia-Yi Teacher College, Taiwan, R.O.C.

It is known that determination of the profile for arbitrary graphs is NP-complete. The corona $G = G_1 \wedge G_2$ of graphs G_1 and G_2 with n_1 and n_2 vertices, respectively, is the graph containing one copy of G_1 and n_1 copies of G_2 such that each distinct vertex of G_1 is joined to every vertex of the corresponding copy of G_2. A tight upper bound and a tight lower bound for the profile of the corona of two graphs is determined. Also, exact values are determined for the profile of the corona of several families of graphs.

Keywords: corona, profile, graphs

37

On (p, Δ) Covering Systems

Nathan S. Wallach, Technion—Israel Institute of Technology

A *covering system*, $C = \{a_i m_i : i = 1, 2, \ldots, t\}, (t \geq 1)$, is a finite set of congruences, such that every integer satisfies at least one of these congruences. introduced this concept in 1950, and posed many questions relating to covering systems. A covering system is said to be *distinct* if its moduli are distinct and *exact* if its congruences are (pairwise) disjoint. A covering system cannot be both distinct and exact; in fact, the multiplicity (of the largest modulus) of an exact covering system must be at least its smallest prime divisor (Newman and Znám). Most research on covering systems has been focused on these disjoint classes of covering systems.

Our research focused on the class of (p, Δ) covering systems. A (p, Δ) *covering system* is a covering system in which the smallest prime dividing any of the moduli is at least p (which itself is a prime) and in which each modulus has a multiplicity not exceeding Δ. Newman and Znám's result establishes that for an exact (p, Δ) covering systems, $\Delta \geq p$. Our work focuses on the case where $\Delta < p$. Of particular interest is the case where $p = 3$, and $\Delta = 1$, a so-called *odd covering system*. conjectured that such a system exists, while Selfridge conjectured that no such system exists. further conjectured that for every prime p, a $(p, 1)$ covering system with square-free moduli exists. Such covering systems are called *square-free*.

J. A. Dewar was the first to study the class of (p, Δ) covering systems. He established the existence of $(p, p-2)$ covering systems for all primes $p > 3$, and empirically investigated the smallest value of Δ for which a (p, Δ) covering system exists for the primes $p = 3$, 5, 7, 11, 13, and 17. We present constructions establishing the existence of (p, Δ) covering systems when Δ is sufficiently large (relative to p) in both the square-free case and the general case. In particular if $\gamma > 0$ then for every sufficiently large prime, p, there exists a $(p, \lfloor (1 - \frac{\gamma}{\log p})p \rfloor)$ covering system. For each of the primes considered by Dewar, our construction implies the existence of (p, Δ) covering system with the same value of Δ that Dewar obtained.

38

LineGCS of Trees

Robert C. Brigham (University Central Florida) and Kathleen G. Cherry* (George Washington University, WDC)

Keywords: Line graph of G or L(G), Greatest Common Subgraph of G_1 and G_2 or GCS(G_1, G_2), symmetry number.

Recall that a LineGCS of two graphs, G_1 and G_2, is defined to be a graph G such that L(G) is a member of L(GCS(G_1, G_2)) and is also a member of GCS(L(G_1), L(G_2)). Although it is obvious that two graphs always have corresponding line graphs and that two graphs have at least one greatest common subgraph, it is not always the case that two graphs have a LineGCS. This talk completely answers the question of whether, for a given tree T, there exists a pair of supertrees T_1 and T_2 such that T is a LineGCS of T_1 and T_2. The approach is to partition the trees according to the value of the s+1 symmetry number (that symmetry number *s* resulting from adding pendant edges to leaves, ie., degree 1 vertices). Interestingly enough the proofs can be reduced to two primary cases—trees with s+1 symmetry number where *s* is at least 2 and trees with a 1+1 symmetry number.

39

Hamming colorings

D.G. Hoffman, Auburn University

We investigate generalized Hamming codes, and Hamming colorings (colorings more restrictive than Grundy colorings) of generalized cubes.

Towers, Beads, and Loopless Generation of Trees With Specified Degree Sequences

40

Paul LaFollette* and James Korsh, Temple University

Various representations for specifying the structure of a tree have been devised and used in the development of loopless generation algorithms. The authors propose a new tree representation, consisting of an array of towers corresponding to the tree O's external (leaf) nodes, and a collection of beads corresponding to the internal nodes. An ordered tree with specified degree sequence and K internal nodes has p_i nodes of degree i, where K is the sum of the p_i. Rules for placing the beads on the towers, and for moving the beads from tower to tower are given which ultimately result in the development of a loopless algorithm for generation of ordered trees with specified degree sequences (trees of mixed arity.)

Keywords: Combinatorial Problems, Loopless Generation, Trees

Monday, March 8, 1999
5:00 PM

EDGE CLIQUE GRAPHS OF SOME CLASSES OF CHORDAL GRAPHS

46

Márcia R. Cerioli Jayme L. Szwarcfiter*
Universidade Federal do Rio de Janeiro, Brazil

The edge clique graph of a graph G is one having as vertices the edges of G, two vertices being adjacent if the corresponding edges of G belong to a common clique. This class of graphs has been introduced by Albertson and Collins (1984). Although many interesting properties of it have been since studied, we do not know complete characterizations of edge clique graphs of any non trivial classes of graphs. In this paper, we describe characterizations of edge clique graphs of some classes of chordal graphs, such as starlike, starlike-threshold, split and threshold graphs. In special, a known necessary condition for a graph to be an edge clique graph is that the sizes of all maximal cliques and maximal clique intersections ought to be triangular numbers. We show that this condition is also sufficient for starlike-threshold graphs.

Key Words: chordal graphs, cliques, edge clique graphs, intersection graphs.

Profile minimization on triangulated mesh graphs

48

Yuqiang Guan (yguan@cs.wmich.edu)
*Kenneth L. Williams (williams@cs.wmich.edu)
Western Michigan University

Given graph $G = (V, E)$ with n vertices, the *profile minimization problem* is to find a one-to-one function $f : V \to \{1, 2, \dots, n\}$ such that $\sum_{v \in V(G)} \{f(v) - \min_{x \in N(v)} f(x)\}$ is as small as possible, where $N(v) = \{v\} \cup \{x : x \text{ is adjacent to } v\}$ is the *closed neighborhood of v in G*. Consider the planar graphs formed as follows: In a plane, take the set of h parallel lines (each an equal distance from its neighbors) in the East-West direction, the set of r such lines angling from Northeast-Southwest, and the set of l such lines angling from Northwest-Southeast. The triple intersection points are the vertices of triangulation mesh graph $T_{h,r,l}$ and the line segments joining points are the edges. Several different shapes of triangulation mesh graphs may be achieved, including the triangle (with $h = r = l$), near triangle, trapezoid, rhombus, and others. The bandwidth minimization problem was solved for triangulated triangles in 1995 (Hochberg, McDiarmid and Saks/Discrete Mathematics 138 (1995) 261-265) and for general triangulated meshes in 1997 (Lam, Shiu, Chan and Lin/ Discrete Mathematics 173 (1997) 285-289). We show that the profile minimization problem for these graphs requires a considerably more complex numbering than that for bandwidth, and that a single numbering strategy for all shapes cannot suffice. A solution for some of the shapes, including the triangulated triangle, is developed.

Key words: graph, profile, bandwidth, triangulated mesh.

41

Equivalence Classes of Additive Permutations

(Mr.) Jean M. Turgeon (Math.)
University of Montreal, CANADA H3C 3J7

Given a sequence of additive permutations, other sequences can be constructed from it, but not all. We obtain some equivalence classes for sequences of lengths two, three and four. We shall describe the construction process.

42

On the Caccetta-Häggkvist Conjecture

Jian Shen, University of Wisconsin

Let c be the smallest possible value such that every digraph on n vertices with minimum outdegree at least cn contains a directed triangle. In 1978, L. Caccetta and R. Häggkvist conjectured that $c = 1/3$. This conjecture, if true, can be considered as an extension of the well-known Mantel-Turán theorem from graphs to digraphs. Recently, J. A. Bondy showed that $c \leq (2\sqrt{6}-3)/5 = 0.3797\cdots$ by using some counting arguments. In this talk, we will prove that $c \leq 3-\sqrt{7} = 0.3542\cdots$.

Key words: directed triangle, minimum outdegree

On the probability that a t-subset of a finite vector space contains an r-subspace—with applications to short, light codewords in a BCH code

43 W. Edwin Clark* and Stephen Suen; University of South Florida

Motivated by the problem of finding *light* (*i.e.*, low weight) and *short* (*i.e.*, low degree) codewords in narrow-sense, primitive BCH codes we consider the problem of determining the probability that a random t-set of vectors in an n dimensional vector space over $GF(q)$ contains an r-dimensional subspace (or affine subspace). We find some bounds for this and similar probabilities and apply these techniques to estimate how short a minimum weight codeword can be in a narrow-sense BCH code.

Keywords: Vector space, finite field, BCH code, light codeword, short codeword, random subset.

Characterization and Properties of Maximal Reducible Flowgraphs

Oswaldo Vernet*, Universidade Federal do Rio de Janeiro, RJ, Brasil
44 Lilian Markenzon, Instituto Militar de Engenharia, RJ, Brasil

Reducible flowgraphs are digraphs that model the control flow of computer programs. Due to their importance in code optimization, several theoretical and applied problems have been solved for the family and its subclasses. A reducible flowgraph is maximal when no new edge can be added to it without violating reducibility. In this paper, maximal reducible flowgraphs are characterized: a product $\otimes$ between flowgraphs is defined and every maximal reducible flowgraph is proved to be uniquely decomposable into two other ones with respect to $\otimes$. The decomposition relies on the existence of two vertices with maximum input degree. As a consequence of the characterization theorem, a one-to-one association with extended binary trees is established, allowing to deduce important numerical properties of the family. For a given n, the lower and upper bounds for the number of edges and the number of non-isomorphic n-vertex maximal reducible flowgraphs are obtained. Finally inclusion relations among maximal reducible flowgraphs and other well known subclasses of reducible flowgraphs (spiral graphs and tree reducible flowgraphs) are shown.

Keywords: flowgraphs, reducibility, maximality.

49

On Injectivity of Discrete Radon Transform

Melkamu Zeleke, William Paterson University.

The problem of determining members of a set by their sums of a fixed order was posed by Leo Moser and partially settled by Ewell, Fraenkel, Selfridge and Straus. For any given $(k,n) \in Z \times Z$ with $2 \le k \le n$, we choose arbitrarily a multiset of complex numbers $X_n = \{x_1, x_2, \ldots, x_n\}$ then form the set $W_n^k(X_n) = \{\sigma_i\}$ of all sums of k distinct elements of X_n and try to determine whether W_n^k is injective.

50

Tilings and Patterns of Enumeration

Ralph P. Grimaldi, Rose-Hulman Institute of Technology*
Phyllis Z. Chinn, Humboldt State University

For $n \ge 0$, let a_n count the number of ways one can tile a $2 \times n$ chessboard with 2×1 tiles (which are also 1×2 tiles) and 2×2 tiles. Then, for $0 \le k \le \lfloor n/2 \rfloor$, we let $a_{n,k}$ count those tilings (determined by a_n) where we use exactly k 2×2 tiles.

Next, we identify the right edge of the n th column of the $2 \times n$ chessboard with the lift edge of the first column of the chessboard. Now we let c_n count the number of ways we can tile this $2 \times n$ tubular chessboard with 2×1 tiles and 2×2 tiles. For $0 \le k \le \lfloor n/2 \rfloor$, $c_{n,k}$ counts the tubular tilings where exactly k 2×2 tiles are used.

Properties and enumerative patterns are developed for a_n, $a_{n,k}$, c_n and $c_{n,k}$.

51

Symmetry Breaking in Graphs - 5 Years Later

Michael O. Albertson, Smith College

A labeling of the vertices of a graph G, $\ell : V(G) \to \{1, 2, \ldots, r\}$, is said to be *r-distinguishing* if no non-trivial automorphism of G preserves all of the vertex labels. $D(G)$, the distinguishing number of a graph G, is the minimum r such that G has an r-distinguishing labeling. Of course the automorphism group plays a critical part. Given a group Γ, $D(\Gamma)$, the distinguishing set of Γ is $\{D(G) | \mathrm{Aut}(G) \cong \Gamma\}$. These ideas were introduced by Albertson and Collins and have been studied by Chan and Collins, Cheng and Cowen, and Russell and Sundaram with contributions by many others. This talk will give some highlights of the different types of problems that have been solved and suggest a few tantalizing open questions.

52

ALMOST REGULAR GRAPHS

Lisa R. Fischer, Department of Mathematics
University of Central Florida, Orlando, FL 32816

A regular graph is one in which all vertices have the same degree. An almost regular graph is a graph that is not regular, but differs from regular graphs in a carefully prescribed manner. We define a class of almost regular graphs and present a set of formulae to compute the minimum number of vertices needed to form such a graph with defined parameters. **Keywords:** almost regular, extremal

53

Fault-Tolerant Multicasting in 3-D Meshes Using Extended Safety Levels

Xiao Chen* and Jie Wu
Department of Computer Science
Florida Atlantic University
Boca Raton, FL 33431
xchen@cse.fau.edu jie@cse.fau.edu

We propose a fault-tolerant multicast algorithm for 3-dimensional (3-D) meshes based on the concepts of a faulty cube and extended safety levels. This algorithm can achieve time-step optimal. In order to minimize the total number of traffic steps, a heuristic strategy is proposed and a simulation is conducted to test the strategy. Our approach is the first attempt to address the fault-tolerant multicast problem in 3-D meshes. We first discuss the no fault case and then fault case based on limited global information with a simple model and succinct information.

Keywords: fault tolerance, faulty cube, mesh, minimal routing, safety level, time step, traffic step

54

Tiling an m-by-n Area with Squares of Size up to k-by-k (m ≤ 5)

This paper derives formulas for the number of tilings $T_{m,n}$ of an m-by-n rectangle with squares of size up to k-by-k, for m ≤ 5. Two cases are considered: 1) Tilings that use only squares of size 1-by-1 and 2-by-2, and 2) Tilings that use squares of size up to k-by-k, where k = min{m,n}. Using the idea of basic blocks $B_{m,k}$ of size m-by-k (tilings that cannot be vertically split into smaller rectangles of size m-by-l, l ≤ k), a general recursive formula for the number of tilings is derived: $T_{m,n} = \sum_{i=1}^{n} T_{m,i} B_{m,n-i}$. Explicit formulas for the number of basic blocks of size m-by-k for m = 3 and m = 4 are derived for both cases; an explicit formula for $T_{3,n}$ involving sums of odd and even powers of 2 is proved for the case of tilings with only 1-by-1 and 2-by-2 squares. Finally, for m = 5, the number of basic blocks of size 5-by-k is determined recursively.

55

Extending graph colorings using no extra colors

Michael O. Albertson, Smith College; Emily H. Moore*, Grinnell College

Let G be a graph, $r = \chi(G)$, and $P \subset V(G)$. Can any coloring of P be extended to a coloring of all of G? For an arbitary graph there is no distance condition on P such that if we pre-color P with r colors, there is an r-coloring extension. In an earlier paper, the authors discuss distance conditions on P that guarantee any $(r+1)$-precoloring of P extends to an $(r+1)$-coloring of G.

In this current paper we restrict G so that distance conditions on P guarantee that any r-coloring of P extends. Specifically, if G can be r-colored using the rth color on vertices no closer than distance $d_r = 3$, then a minimum distance of $d = 12$ between any two vertices in P guarantees that any r-coloring of P extends to an r-coloring of G. If $d_r \geq 4$, then $d = 8$ guarantees a color extension. Example graphs give lower bounds for the critical distance that guarantees a color extension. If P induces a set of k-cliques, similar results hold.

Finally we explore families of graphs whose structure guarantees they meet the hypotheses of our main result.

Key words: graph coloring extensions, outerplanar graphs

56

The Clique-Width of Unit Interval Graphs is Unbounded

Martin Charles Golumbic* and Udi Rotics (Bar-Ilan University, Israel)

The notion of clique–width of graphs was first introduced by Courcelle, Engelfriet and Rozenberg (1993) as graphs which can be defined by k-expressions based on graph operations which use k vertex labels. The clique–width of a graph G, denoted by $cwd(G)$, is defined as the minimum number of labels needed to construct G, using the 3 graph operation: $\oplus$ (disjoint union), η (connecting all vertices labelled i and j), and ρ (changing labels from i to j).

Clique–width has analogous properties to tree–width: If the clique–width of a class of graphs C is bounded by k (and the k–expression can be computed from its corresponding graph in time $T(|V|+|E|)$, then every decision, optimization, enumeration or evaluation problem on C which can be defined by a Monadic Second Order formula ψ can be solved in time $c_k \cdot O(|V|+|E|) + T(|V|+|E|)$ where c_k is a constant which depends only on ψ and k, where $|V|$ and $|E|$ denote the number of vertices and edges of the input graph, respectively. Details can be found in recent papers by Courcelle, Makowsky, and Rotics.

Our research has focused on studying the clique-width of perfect graph classes. In recent work, we have shown that for every distance–hereditary graph G, $cwd(G) \leq 3$, and a 3–expression defining it can be constructed in time $O(|V|+|E|)$. It is also known the cographs are exactly the graphs with $cwd(G) \leq 2$. In this paper we show that the classes of unit interval graphs and permutation graphs are not of bounded clique-width. These results allow us to see the border of tractability for the clique-width on the hierarchy of perfect graph classes.

key-words: clique-width, unit interval graphs, permutation graphs, graph operations

The Complexity of Two Processor Scheduling with Deadline and Communication Delay

57

Wing Ning Li*, Rex Foust, University of Arkansas

Given a set of tasks ti, with each ti having unit execution time, a deadline di for each task, a communication delay r, and a set of precedence constraints with restrict allowable schedules, the problem of determining whether there exists a schedule on two identical processors that executes each task before its deadline is examined. Without considering the communication delay r, and O(n3) algorithm exists to solve the problem. We show that with communication delay the problem is NP-complete.

Keywords: scheduling, complexity, NP-complete, parallel computing

Polynomial Time Recognition of Totally Bounded Bitolerance Digraphs

58

Randy Shull* and Ann N. Trenk, Wellesley College

Interval digraphs and bounded bitolerance digraphs represent two important directed graph analogues to the well-known interval graphs. In 1997, Haiko Müller published a polynomial-time recognition algorithm for interval digraphs. In this paper, we modify Müller's algorithm in order to recognize the class of totally bounded bitolerance digraphs in polynomial time.

Keywords: Tolerance graph; bitolerance digraph; interval nest digraph.

Large Multicolored forests in complete bipartite graphs

59

Richard A. Brualdi and Susan Hollingsworth*, University of Wisconsin

If the edges of a graph G are colored using k colors, we may consider the *color distribution* for this coloring $\vec{a} = (a_1, a_2, \ldots, a_k)$, in which a_i denotes the number of edges of color i for $i = 1, 2, \ldots, k$. We find inequalities and majorization conditions on color distributions of the complete bipartite graph $K_{n,n}$ which guarantee the existence of multicolored subgraphs: in particular, multicolored forests and trees. We end with a conjecture on partitions of $K_{n,n}$ into multicolored trees inspired by a long-standing conjecture of Gyárfás and Lehel.

Keywords: complete bipartite graph, majorization, multicolored subgraphs, trees

Another Proof for Interval Order Representation

60

Garth Isaak, Lehigh University

We give alternative proofs of the representation theorems for interval orders and semiorders: An order is an interval order if and only if it does not contain a $\underline{2}+\underline{2}$ and an order is a semiorder if and only if it does not contain $\underline{2}+\underline{2}$ or $\underline{3}+\underline{1}$. These are conceptually simple proofs based on associated digraphs for which either shortest paths are defined (to give a representation) or there is a negative cycle (to give a forbidden suborder).

Keywords: Interval order, duality

61 **On Scheduling Partially Ordered Tasks in Distributed Systems**

Hesham H. Ali* and Raj Vemulapalli, University of Nebraska at Omaha

The problem of finding optimal schedules for partially ordered tasks in distributed systems has been proven to be NP-hard in its general form as well as in several restricted versions. The problem becomes even harder when the communication cost among the tasks is considered. Therefore, attention has been directed towards developing heuristics and identifying special versions of the problem for which optimal algorithms can be obtained. In this paper, we explore the approach of augmenting the partial order describing the relations among the tasks with additional relations to form a special class of partial orders. An optimal schedule of the augmented partial order can then be obtained. In particular, we focus on augmenting the input partial order with minimum relations to form an interval order since optimal algorithms for scheduling interval ordered tasks have been developed. We study the effectiveness of this approach on solving the scheduling problem when some communication cost among the input tasks is considered. A class of partial orders for which optimal schedules can be obtained using the proposed approach is identified. Also, several experimental studies are conducted to compare the approach with other known heuristics in the cases when the communication cost among the systems tasks is ignored or considered.

Key Words: Partial orders, interval orders, optimal schedules, distributed systems.

62 **Tolerance Intersection Graphs on Subtrees of a Tree**

R.E. Jamison, Clemson University
Henry Martyn Mulder*, Erasmus University

A chordal graph is the intersection graph of subtrees of a tree. We generalize this concept by assigning tolerances to the subtrees, and by joining two vertices if and only if the intersecion of the corresponding subtrees exceeds the minimum of their two tolerances. We call the resulting graph representable. By imposing restrictions on the host tree, on the subtrees, or on the tolerances, one gets various types of representability. In this talk we present representability and non-representability results, as well as open problems.

63 **4-coloring Triangle Free Graphs on N3**

Barrett Walls, Georgia Institute of Technology

We prove that every graph on a surface with three crosscaps which does not contain any triangles is 4-colorable.

KEY WORDS: Graph Theory, Coloring Problems

64 **Finite Linear Spaces I**

Malcolm Greig
Greig Consulting, West Vancouver, BC, CANADA
e-mail: greig@sfu.ca

In this presentation we illustrate a structured approach to determining the basic structure of Finite Linear Spaces. We illustrate this approach by looking at FLSs on 25 points with minimum line size 4. In this case, after our approach has determi ned the basic structure, we examine 3 of the potential spaces in detail, showing en passant that the bound for the embeddabilty of the pseudo-complement of a triangle of lines in a plane of order n can be improved from $n \geq 7$, to $n \geq 5$; (this bound is sharp). Batten and Beutelspacher have classified Finite Linear Spaces with lines of size n, $n+1$ and $n+2$, and with $v \leq (n+1)^2$ for $n \geq 22$, and many of the cases with smaller n. As further illustrations of our structured approach, we extend their classification to include FLSs containing at least one still larger line(s) for $n \geq 4$, and to FLSs without an $(n+2)$-line, but with an $(n+3)$-line, for $v < n(n+3)$ and $n \geq 4$. For FLSs with minimum line size n containing a line of size k, there is a standard inequality $v \geq (n-1)k+1$. We present a variant inequality which allows us to eliminate values of v close to this bound. This allows us to deal with FLSs with $k \geq n+4$ for $n \geq 4$ and $v < n(n+3)$.

AMS Classifications: 51E26, 05B05, 51E20.
Keywords: Finite Linear Spaces, Pairwise Balanced Designs, Planar Embeddability.

Finding a Low-Diameter and Low-Weight k-Connected Subgraph

65

WeifaLiang[†] GeorgeHavas[*‡] AnneStreet[‡]
†Australian National University ‡The University of Queensland

Low-diameter, low-weight subgraphs have useful applications in networks. [T]he problem is how to find them. Given a weighted undirected k-connected [g]raph $G = (V, E)$, let δ denote the diameter of G and let $w(G)$ the sum of [t]he weights of the edges in G. In this context, the problem is to find a k-[c]onnected spanning subgraph $G' = (V, E')$ of G such that the diameter and the [w]eight of G' are minimized simultaneously. Since this problem is NP-hard, it is [u]nlikely that there is a polynomial algorithm for it. We present an approximate [s]olution with a performance guarantee. That is, we find a k-connected spanning [s]ubgraph G' in polynomial time such that $\delta \leq \alpha\delta$ and $w(G') \leq \beta w(G^*)$ for any [fi]xed $\alpha > 1$, where G^* is a minimum k-connected spanning subgraph of G and [β] is a constant depending on α and k.

Keywords: *approximation algorithm, combinatorial optimization, k-connectivity, [N]P-hard problem.*

Path Tolerance Graphs

66

Robert E. Jamison,Clemson University

A special subclass of chordal graphs is the class of path graphs: the intersection graphs of subpaths of a tree. This concept may be generalized by choosing a tolerance t, and by declaring two paths adjacent if and only if their intersecion contains at least t vertices. The case t=3D2 was previously studied by Golumbic, Syslo, and the presenter. In this talk we present a number of representability and non-representability results, as well as open problems. Among the more striking results is the fact that there exist chordal graphs (if fact split graphs) that are not representability as path tolerance graphs for any tolerance. Among the most puzzling of the open problems is the question of whether or not path tolerance graphs can be recognized in FINITE time (let alone polynomial time).

Quasi-uniquely edge colorable graphs - A preliminary report

67

Mark Ginn, Appalachian State University

An edge k-coloring of a graph $G = (V, E)$ is a function $f : E(G) \rightarrow [k]$ such that $f(e_1) \neq f(e_2)$ whenever e_1 and e_2 share an endpoint. A graph G is said to be *uniquely k-edge colorable* if for any two k-coloring f and g of G, there is a bijection $\psi : [k] \rightarrow [k]$ such that $f(e) = \psi(g(e))$ for all $e \in E(G)$. We extend this definition to say that a graph G is *quasi-uniquely k-edge colorable* if for any two k-colorings f and g of G, there exist bijections $\psi : [k] \rightarrow [k]$ and $\phi : V(G) \rightarrow V(G)$ such that ϕ is a graph automorphism of $G$and $f(\{x, y\}) = \psi(g(\{\phi(x), \phi(y)\}))$. We will give our motivation for this definition, contrast some known results about uniquely edge colorable graphs with their counterparts for quasi-uniquely colorable graphs and give some open problems in the area.

On Balance Preserving Matroid Operation

68

Jenny McNulty*, The University of Montana
Jenifer Corp, The University of Montana

Most matroid operations do not in general preserve balance. Conditions are given for the direct sum of balanced matroids to be balanced. An obvious necessary condition for the amalgam A of the balanced matroids M_1 and M_2 to be balanced is that the density of M_i, $i = 1, 2$ is at most the density of A. For the class of uniform matroids these conditions are also sufficient.

69 **How deleting and adding channels to a network affect its diameter**

Yuqiang Guan*, Kenneth L. Williams; Western Michigan University

We consider a connected network $T = (N, C)$ where N is a set of nodes and C is a set of channels connecting some of the nodes. It may be desirable to modify the network by adding or deleting channels such that the resulting network has expected diameter. Although it has been proved that the general edge augmentation problem is NP-complete(by Y. Lai and K.L. Williams) and a very closely related edge deletion problem is also NP-hard(by Y. Guan and K.L. Williams), techniques for deleting channels while maintaining a given diameter for several network architectures will be demonstrated. Some results on adding channels will also be exhibited. **Key words:** graph, diameter.

70 **Threshold representations of ordered sets and unbounded tolerance graphs**

Kenneth P. Bogart, Dartmouth College

A *threshold representation* of an ordered set (X, P) consists of a function c from X to the real numbers such that $x \preceq y$ in P if and only if $c(x) + T(x,y) \leq c(y)$. We show that every ordered set has a threshold representation in which T is a metric and any metric T defines an ordered set. Interval orders and statistical confidence orders arise naturally in this way. We use this idea to give yet another proof that bounded tolerance graphs are cocomparability graphs. Finally, we use threshold representations to prove that an (unbounded) tolerance graph is the intersection of two cocomparability graphs.

Key Words: Ordered set, Tolerance graph, Interval order

71 **A NOTE ON THE DIVISIBILITY OF GRAPHS**

Chính T. Hoàng*, Lakehead University, C. McDiarmid, University of Oxford

A graph G *k-divisible* if for each induced subgraph H of G with at least one edge there is a partition of the vertex-set of H into sets $V_1, \ldots, V_k$ such that no V_i contains a maximum clique of H. For every fixed k, k-divisible graphs are χ-bounded, that is, the chromatic number χ is bounded in terms of the clique number ω. We had earlier proposed a number of conjectures on the divisibility of a graph, one of which is the following: a graph is 2-divisible if it does not contain an odd chordless cycle with at least five vertices. We shall present some partial results on this conjecture.

Key words. graph colouring, perfect graph.

A Higher-Dimensional Generalization of Rota's Colorful Conjecture

72 Glenn G. Chappell, Southeast Missouri State University

Consider a square matrix with entries in a matroid. G.-C. Rota has conjectured that, if the entries of each column form a base of the matroid, then we can permute the entries of each column so that the entries of each row form a base as well. In the special case of linear matroids, S. Onn has named this "Rota's Colorful Conjecture".

We generalize Rota's conjecture in a number of ways: allowing the matrix to be nonsquare, allowing each entry of the matrix to contain more than one element of the matroid, and replacing the (2-dimensional) matrix with a higher-dimensional analogue. We prove our generalizations for the special cases of strongly base orderable matroids and of sets, and we discuss the relationship of our conjectures to other matroid conjectures.

Keywords: matroids, strongly base orderable, independence

73

On Massive Interconnection Graphs

German Hernandez, The University of Memphis and Universidad Nacional de Colombia

With the extremely fast growth of the communication networks, telephone and computer networks, a strong interest in the study of the interconnection graphs that arise in this communication systems had been shown recently [1] [3]. Another field in which interconnection graphs arise naturally is to model the interconnection between neurons in the brain. In this types of networks every device: neuron, telephone or computer, is modelled as a node; and every connection: synapsis, call or link is represented as a directed edge. We will consider that the connections between devices are established, maintained and terminated dynamically and, in general, in a random fashion. The interconnection graphs are directed graphs, are called here *massive* because the number of nodes is in the order of millions in computer netwoks, hundreds of millions telephone networks and thousens of billions in nerural netwoks. This graphs are *stochastic* because the set of directed edges is consider here as a stochastic process. In this paper definition of massive stochastic graphs that model the interconnection graphs is presented and some connections with random graphs [2] and limit theorems for functionals of random graphs [4] are explored.

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74

Future Directions: an Open Discussion on Tolerance Graphs

Martin Charles Golumbic, Bar-Ilan University; Ann Trenk, Wellesley College

Concluding the special session on tolerance graphs, we will lead an open discussion on future research directions and open problems.

75

Chromatic spectrum of mixed hypergraphs

Tao Jiang (Univ. Illinois), Dhruv Mubayi (Georgia Inst. Tech.),
Zsolt Tuza (Hungarian Acad. Sci.), Vitaly Voloshin (Moldovan Inst. Sci.),
Douglas B. West* (Univ. Illinois)

A mixed hypergraph is a triple $H = (X, \mathcal{C}, \mathcal{D})$, where X is the vertex set and each of $\mathcal{C}$, $\mathcal{D}$ is a list of subsets of X. A strict k-coloring of H is a surjection $c : X \rightarrow [k]$ such that each member of $\mathcal{C}$ has two vertices assigned a common value and each member of $\mathcal{D}$ has two vertices assigned distinct values. The feasible set of H is the set of integers k for which H has a strict k-coloring.

We prove constructively that a finite set of positive integers is the feasible set of some mixed hypergraph if and only if it omits 1 or is an interval containing 1. We give bounds on the size of the smallest realization; for the set $\{s, t\}$ with $2 \leq s \leq t-2$, the smallest realization has $2t - s$ vertices.

Keywords: coloring, hypergraph

76

Uniform Antimatroid Closure SpacesResearch supported in part by DOE grant DE-FG05-95ER25254

John E. Karro*, John L. Pfaltz ; University of Virginia

Often the structure of discrete sets can be described in terms of a closure operator. Recently, some study have been done of *antimatroid* closure spaces – closure spaces for which each closed set has a unique minimal generating set. One such example is that of convex geometries, in which the extreme points of a convex set generate the closed set. In this research we have been focused on *uniform* antimatroid closure spaces, in which all non-trivial closed sets have the same sized generator. We have shown there exist antimatroid closure spaces of any size, of which convex geometries are only a sub-family. Furthermore, while there is a good deal of variety between these uniform spaces, they are all structured in interesting ways. We have worked on a general analysis of these structures, and proved some characteristcs shared by all such spaces.

The issue of whether a planar convex geometry, that is a discrete set of points in in the plane, exists in which all convex configurations are triangles, with no quadrilaterals; or as quadrilaterals and triangles, with no pentagons; *etc.*, has fascinated combinatorial mathematicians for many years. Our results throw light on these kinds of questions, potentially leading to new approaches to problems such as the well-known Erdös-Szekeres conjecture concerning planar convex geometries.

Key Words: uniform antimatroid closure spaces convex geometries Erdös Szekeres

77

Just How Good Is That Bound?

Gary Chartrand, Western Michigan University
Dedicated to the Memory of E. A. Nordhaus

There are many results in graph theory that give, for a given graph G of order n and a graphical parameter f, bounds for $f(G)$ in term of n. There are also bounds in terms of n for combinations of $f(G_1)$ and $f(G_2)$ for various related graphs G_1 and G_2 of the same order n. Furthermore, results of this kind exist when more than one parameter is involved. Some results of this type are described when the parameters involve distance concepts, namely geodesics and convexity, and the bounds are investigated for varying degrees of sharpness.
Keywords: distance, geodesic, convexity

78

The Multiplicity of Parts in an Integer Composition and a Theorem of Szekeres

Nils R. Barth*, Harvard University
Anant P. Godbole, Michigan Technological University
Camillia Smith*, Michigan State University

Given a composition of n into k parts, chosen uniformly at random, let X be the number of part sizes that occur with multiplicity greater than 1. We show that for $k = o(n^{1/2})$, X asymptotically has a Poisson distribution with $\lambda = k^3/4n$. From this, we derive a formula of Szekeres regarding partitions of integers:

$$P(n,k) = \binom{n-1}{k-1} \exp\left\{\frac{-k^3}{4n}(1+O(k^2/n))\right\}.$$

Keywords: asymptotic distribution, composition, Poisson, Stein-Chen

Cyclic Mendelsohn triple systems with a cyclic resolution or a cyclic almost resolution

79

Miwako MISHIMA*, Gifu University

Constructions for a cyclic Mendelsohn triple system with a cyclic resolution or a cyclic almost resolution are provided. Furthermore, as an ingredient of such a design, some examples, and direct and recursive constructions for a μ-ply disjoint Mendelsohn difference family are given.
Keywords: BIB design; generalized triple system; Mendelsohn triple system; cyclic; resolvable; almost resolvable, Mendelsohn difference family; μ-ply disjoint.

80

To Change or Not to Change

Jean E. Dunbar*, Converse College
Teresa W. Haynes, Linda Lawson; East Tennessee State University

For a graph G, let $\gamma(G)$ represent the cardinality of a minimum dominating set. We say an edge in $E(G)$ ($E(gbar)$, respectively) is *stable* if its removal (respectively, addition) does not change $\gamma(G)$, and it is *critical* if its removal (addition) changes $\gamma(G)$. Similarly, a vertex is stable if its removal does not alter $\gamma(G)$ and critical if its removal changes $\gamma(G)$. Let $\mathcal{F}$ be the family of graphs such that every edge in $E(G)$ is stable. We investigate a subset of graphs in $\mathcal{F}$ for which each graph in the subset has at least one stable and one critical edge in $E(gbar)$ and at least one stable and one critical vertex. keywords: domination, critical,stable

81

The Steiner Distance Dimension of Graphs

Michael Raines* and Ping Zhang, Western Michigan University

For a nonempty set S of vertices of a connected graph G, the Steiner distance $d(S)$ of S is the minimum size among all connected subgraphs whose vertex set contains S. For an ordered set $W = \{w_1, w_2, \ldots, w_k\}$ of vertices in a connected graph G and a vertex v of G, the Steiner representation $s(v \mid W)$ of v with respect to W is the $(2^k - 1)$-vector

$$s(v \mid W) = (d_1(v), d_2(v), \ldots, d_k(v), d_{1,2}(v), d_{1,3}(v), \ldots, d_{1,2,\ldots,k}(v))$$

where $d_{i_1,i_2,\ldots,i_j}(v)$ is the Steiner distance $d(\{v, w_{i_1}, w_{i_2}, \ldots, w_{i_j}\})$. The set W is a *Steiner resolving set* for G if, for every pair u, v of distinct vertices of G, u and v have distinct representations. A Steiner resolving set containing a minimum number of vertices is called a *Steiner basis* for G, and the cardinality of a Steiner basis is the *Steiner distance dimension* of G, $\dim_S(G)$. In this talk, we present some results on the Steiner dimensions of several classes of graphs.

Keywords: Steiner distance, representation, dimension

82

Distances on subsets of a finite set

Kaddour Boukaabar, California University of PA

The comparison of hierarchical or non-hierarchical classifications is an important problem in the behavioral sciences. This comparison is usually done with distance functions that quantify the agreement or disagreement between two classifications of the same type. Several metrics on subsets and on partitions of a finite set, some of which are new, will be presented and discussed.

Key words: Finte set; partition; metric; median; remoteness.

83

Constructions of balanced incomplete block designs with nested rows and columns

Takaaki HISHIDA, Gifu University

A balanced incomplete block design with nested rows and columns, denoted by BIBRC(v, p, q, λ) is a block design in which the plots of the blocks are arranged in a $p \times q$ array such that

$$\lambda_{ij} = p\lambda_{ij}^{(R)} + q\lambda_{ij}^{(C)} - \lambda_{ij}^{(D)}$$

is constant not depending on the choice of points i, j, where $\lambda_{ij}^{(R)}$, $\lambda_{ij}^{(C)}$, $\lambda_{ij}^{(D)}$ are the number of times that the pair i, j occurs in the same row, column and array of blocks, respectively.

In this talk we will give some constructions of BIBRC.

Keywords: BIB design; nested rows and columns design.

84

REALIZABILITY OF (j,t)-CRITICAL GRAPHS

Teresa W. Haynes, East Tennessee State University
James B. Phillips* and Peter J. Slater, University of Alabama in Huntsville

In vertex-domination critical graphs, the removal of any single vertex causes the domination number of the graph to be reduced by one. For a connected graph G with domination number $\gamma(G) = j$, we have defined G to be (j,t)-critical if the removal of any t vertices from a packing reduces the domination number by exactly t. It will be shown that, given any integers j and t where $t \leq j$, there exists a (j,t)-critical graph. We consider the following general realizability question, given an integer j and a set of integers A, does there exist a graph that is (j,t)-critical if and only if $t \in A$? Some results involving restrictions of the general realizability question will be presented.Keyworks: Domination, Domination-critical

85

Which Sequences of Iterated Jump Graphs are Planar?

Donald W. VanderJagt, Grand Valley State University

For a graph G of size $m \geq 1$ and edge-induced subgraphs F and H of size r $(1 \leq r \leq m)$, the subgraph H is said to be obtained from F by an edge jump if there exist four distinct vertices u, v, w, and x in G such that $uv \in E(F)$, $wx \in E(G) - E(F)$, and $H = F - uv + wx$. The minimum number of edge jumps required to transform F into H is the jump distance from F to H. For a graph G of size $m \geq 1$ and an integer r with $1 \leq r \leq m$, the r-jump graph $J_r(G)$ is that graph whose vertices are the edge-induced subgraphs of size r of G and where two vertices of $J_r(G)$ are adjacent if and only if the jump distance between these subgraphs is 1. An infinite sequence $\{G_k\}$ of graphs is planar if every graph G_k is planar. For every positive integer r, all connected graphs G are determined for which the sequence $\{J_r^k(G)\}$ of iterated r-jump graphs of G is planar.

Keywords: distance, planar graph, jump graph

86

Light subgraphs of multigraphs embedded in compact 2-manifolds

Stanislaw Jendrol, P.J.Safárik University Kosice
Heinz-Juergen Voss*, Technical University Dresden

Fabrici and Jendrol' proved that each 3-connected plane graph with a k-path (a path of k vertices), contains a k-path such that each vertex has a degree at most $5k$. Further they showed that each 3-connected plane graph with at least k vertices contains a connected subgraph on k vertices such that each vertex has a degree at most $4k+3$ for each $k \geq 3$. Both bounds are sharp. We generalized these results to compact 2-manifolds M of Euler characteristic $\chi(M) < 0$. Three of our results are: each polyhedral map of M with a k-path contains a k-path such that each vertex has a degree at most $k\lfloor(5+\sqrt{49-24\chi(M)})/2\rfloor$. Equality for even $k \geq 2$. If G is a 3-connected multigraph on M without loops and 2-faces then the degree bound is $(6k-2\epsilon)(1+\chi(M)/3)$, where $\epsilon = 0$, if k is even, and $\epsilon = 1$, if k is odd. If G is a 3-connected graph on M, then the degree bound is $2 + \lfloor(6k-6-2\epsilon)(1+\chi(M)/3\rfloor$ for $k \geq 4, \chi(M) < 0$ and for $k \geq 2, \chi(M) < -3$. All the bounds for G are the best possible.

Keywords: path, light graph, compact 2-manifold, embeddings of graphs.

88

Realizability of the Criticality Index of Total Domination

L.C. van der Merwe* and C.M. Mynhardt, University of South Africa;
T.W. Haynes, East Tennessee State University

For a graph $G = (V, E)$, a set $S \subseteq V$ is a *total dominating set* if every vertex in V is adjacent to some vertex in S. The smallest cardinality of any total dominating set is the total domination number $\gamma_t(G)$. For an arbitrary edge $e \in E(\overline{G})$, $\gamma_t(G) - 2 \leq \gamma_t(G+e) \leq \gamma_t(G)$. The *criticality index* of an edge $e \in E(\overline{G})$ is $ci(e) = \gamma_t(G) - \gamma_t(G+e)$. Let $E(\overline{G}) = \{e_1, \ldots, e_{\overline{m}}\}$ and let $S = \sum_{j=1}^{\overline{m}} ci(e_j)$. Then the *criticality index* of G is $ci(G) = S/\overline{m}$. We construct graphs G such that $ci(G) = k$ for any rational number k, $0 \leq k \leq 2$. Keywords. Total domination, total domination critical graphs, criticality index.

89

A High Girth 4-Chromatic Unit Distance Graph in the Plane

Paul O'Donnell, Rutgers University

In a relative of the chromatic number of the plane problem, Erdős asked if there were triangle-free, 4-chromatic unit distance graphs in the plane. In 1979, Wormald showed that Tutte's construction of a girth 5, 4-chromatic graph was such a graph. This idea can be generalized to show the existence of arbitrary girth, 4-chromatic unit distance graphs in the plane. Hypergraphs with large girth and chromatic number are used in the construction. The simple structure of the graph permits a straightforward embedding as a unit distance graph in the plane.
Keywords: unit distance graph, girth, chromatic number

90

Quadrangular Cayley Graphs Embedded on the Torus

Linda Valdés, San José State University

All groups with Cayley graphs that can be embedded on the torus so that all faces are quadrangles are found. The groups are distinguished by the symmetry of their embeddings: that is, strong or weak symmetry or nonsymmetry.

Keywords: Cayley graphs, embeddings, torus

91

Results on Lotto Designs

Ben Li* and John Van Rees, University of Manitoba

There are many lotteries in the world. People are interested in devising efficient betting schemes in playing these lotteries. The way that a lottery works is the following : From n numbers, the better chooses k numbers on a ticket and at a later time, the governement picks p numbers from the n possible numbers. If any of the tickets match t or more of the p numbers chosen by the government, then a prize is given to the ticket holder. The larger the value of t, the larger the prize. For very small t, no prize is given. We formally define a $LD(n,k,p,t;b)$ Lotto Design to be a set of b k-sets (blocks or tickets) of an n-set such that any p-set intersects at least one k-set in t or more elements. Let $L(n,k,p,t)$ denote the minimum number of blocks in any $LD(n,k,p,t;b)$. We introduce construction techniques for determining upper bounds for $L(n,k,p,t)$ and lower bound formulas for determining lower bounds for $L(n,k,p,t)$. We also provide tables for lower and upper bounds on $L(n,k,p,t)$ for $5 \leq n \leq 20$, $2 \leq k \leq 20$, $2 \leq p \leq 20$ and $2 \leq t \leq min\{k,p\}$. **Keywords** : Lotto Designs, construction.

92

Total Irredundance in Graphs

O. Favaron, T.W. Haynes*, S.T. Hedetniemi, D.J. Knisley

For a graph $G = (V, E)$, a set $S \subseteq V$ is a total irredundant set if for every vertex $v \in V$, $N[v] - N[S - \{v\}]$ is not empty. The total irredundance number $ir_t(G)$ is the minimum cardinality of any maximal total irredundant set of G and the upper total irredundance number $IR_t(G)$ is the maximum cardinality of any total irredundant set of G. We investigate total irredundance numbers of graphs.
Keywords. Irredundance, total irredundance, domination, total domination.

93

Wiener Polynomials of Recursively Defined Trees

John Frederick Fink, University of Michigan-Dearborn, Dearborn, MI 48128-1491

The *Wiener polynomial* of a connected graph G is the polynomial $W(G; q) = \sum q^{d(u,v)}$, where the sum is taken over all unordered pairs $\{u, v\}$ of distinct vertices in G, and $d(u, v)$ is the distance between vertices u and v. Thus, $W(G; q)$ is a generating function for the distance distribution $dd(G) = (D_1, D_2, \ldots, D_t)$, where D_i is the number of unordered pairs of distinct vertices at distance i from one another and t is the diameter of G. The derivative $W'(G; 1)$ is the much studied Wiener index $W(G)$. If u is a specified vertex of a connected graph G, the *Wiener polynomial of G relative to u* is the polynomial $W_u(G; q) = \sum q^{d(u,v)}$ where the sum is taken over all vertices v in G, including $v = u$. Among other things, we find the Wiener polynomial for full k-ary trees and the Wiener polynomial relative to the root vertex for Fibonacci trees.

Key words: Wiener index, Wiener polynomial, distance, tree.

94

Embedding Digraphs on Orientable Surfaces

C. Paul Bonnington, Marston Conder, Patricia McKenna*, Margaret Morton
University of Auckland, Auckland, New Zealand

We consider a notion of embedding digraphs on orientable surfaces, applicable to digraphs in which the indegree equals the outdegree at each vertex, i.e., Eulerian digraphs. This idea has been considered before, in the context of "compatible Euler tours" or "orthogonal A-trails", by Andersen et al and by Bouchet. This prior work has mostly been limited to embeddings of Eulerian digraphs on predetermined surfaces, and to digraphs with underlying graphs of maximum degree at most four. In our work, a foundation is laid for the study of all Eulerian digraph embeddings. Results are proved which are analogous to those fundamental to the theory of undirected graph embeddings, such as Duke's Theorem, and an infinite family of digraphs which demonstrates that the genus range for an embeddable digraph can be any nonnegative integer is given. We show that it is possible to have genus range equal to one, with arbitrarily large minimum genus, unlike in the undirected case. The difference between the minimum genera of a digraph and its underlying graph is considered, as is the difference between maximum genera. We say that a digraph is upper-embeddable if it can be embedded with two or three regions, and prove that every regular tournament is upper-embeddable.

Key words: digraph embeddings, compatible Euler tours, orthogonal A-trails

ENUMERATION OF 1-ROTATIONAL STEINER 2-DESIGNS OF SMALL ORDER

95

Marco Buratti and Fulvio Zuanni*, Universita' de L'Aquila, Italy

By S(2,k,v) we denote a Steiner 2-design of order v and block size k while by RS(2,k,v) we denote a "resolved" S(2,k,v), i.e., a resolvable S(2,k,v) together with a specific resolution of it. We say that a S(2,k,v) or RS(2,k,v) is 1-rotational if it admits an automorphism group fixing one point and acting regularly on the others. About the enumeration of such designs we recall that: (PR) (Phelps and Rosa) there are exactly 35 1-rotational S(2,3,27)'s; (JV) (Jimbo and Vanstone) there are exactly 47 1-rotational RS(2,3,27)'s. In this talk we briefly illustrate how we have got the following results: (BZ1) there are exactly 500 1-rotational RS(2,3,33)'s (436 are over the cyclic group and the remaining 64 are over the dicyclic group); (BZ2) there are exactly 4 abelian 1-rotational S(2,4,49)'s; (BZ3) there are exactly 22 1-rotational RS(2,4,52)'s.

Nordhaus-Gaddum results for CO-irredundance in graphs.

96

Ernie Cockayne,Devon McCrea(Univ. of Victoria,Canada)
Christine Mynhardt(Univ.of South Africa)

We prove that the sum and product of upper CO-irredundance numbers of an n-vertex graph and its complement are bounded above by $n + 2$ and $((n + 2)^2)/4$ respectively and exhibit extremal graphs for these inequalities.
Keywords: graph,CO-irredundance,Nordhaus-Gaddum.

Tuesday, March 9, 1999
5:00 PM

97

Orientation distance grap

Gary Chartrand, David Erwin*, Michael Raines, Ping Zhang

For two nonisomorphic orientations D and D' of a graph G, the orientation distance $d_o(D, D')$ between D and D' is the minimum number of arcs of D whose directions must be reversed to produce an orientation isomorphic to D'. The orientation distance graph $\mathcal{D}_o(G)$ of G has the set $\mathcal{O}(G)$ of pairwise nonisomorphic orientations of G as its vertex set and two vertices D and D' of $\mathcal{D}_o(G)$ are adjacent if and only if $d_o(D, D') = 1$. For a nonempty subset S of $\mathcal{O}(G)$, the orientation distance graph $\mathcal{D}_o(S)$ of S is the induced subgraph $\langle S \rangle$ of $\mathcal{D}_o(G)$. A graph H is an orientation distance graph if there exists a graph G and a set $S \subseteq \mathcal{O}(G)$ such that $\mathcal{D}_o(S)$ is isomorphic to H. In this case, H is said to be an orientation distance graph with respect to G. This paper deals primarily with orientation distance graphs with respect to paths. For every integer $n \geq 4$, it is shown that $\mathcal{D}_o(P_n)$ is hamiltonian if and only if n is even. Also, the orientation distance graph of a path of odd order is bipartite. Furthermore, every tree is an orientation distance graph with respect to some path, as is every cycle, and for $n \geq 3$ the clique number of $\mathcal{D}_o(P_n)$ is 2 if n is odd and is 3 otherwise. Keywords: distance, digraphs, orientations

98

New Results for the Martin Polynomial

Joanna A. Ellis-Monaghan St. Michael's College

This talk will discuss the Martin Polynomial and some of its properties. Recall that an Eulerian graph can be covered by a cycle which traverses each of its edges exactly once. The Martin polynomial of a graph is a one variable polynomial whose coefficients enumerate the families of Eulerian subgraphs in the original graph. More information about the graph is encoded in the polynomial, but as with many other graph polynomials, extracting it can be very difficult. A new identity for the Martin polynomial makes it possible to get combinatorial interpretations for valuations of it fairly easily by using induction. The talk will conclude with a brief discussion of how the new results for the Martin polynomial derive from its being a special case of a much more general new graph polynomial with especially rich algebraic properties.

Keywords: Martin polynomial, graph invariants, Eulerian graphs and orientations, Hopf algebras, graph polynomials, algebraic combinatorics.

99

BTD Configuration Dependencies

Margaret Francel The Citadel

A **balanced ternary design** (BTD), with parameters (V;B; ρ_1,ρ_2,R;K, Λ) is a collection of B blocks (multisets) on V elements such that each element occurs R times in the desgin, singly in ρ_1 blocks and doubly in ρ_2 blocks, and each pair of distinct elements occurs Λ times in the design.

A **n-block design configuration** is a set of n distinct blocks of a design. The complete set of n-block configurations of a design can be partitioned into configuration classes according to the relationships that exist between elements and blocks within the configuration. Each class can be represented by a template that describes these relationships.

Given an n-configuration class in a BTD(V;B;R,ρ_1,ρ_2; K;Λ), a **n-configuration formula** for the class is a formula that calculates the number of n-configurations in the class. A set of m-configurations, $m \leq n$, is said to be generating set for the n-configuration formulae, if every n-configuration formula can be expressed as a rational expression in the design parameters, plus a linear combination of the counts of the configurations in the generating set. Configuration formulae can be classified by the number and type of configurations from the generating set that appear in the formula.

The talk will describe 2 and 3 block configuration templates, formulae, and generating sets in BTD's with block size three and repetition factor two or greater. The classification of the formulae will also be discussed.

Key Words: balanced ternary design, design configuration

100

Graphs with large least domination number

Michael A. Henning, University of Natal, South Africa

Let $G = (V, E)$ be a graph. A set $S \subseteq V$ is a dominating set if every vertex of $V - S$ is adjacent to some vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set of G. A dominating set D is a least dominating set if $\gamma(\langle D \rangle) \leq \gamma(\langle S \rangle)$ for any dominating set S, and $\gamma_\ell(G)$ is the minimum cardinality of a least dominating set. Sampathkumar (1990) conjectured that $\gamma_\ell(G) \leq 3n/5$ for every connected graph on $n \geq 2$ vertices. This conjecture was proven by Favaron (1996). We shall characterise graphs G of order n that are edge-minimal with respect to satisfying G connected and $\gamma_\ell(G) = 3n/5$. Furthermore, we construct a family of graphs G of order n that are not cycles and are edge-minimal with respect to satisfying G connected, $\delta(G) \geq 2$ and $\gamma_\ell(G) = 3n/5$.

101 **The Steiner Number and Steiner Sets in Graphs**

Gary Chartrand and Ping Zhang*, Western Michigan University

For a connected graph G of order at least 3 and a set W of vertices in G, a tree T contained in G is a Steiner tree with respect to W if T is a tree of minimum order whose vertex set contains W. The Steiner interval $S(W)$ consists of all vertices in G that lie on some Steiner tree with respect to W. The set W is a Steiner set for G if $S(W) = V(G)$. The minimum cardinality among the Steiner sets of G is the Steiner number $s(G)$. We discuss some results in this area.

Keywords: distance, Steiner distance

102 **Drawing Graphs on a Sphere**

Nathaniel Dean, Rice University

This paper reports on heuristics for drawing graphs on the surface of a sphere. Here are some applications. (1) How do you pack n spherical caps of equal and largest possible radius on the surface of a sphere? (2) How do you minimize the energy of a system of electrons constrained to lie on the surface of a sphere? (3) How should you draw a 3-connected planar graph on the surface of a sphere? Keywords: sphere, embedding, layout, drawing, optimization

103 **Packing Trees into the Complete Graph**

Edward Dobson, Mississippi State University

Gyárfás and Lehel conjectured that every sequence of trees $T_1, \ldots, T_n$ such that $|V(T_i)| = i$ can be packed into the complete graph. We prove that every sequence of trees $T_1, \ldots, T_n$ for which $|V(T_i)| \leq 14i/15$ and there exists $x_i \in V(T_i)$ such that $T_i - x_i$ has at most $i/15$ vertices of degree at least 1 can be packed into K_n.

104 An upper bound for the domination number of the queens graph

A. P. Burger and C. M. Mynhardt*, University of South Africa

The queens graph Q_n has the squares of the $n \times n$ chessboard as its vertices; two squares are adjacent if they are in the same row, column or diagonal. It is shown that the domination number of Q_n, *i.e.*, the smallest number of queens necessary to cover the $n \times n$ board, is at most $\frac{16}{30}n + \mathcal{O}(1)$.

Key words: chessboard, queens graph, domination

A New Proof Of The Independence Ratio Of Triangle-Free Cubic Graph

105

Christopher Carl Heckman*, Robin Thomas, Georgia Institute of Technology

Staton proved that every triangle-free graph on n vertices with maximum degree three has an independent set of size at least $\frac{5}{14}n$. A simpler proof was found by Jones. We give a yet simpler proof, and use it to design a linear-time algorithm to find such an independent set.

Key words: Triangle-free, independent set, linear-time algorithm

106

The Isometric Path Number

Shannon L. Fitzpatrick, Acadia University, Nova Scotia

An isometric path is merely any shortest path between two vertices.The **isometric path number** of a graph G is the minimum number of isometric paths required to cover the vertices of G. The isometric path number has been determined for grid graphs and the hypercubes Q_n where $n = 2^t + 1$. I will discuss these results as well as upper and lower bounds on the isometric path number for 3-dimensional grids.

THE GROWTH OF HEREDITARY PROPERTIES OF GRAPHS (work in progress)

107

Jozsef Balogh, Bela Bollobas, and David Weinreich*
University of Memphis

A hereditary graph property is an infinite class of graphs closed under isomorphism and under taking induced subgraphs. The most natural measure of the 'size' of a property P is $|P^n|$, the number of graphs in P with vertex set $[n] = \{1, \ldots, n\}$. Scheinerman and Zito [1] proved in 1994 that the growth of $|P^n|$ is severely constrained: it can be constant, polynomial, exponential, factorial or superfactoral. We were able to describe the functions that occur for properties P where $|P^n| \leq n^n$. Further, we can characterize the general structure of these hereditary properties.

The talk "Minimal and Aberrant Hereditary Properties of Graphs" contains further results.

References

[1] E.R. Scheinerman and J. Zito, On the size of hereditary classes of graphs, J. Combinatorial Theory Ser. B. 61 (1994) 16–39.

108

On Minimum Spans of No-hole T-Colorings

Daphne Der-Fen Liu*, California State University
Roger K. Yeh, Feng Chia University, Taiwan

We study a variation of T-coloring which arose from the channel assignment pro blem introduced by Hale [1980]. Given a non-negative integral set T containing 0, a T-coloring on a graph G assigns a non-negative integer (color) to each vertex with the restriction that the difference of colors of any two adjacent vertices does not fall in T. A no-hole T-coloring is a T-coloring such that the colors used must be consecutive. Roberts [1993] and Sakai and Wang [1993] studied no-hole T-colorings for $T = \{0, 1\}$ and $\{0, 1, 2, \ldots, r\}$, respectively. We explore this problem to other sets T. A characterization of the existence of no-hole T-colorings is obtained immediately by combining graph homomorphism and a special family of gra phs named T-graphs. For given T and G, the consecutive T-span, $\text{csp}_T(G)$, is the minimum span among all possible no-hole T-colorings of G if there exists one; otherwise $\text{csp}_T(G) = \infty$. If G has a no-hole T-coloring, then $|V(G)| - 1$ is an upper bou nd of $\text{csp}_T(G)$; we show for some families of sets T, this upper bound is attained by large T-graphs.

109

Hamilton Path Heuristics and the Middle Two Levels Problem

Ian Shields*, IBM, Research Triangle Park, NC
Carla D. Savage, North Carolina State University

We describe a polynomial-time heuristic for searching for Hamilton paths and an application of the heuristic. The notorious middle two levels problem is to find a Hamilton cycle in the middle two levels of the Hasse diagram of $\mathcal{B}_{2k+1}$ (the partially ordered set of subsets of a $2k+1$-element set ordered by inclusion). Previously, the best known result, due to Moews and Reid in 1990, was that $\mathcal{B}_{2k+1}$ is Hamiltonian for all positive k through $k = 11$. We describe the application of the heuristic to finding Hamilton cycles in $\mathcal{B}_{2k+1}$ and an extension of the previous best known results. This also improves the best lower bound on the length of a longest cycle in $\mathcal{B}_{2k+1}$ for any k.

110

A Result about dense k-linked Graphs

Florian Pfender, Emory University

A graph on n vertices is said to be k-linked, if $n \geq 2k$ and for every set $\{x_1, \ldots, x_k, y_1, \ldots, y_k\}$ of $2k$ distinct vertices there are vertex disjoint paths $P_1, \ldots, P_k$, such that P_i joins x_i to y_i for all $i \in \{1, \ldots, k\}$. Recently Bollobás and Thomason proved, that every $22k$-connected graph is k-linked. In this talk it is shown how this bound can be reduced to $2k$, if the minimum degree $\delta(G) \geq n/3 + 16k$.

111

Minimal and aberrant hereditary properties of graphs

Jozsef Balogh*, Bela Bollobas, and David Weinreich; University of Memphis

A hereditary graph property is an infinite class of graphs closed under isomorphism and under taking induced subgraphs. The most natural measure of the 'size' of a property $\mathcal{P}$ is $|\mathcal{P}^n|$, the number of graphs in $\mathcal{P}$ with vertex set $[n] = \{1, \ldots, n\}$. Scheinerman and Zito [1] proved in 1994 that the values which $|\mathcal{P}^n|$ can take are severely constrained, and the authors have been able to describe the functions allowed. In this talk, we describe minimal hereditary properties at each type of growth: constant, polynomial, exponential, factorial and superfactoral. We also answer a question of [1], showing that there are classes of properties for which

$$\lim_{n\to\infty} \frac{\log|\mathcal{P}^n|}{n \log n}$$

need not exist. Other similar results will be mentioned.

Note: This is work in progress.

The talk "The growth of hereditary properties of graphs" introduces our results.

References

[1] E.R. Scheinerman and J. Zito, On the size of hereditary classes of graphs, J. Combinatorial Theory Ser. B. 61 (1994) 16–39.

112

Aspects of edge-list colourings

A.J.W. Hilton* and D.S.G. Stirling (Reading University, England)

Recently Borodin, Kostochka and Woodall improved Galvin's seminal result that the edge list chromatic number $\chi'_\ell(G)$ of a bipartite multigraph is $\Delta(G)$ by showing that if each edge $e = uv$ of G has a list of length $\max\{d(u), d(v)\}$ then G is list-colourable. Also recently, Hilton, Slivnik and Stirling used Galvin's result to show that, for any multigraph G, the 2-improper edge list chromatic number of G is $\left\lceil \frac{\Delta(G)}{2} \right\rceil$, and that, if the lists are all of size at least $\left\lceil \frac{\Delta(G)}{2} \right\rceil$ then G has a 2-improper edge list colouring that is edge-bounded (meaning that no multiple edge has "too many" edges of any colour). Here we strengthen this result by using the Borodin, Kostochka and Woodall result instead of Galvin's result. We also give a number of other analogues and generalizations.

We also give a number of simple examples to show how the existing results on edge-list colouring can be used in school timetabling as well as in some other scheduling problems.

Key words. List colouring, edge colouring, timetabling, scheduling.

113

Nonconstructive is BETTER for sorting

W.Gasarch*, E.Golub, C.Kruskal, U. of MD at College Park

There are several known parallel algorithms for sorting n elements in k rounds, where k is a constant. The key parameter to minimize is the number of comparisons per round. In all of the algorithms a graph is used to specify which comparisons to make. The number of edges in the graph is the number of processors needed. In some of the algorithms (due to Pippenger) the probabilistic method is used to show that graphs G with the following two properties exist: (1) G has a 'small' number of edges, and (2) if G is used to compare elements then 'a lot' of information can be extracted. We have coded up these 'nonconstructive' algorithms as well as several known constructive ones. The nonconstructive algorithms are easier to code and use fewer comparisons-per-round.

Keywords: parallel sorting, expander graph, probabilistic method

114

A Note on Graph Pebbling

A. Czygrinow, G. H. Hurlbert (*), H. A. Kierstead, W. T. Trotter Arizona State University

For a positive integer d, let k(d) denote the least positive integer so that for every graph G, with diameter at most d and connectivity at least k(d), the pebbling number of G equals the number of vertices of G. The existence of the function k was conjectured by Clarke, Hochberg and Hurlbert, who showed that if the function k exists, then it must satisfy $k(d) = Omega(2^d/d)$. We show that k exists and satifies $k(d) = O(2^{2d})$.

key words: connectivity, diameter, pebbling

EXCLUDING MINORS IN NONPLANAR GRAPHS OF GIRTH AT LEAST FIVE

115

Robin Thomas and Jan Thomson*, Georgia Institute of Technology

We show that any nonplanar graph with minimum degree at least three and no cycle of length less than five has a minor isomorphic to the Petersen graph with one edge deleted (P_10). We deduce the following weakening of Tutte's Four Flow Conjecture: every 2-edge-connected graph with no minor isomorphic to P_10 has a nowhere zero four flow. This extends a result of Kilakos and Shepherd who proved the same for cubic graphs.

116

Progress on the Hall-Number-Two Problem

Ch. Eslahchi, Institute for Studies in Theoretical Physics and Mathematics (Tehran), A. J. W. Hilton, The University of Reading (England), and P. D. Johnson Jr.*, Dept. of Discrete and Statistical Sciences, Auburn University

Throughout, G is a simple graph and L is a list of assignment to the vertices of G. The Hall number of G, denoted $h(G)$, is the smallest positive integer among those m such that Hall's condition on G and L and the requirement that $|L(v)| \geq m$ for all $v \in V(G)$ are sufficient for the existence of a proper L-coloring of G. The Hall number turns out to be useful in investigating when the chromatic number and the choice (list-chromatic) number are equal. In this investigation, it would be a powerful good if the graphs with Hall number $\leq k$ were characterized, for each k. It is known that these have a forbidden-induced-subgraph characterization, for each k. For $k = 1$ the minimal forbidden induced subgraphs are C_n, $n \geq 4$, and K_4-minus-an-edge. In this work we add the following to an already sizeable list of minimal forbidden induced subgraphs for the case $k = 2$:

(i) K_5 with an "ear" (a path with its end-vertices, only, in K_5) of length 2; (ii) K_4 with an ear of length > 2; (iii) an odd cycle with two more vertices making two triangles based on non-incident edges on the cycle; (iv) an even cycle with three more vertices making three triangles based on mutually non-incident edges on the cycle; and (v) any cycle with two more vertices making two triangles based on two incident edges of the cycle.

Key words and phrases: Hall's condition, independence, choice number, chromatic number, system of distinct representatives, list coloring.

117

Stop Minding Your P's and Q's: A Simplified $O(n)$ Planar Embedding Algorithm

John Boyer and Wendy Myrvold*, University of Victoria

A graph is *planar* if it can be drawn on the plane with no crossing edges. This paper presents a new method for performing linear time planar graph embedding which avoids some of the complexities of previous approaches (including the need to first *st*-number the vertices). Our algorithm is similar to the algorithm of Booth and Lueker which uses a data structure called a PQ-tree. The P-nodes in a PQ-tree represent parts of the partially embedded graph that can be permuted, and the Q-nodes represent parts that can be flipped. We avoid the use of P-nodes by not connecting pieces together until they become biconnected. We avoid Q nodes by using a simple data structure which allows biconnected components to be flipped in $O(1)$ time.

Keywords: planar graph embedding algorithm.

118

The Average Connectivity of a Graph

L.W. Beineke, Indiana University Purdue University at Fort Wayne
O.R. Oellermann*, The University of Winnipeg, Manitoba, Canada
R.E. Pippert, Indiana University Purdue University at Fort Wayne

The average connectivity of a graph is the average, over all pairs of vertices, of the maximum number of internally disjoint paths that connect such a pair of vertices. Sharp bounds on this parameter are given and necessary conditions for the average connectivity to equal its connectivity are presented and constructions of graphs with equal connectivity and average connectivity are presented. We close with a number of open problems.

119

Labeled $K_{2,t}$-Minors in 3-Connected Planar Graphs

Thomas Böhme*, Technical University Ilmenau, Germany
Bojan Mohar, University of Ljubljana, Slovenia

Let G be a 3-connected planar graph and let $U \subseteq V(G)$. It is shown that G contains a minor isomorphic to the complete bipartite graph $K_{2,t}$ such that t is large and each vertex of degree 2 in $K_{2,t}$ corresponds to some vertex of U if and only if there is no small face cover of U. This result cannot be extended to 2-connected planar graphs.

Keywords: planar graphs, minors

120

A Coloring Problem

Hong-Jian Lai* and Xiankun Zhang; West Virginia University

Let G be a graph with a fixed orientation and let A be an Abelian group. Let $F(G,A)$ denote the set of all functions f: $E(G) \mapsto A$. The graph G is A-colorable if for any function $f \in F(G,A)$, there is a function c: $V(G) \mapsto A$ such that for every directed $e = uv \in E(G)$, $c(u) - c(v) \neq f(e)$. It is shown that whether G is A-colorable is independent of the choice of the orientation. The **group chromatic number** $\chi_1(G)$ of a graph G is the minimum m such that G is A-colorable for any Abelian group A of order at least m under a given orientation D. In [J. Combin. Theory Ser. B, 56 (1992), 165-182], Jaeger *et al* proved that if G is a simple planar graph, then $\chi_1(G) \leq 6$. We prove in this paper that if G is a simple graph without a K_5-minor, then $\chi_1(G) \leq 5$.

Keywords: chromatic number, coloring.

Wednesday, March 10, 1999
11:30 AM

121 A Fast Algorithm for the Maximum Clique Problem

Patric R. J. Östergrd, Helsinki University of Technology

We present an exact branch-and-bound algorithm for the maximum clique problem—which is computationally equivalent to the maximum independent set problem—with a new, effective pruning strategy. Comparisons carried out with previously published algorithms show a performance that is slightly better for random graphs, and considerably better for some special graphs.

The algorithm works particularly well in the search for binary error-correcting codes when the words of a Hamming space are listed in lexicographic order (these words are the vertices of the graph, and two vertices are adjacent exactly when they are at Hamming distance greater than or equal to d—which is fixed—apart). The algorithm played a central role when we recently proved that the maximum sizes of binary one-error-correcting codes of length 10 and 11 are 72 and 144, respectively.

We finally show how the algorithm can be generalized to find maximum-weight cliques in weighted graphs.

Keywords: Branch-and-bound algorithm, graph algorithm, maximum clique problem.

122 Toughness and (k, r)-Factor-Critical Graphs

Rui Xu, Shandong University, P. R. China
Qinglin Yu*, University College of The Cariboo, Canada

A graph G is said (k, r)-factor-critical if for any vertex set S of G with $|S| = r$, $G - S$ has a k-factor. In this paper we prove that if a graph G has toughness $t(G) \geq k \geq 2$, then G is (k, r)-factor-critical for any non-negative integer r such that $r \leq 2t(G) - k$. This confirms a conjecture by Shi et al.

On Possible Counterexamples to Negami's Planar Cover Conjecture

Petr Hliněný* and Robin Thomas, Georgia Institute of Technology

123 A graph H is a cover of a graph G if there exists a mapping φ from $V(H)$ onto $V(G)$ such that for every vertex v of G, φ maps the neighbours of v in H bijectively to the neighbours of $\varphi(v)$ in G. Negami conjectured in 1987 that a connected graph has a finite planar cover if and only if it embeds in the projective plane.

It follows from the results of Archdeacon, Fellows, Negami, and the first author that the conjecture holds as long as $K_{1,2,2,2}$ has no finite planar cover, but those results do not seem to imply anything about possible counterexamples. We show that that there are, up to obvious constructions, at most 16 possible counterexamples to Negami's conjecture.

Keywords: planar covers; projective planar graphs; Negami's conjecture; graph minors

Colouring Graphs with Prescribed Induced Cycle Lengths

Bert Randerath, University at Cologne, Germany
124 Ingo Schiermeyer*, Technical University Cottbus, Germany

In this talk we study the chromatic number for graphs with two prescribed induced cycle lengths. It is due to Sumner that triangle-free and P_5-free or triangle-free, P_6-free and C_6-free graphs are 3-colourable. A canonical extension of these graph classes is $\mathcal{G}^I(4,5)$, the class of all graphs whose induced cycle lengths are 4 or 5. Our main result states that all graphs of $\mathcal{G}^I(4,5)$ are 3-colourable. Moreover, we present polynomial time algorithms to 3-colour all triangle-free graphs G of this kind. Thus, every $G \in \mathcal{G}^I(n_1, n_2)$ with $n_1, n_2 \geq 4$ is 3-colourable.

Furthermore, we consider the related problem of finding a χ-binding functions for the class $\mathcal{G}^I(n_1, n_2)$. Because of our previous results we only have to consider $\mathcal{G}^I(3, n)$. Recently, Rusu proved that all graphs of $\mathcal{G}^I(3, 2q)$ are perfect $(f(\omega) = \omega)$ for any $q \geq 3$. [Subclass $\mathcal{G}^I(3) \simeq$ chordal graphs.] Gyárfás conjectured in 1987, motivated by the Strong Perfect Graph Conjecture, that there exists a χ-binding function for $\mathcal{G}^I(3,4)$. We have shown that there exists no linear χ-binding function for $\mathcal{G}^I(3,4)$. For the remaining case n odd we expect that there exists a linear χ-binding function for $\mathcal{G}^I(3, 2p+1)$.

125

FINDING SHORT PATHS IN THE ROTATION GRAPH OF BINARY TREES

Rodney O. Rogers*
University of Central Florida, Embry-Riddle Aeronautical University

Finding the minimum rotation distance between two n-node binary trees is an interesting and difficult problem whose time complexity is unknown. The problem is equivalent to the finding a shortest path between two nodes in the Rotation Graph R(n) of Binary Trees, where each vertex represents an n-node binary tree, and two vertices are connected by an edge if and only if the corresponding trees differ by a single rotation. The vertices in R(n) can also be labelled with triangulations of convex (n+2)-gons, with adjacent triangulations differing by a single diagonal flip. We present an algorithm to find short paths in R(n) using the correspondence between binary tree edge rotations and triangulation diagonal flips. The time complexity of the algorithm is O(n log n), and the path it constructs always has less than twice the optimal length. We also adduce triangulation pairs where the algorithm does not perform well, and suggest how the algorithm might be modified to improve its performance.

126

On 2-factors containing specified edges

Ralph Faudree, University of Memphis;
Ervin Győri*,Rényi Inst. Math., Budapest;
Richard Schelp, University of Memphis

We find sufficient degree and degree sum conditions for the existence of 2-factors of $k \geq 2$ cycles in a graph G, each cycle containing exactly one of k arbitrarily prescribed independent edges. Let us mention that depending on the ratio of the number of prescribed edges and the number of vertices, we have degree conditions of different form. We present examples showing that these degree conditions are sharp.

127

Biclique Covers and Partitions of Bipartite Graphs and Digraphs and Related Matrix Ranks of {0,1}-Matrices

Faun Doherty, J. Richard Lundgren*, & Daluss Siewert
University of Colorado at Denver

During the past twenty years there has been considerable research on the biclique cover and partition numbers of bipartite graphs and digraphs and several related matrix ranks including the boolean rank, nonnegative integer rank, term rank, and the real rank. In this talk we will focus on classes of graphs and digraphs where the numbers are equal as well as classes of matrices where some or all of the ranks are equal. Bipartite graphs considered will include 4-cycle-free, domino-free, and minimal elementary. Digraphs considered will include regular and near-regular tournaments, unipathic, and min-strong. Several open problems will be presented.

Key Words Biclique Cover and Partition, Tournament, Min-Strong and Unipathic Digraphs, Domino-Free

128

Binary Trees Partitions

Ronald D. Dutton, School of Computer Science, and Robert C. Brigham*, Department of Mathematics, University of Central Florida, Orlando Florida 32816

The vertices of a graph are partitioned into sets R, B, and C such that the subgraphs induced by V(G) – R and V(G) – B are isomorphic and |C| is as small as possible. This optimization problem is solved when G is a binary tree having exactly one vertex of degree two or G is any tree all of whose vertices have degree one or three. Two solutions are possible in the latter case and the trees associated with each are characterized. Key words: trees, coloring

129 **The Tower of Hanoi**

Andreas M. Hinz, Technical University Munich

Invented more than a hundred years ago by the French number theorist Edouard Lucas as a mathematical puzzle, the *Tower of Hanoi* (TH) has recently attracted computer scientists in connection with recursion and complexity of algorithms. This revived interest resulted in a number of new discoveries about its rich mathematical structure. The problem consists in transferring n discs of mutually different diameter, originally stacked in natural order on one or more of three pegs, to a designated final position, usually a "tower" on one of the pegs, subject to the *devine rule* never to place a larger disc on a smaller one. The resulting *state graph* H_n and optimal sequence of (triples of) moves lead to surprising connections to other mathematical objects when n tends to infinity, in the latter case to *square-free sequences*, in the former case to the fractal structure of the *Sierpiński gasket* (SG) and *Pascal's arithmetical triangle* (AT). It is, for instance, possible to deduce the *average distance* on the SG from the corresponding values on H_n; it turns out to be $\frac{466}{885}$ (of the diameter). On the other hand, the fractal structure of SG is the same as that of the distribution of odd entries in AT and so, by transitivity, there are links between TH and AT mod 2, essentially through some intriguing combinatorial functions, which in turn are connected to the so-called *Stern-Brocot array*. They are related to an unsolved decision problem for finding a shortest path between two arbitrary states of TH. The most challenging open question, however, is the solution for TH with more than three pegs.

130 **The Periphery of Edge-Deleted Graphs**

John Koker, Hosien Moghadam, Steven J. Winters * University of Wisconsin

The eccentricity of a vertex v of a connected graph G is the distance from v to a vertex farthest from v in G. For a vertex v in a 2-edge-connected graph G, the edge-deleted eccentricity of v is the maximum eccentricity of v in G - e over all edges e of G. The edge-deleted center of a graph is the subgraph induced by those vertices having minimum edge-deleted eccentricity while the edge-deleted periphery of a graph is the subgraph induced by those vertices having maximum edge-deleted eccentricity. This talk presents necessary and sufficient conditions for a graph to be the periphery of an edge-deleted graph. In addition, we investigate the peripheral appendage number in edge-deleted graphs.

Key words: eccentricity, center, periphery, edge-deleted graphs, appendage number

131 **"Turán Related Problems for Hypergraphs"**

Brendan Nagle, Emory University

For a finite family $\mathcal{F}(r)$ of r-uniform hypergraphs and an integer n, the *extremal number* $ex(n, \mathcal{F}(r))$ is defined to be the maximum size $|\mathcal{H}|$ of a hypergraph $\mathcal{H} \subseteq [n]^r$ not containing any $F \in \mathcal{F}(r)$. For $r = 2$ and $\mathcal{F}(2)$ consisting of the single complete graph K_k on k vertices, these numbers were determined by P. Turán. However, for $r > 2$, and any family $\mathcal{F}(r)$, the Turán type problem of determining the numbers $ex(n, \mathcal{F}(r))$ has proved to be very difficult, and very little about these numbers is known. In this paper, we discuss some recent results and open problems for triple systems which relate to extremal numbers $ex(n, \mathcal{F}(3))$, where $\mathcal{F}(3)$ will consist of single given 3-uniform hypergraphs.

Keywords: Extremal numbers, Turán type problems for hypergraphs, triple systems.

132 **On Parameterizing Circle Graphs**

Ehab S. Elmallah* and Lorna K. Stewart,The University of Alberta

One way of parameterizing circle graphs - so as to obtain useful dynamic programming algorithms - is to ask whether a given circle graph can be embedded in a polygon with a few number of sides (the polygon replaces the circle in an intersection diagram of the graph, and the number of sides is the parameter of interest). In this talk we present both positive and negative results on the above problem. In particular, we outline the structure of a complex algorithm for deciding whether such an embedding is feasible. Keywords: circle graphs, W. Cunningham and J. Edmonds' decomposition, dominating sets, k-colorability, network reliability

133

Ranks of Circulant Graphs

George J. Davis*, Gayla S. Domke, Georgia State University

Let S be any subset of $\{1, 2, ..., n-1\}$ such that $S = -S \bmod n$. A graph G with vertex set $\{0, 1, 2, ..., n-1\}$ is called a circulant graph if two vertices i and j are adjacent iff $(i-j) \bmod n \in S$. The adjacency matrix $A(G)$ is a circulant matrix, i.e., $a_{ij} = a_{i-1,j-1}$ with the subscript calculation done mod n. We completely characterize rank($A(G)$) where $|S| = 3$, and give insight into larger $|S|$.

Key words: rank, adjacency matrix, circulant matrix, circulant graph.

134

ON PACKING 3–VERTEX PATHS IN A GRAPH

Hikoe Enomoto, Keio University; Atsushi Kaneko, Kougakuin University;
Alexander Kelmans*, Rutgers University and University of Puerto Rico;
Tsuyoshi Nishimura, Shibaura Institute of Technology

Let G be a connected graph and let $eb(G)$ and $\lambda(G)$ denote the number of end–blocks and the maximum number of disjoint 3–vertex paths Λ in G.

We prove the following theorems on claw–free graphs:
($t1$) if G is claw–free and $eb(G) \leq 2$ (and in particular, G is2–connected) then $\lambda(G) = \lfloor |V(G)|/3 \rfloor$,
($t2$) if G is claw–free and $eb(G) \geq 2$ then $\lambda(G) \geq \lfloor (|V(G)| - eb(G) + 2)/3 \rfloor$, and
($t3$) if G is claw–free and Δ^*–free then $\lambda(G) = \lfloor |V(G)|/3 \rfloor$ (here Δ^* is a graph obtained from a triangle Δ by attaching to each vertex a new dangling edge).

We also give the following sufficient condition for a graph to have a Λ–factor: Let n and p be integers, $1 \leq p \leq n-2$, G a 2–connected graph, and $|V(G)| = 3n$. Suppose that $G - S$ has a Λ–factor for every $S \subseteq V(G)$ such that $|S| = 3p$ and both $V(G) - S$ and S induce connected subgraphs in G. Then G has a Λ–factor.

Key words: path packings, path factors, claw–free graphs.

135

Bull-Designs on Graphs with Holes

Kimberly S. Kirkpatrick, University of Evansville

A G-design on a graph H is a collection of subgraphs of H, each isomorphic to a graph G, that partitions the edges of H. We consider designs where G is isomorphic to K_3 with two pendant edges (a.k.a. The Bull) and H is $K_d \times \overline{K_v}$. Some of the techniques used to construct these designs are difference methods, edge colorings and orientations.

Key Words: G-design, Decompositions

136

The Hedetniemi Number

Peter J. Slater, University of Alabama in Huntsville

I could think of only one way to make this discussion of (maximal) independent sets, (minimal) dominating sets and recurrence relations (including the ubiquitous Fibonacci numbers) more fun (at least for me). So, a new parameter will be introduced, the Hedetniemi number of a graph.

Keywords: domination, independence

The Tensor Product of Circulant Graphs Need Not Be Circulant

137

John C. George, Tougaloo College

We find a number of conditions that imply that the tensor product of two circulant graphs is another circulant graph, and other conditions that imply that this product is not circulant. In particular: The tensor product of two complete graphs is a circulant graph whenever their orders are relatively prime. The product of any two circulant graphs with relatively prime orders is circulant. The product of two cycles whose orders have greatest common divisor 2 is a (disconnected) circulant graph.

Keywords: Circulant graph, tensor product (weak product, categorical product), automorphism group

Decompositions and Packings of Digraphs with Orientations of a 4-Cycle

138

Robert B. Gardner, East Tennessee State University
Coleen Huff*, East Tennessee State University and University of South Africa
Janie Kennedy,Samford University

We present necessary and sufficient conditions for the decomposition of the complete symmetric bipartite digraph into each of the orientations of a 4-cycle (in the cases for which such decompositions are not already known). We use these results to find optimal packings of the complete symmetric digraph with each of the orientations of a 4–cycle.

Keywords: graph decompositions, designs, directed graphs, graph coverings.

Embedding of incomplete latin squares of order n in an idempotent latin square of order $2n$

139

Atif Abueida *, C.A. Rodger, Auburn University

In 1983, necessary and sufficient conditions were obtained for embedding of an incomplete idempotent latin square of order n in an idempotent latin square of order $2n$, providing $n > 16$. In 1998, Rodger and Grant considered the cases where $10 \leq n \leq 16$. In this paper we consider some of the remaining cases.

Independent Domination in Planar Graphs

140

G. MacGillivray, University of Victoria
K. Seyffarth*, University of Calgary

In some earlier work (*J. Graph Theory* 22 (1996) 213–229), we considered the problem of obtaining an upper bound on the domination number of a planar graph with fixed diameter. Here, we consider the analogous problem for the independent domination number. For planar graphs with n vertices and diameter two, we prove that the independent domination number is at most $\lceil n/3 \rceil$, and furthermore, we characterize those graphs for which the independent domination number is equal to $\lceil n/3 \rceil$.

We also obtain a bound on the independent domination number of a planar graph with diameter at least three. This result has less to do with the diameter than with the chromatic number of the graph, and we relate the independent domination number of an arbitrary graph to its chromatic number.

Keywords: independent domination, planar graphs, diameter, chromatic number.

141 **Recognition of HH-free and HHD-free Graphs**

Chính T. Hoàng, Lakehead University; R. Sritharan*, Indiana State University

A *house* is the complement of an induced path on five vertices. A *hole* is an induced cycle on five or more vertices. A *domino* is the cycle on six vertices with a long chord. A graph is *HH-free* if it does not contain a house or a hole. A graph is *HHD-free* if it does not contain a house, or a hole, or a domino. *HHD-free* graphs and complements of *HH-free* graphs are perfectly orderable. We present $O(n^3)$ algorithms to recognize *HH-free* graphs and *HHD-free* graphs. The previous best algorithms for the problems run in $O(n^4)$ time.

Key words. house, hole, domino, perfectly orderable, algorithm.

142 **Four-cycle Systems with Two-regular leaves**

C.A.Rodger*, Auburn University; H.L.Fu, National Chiao Tung University

Over the years, there have been several results on the existence of partial m-cycle systems with leave being a 2-regular graph. In this paper, this problem is solved in the case where m=4.

143 **NEW RESULTS ON REGULAR AND 1-ROTATIONAL DESIGNS**

Marco Buratti, Universita' de L'Aquila, Italy

A design (V,B) is said to be regular (1-rotational) over a group G if it admits G as an automorphism group acting regularly, i.e., sharply transitively, on V (on V-* for some point *). The definition may be also extended to a "resolved" design (V,B,R). In this talk several new constructions for regular and 1-rotational (possibly resolved) designs are presented.

144 **Achievable Sequences for Independent Domination in Connected Graphs**

Robert C. Brigham
Department of Mathematics, University of Central Florida, Orlando FL 32816
Julie R. Carrington* and Richard P. Vitray
Department of Mathematical Sciences, Rollins College, Winter Park FL 32789
Jay Yellen, Department of Mathematical Sciences
Florida Institute of Technology, Melbourne FL 32901

Let $I(G)$ be a graphical invariant defined for any graph G. If I represents either the domination number or the independent domination number, it is easy to characterize sequences of positive integers $a_1, a_2, \ldots, a_n$ which have an associated sequence of graphs $G_1, G_2, \ldots, G_n$ such that G_i has i vertices, G_i is an induced subgraph of G_{i+1}, and $I(G_i) = a_i$. The problem becomes surprisingly difficult when restricted to connected graphs and remains unsolved in the case of independent domination. Partial results will be given.

145 **An Upper Bound for Disjunct Matrices**

Laura Riccio and Charles J. Colbourn*, University of Vermont

The 'union' of columns in a binary matrix is a column vector whose entries are the componentwise maxima of the individual columns. A d-disjunct matrix is a binary matrix with the property that the union of any set of d or fewer columns does not cover another column. Disjunct matrices arise in nonadaptive group testing problems and in the construction of superimposed codes. The existence question of interest is to determine how many columns a disjunct matrix can have for a specified number of rows, or equivalently, how few rows a disjunct matrix can have for a specified number of columns. Erdös, Frankl, and Füredi established an upper bound on the minimum number of rows in a 2-disjunct matrix. We refine their bound, and establish analogous results for 3- and 4-disjunct matrices.

146 4-Cycle Decompositions of $K_m \times K_n$

Dean G. Hoffman, Auburn University
David A. Pike*, Memorial University of Newfoundland

We establish necessary and sufficient conditions on m and n in order for $K_m \times K_n$, the cartesian product of two complete graphs, to be decomposable into cycles of length 4. The main result is that $K_m \times K_n$ can be decomposed into 4-cycles if and only if either $m, n \equiv 0 \pmod 2$, $m, n \equiv 1 \pmod 8$, or $m, n \equiv 5 \pmod 8$.

Keywords: graph decomposition, cartesian product, line graph

147 **Pooling, Lattice Square, and Union Jack Designs**

Mark A. Chateauneuf*, Michigan Technological University
Charles J. Colbourn, University of Vermont
Donald L. Kreher , Michigan Technological University
E. R. Lamken, California Institute of Technology
David C. Torney, Los Alamos National Laboratory

Simplified pooling designs employ rows, columns, and principal diagonals from square and rectangular plates. The requirement that every two samples be tested together in exactly one pool leads to a novel combinatorial configuration, the union jack design. Existence of union jack designs is settled affirmatively whenever the order n is a prime and n = 3 mod 4.

148 On dominations of product graphs

Cong Fan, Don Lick, and Jiuqiang Liu*
Eastern Michigan University

We will study the domination numbers of graph products, including the Cartesian product and the strong product. We develop some interesting results, some of which can be viewed as extensions of some existing results to the Vizing's Conjecture that the domination number of the Cartesian product of any two graphs is at least as large as the product of their domination numbers.

149 **Factorizing the Generalized Vandermonde Matrix**

Neil Calkin, Clemson University

It is well known that the ratio of the determinants of a generalized Vandermonde matrix and the corresponding Vandermonde matrix gives the Schur symmetric functions. We prove that the generalized matrix factors, one of the factors being the Vandermonde matrix, and thus give a new derivation of the determinantal form for the Schur functions.

Resolvable cycle decompositions of complete multipartite graphs

150 Sarah Heuss *, D.G. Hoffman; Auburn University

We shall consider the problem of resolvably decomposing the complete r-partite graph with parts of size a into cycles of length k.

151 **Orthogonal Double Covers of Complete Graphs by Hamiltonian paths**

Sven Hartmann, Uwe Leck*, Volker Leck, University of Rostock, Germany

An orthogonal double cover (ODC) of the complete graph K_n by some given graph G is a collection $\mathcal{G} = \{G_1, G_2, \ldots, G_n\}$ of spanning subgraphs of K_n such that the following conditions are satisfied:

1. Every edge of K_n is contained in exactly two members of $\mathcal{G}$.
2. Every two distinct members of $\mathcal{G}$ share exactly one edge.
3. G_i is isomorphic to G for $i = 1, 2, \ldots, n$.

We present a new construction for the case that $G = P_n$, the path on n vertices. In combination with a result of Horton and Nonay, our construction provides an ODC of K_n by P_n for all $n = 2^\alpha p_1^{2\alpha_1} \ldots p_m^{2\alpha_m}$ such that the p_i's are primes with $p_i \equiv 3(4)$ and $\alpha \neq 1, 2$.

152 **Multi-Fractional Domination**

Peter J. Slater and Eric L. Trees*, University of Alabama in Huntsville

As an extension of the fractional domination and fractional domatic graphical parameter, multi-fractional domination is introduced. We will demonstrate its Linear Programming (LP) formulation, and to this formulation we apply the Partition Class Theorem (PCT) which is a generalization of the Automorphism Class Theorem (ACT). We investigate some properties of the multi-fractional domination number and its relationship to the fractional domination and fractional domatic numbers.

Key words:domination, domatic, fractional

A Fibonacci Survival Indicator for Efficient Fitness Calculation in Genetic Paradigms

153

Michael L. Gargano * — Pace University, NYC
William Edelson — Long Island University, NYC

Keywords: Fibonacci Scoring Function, Genetic Paradigms, Efficient Fitness Calculation

A Fibonacci survival indicator is a novel, simple, and robust way to efficiently calculate fitness in genetic paradigms that are compute intensive. The Fibonacci scoring function, based upon concepts borrowed from artificial life paradigms, is a heuristic indicator of a population member's future survivability. It enables a fair comparison of population members of different ages so that calculation resources can be concentrated on old, fitter members of the surviving population. This paper will define the Fibonacci scoring function, investigate some of its mathematical properties, and apply the concept to two simple real world examples (a queuing problem and a finite state machine problem).

Holey Steiner Pentagon Systems and Related Designs

154

F. E. Bennett*, Mathematics Department, Mount Saint Vincent University
H. Zhang, Computer Science Department, University of Iowa
L. Zhu, Mathematics Department, Suzhou University

In this paper, it is shown that the necessary conditions for the existence of a holey Steiner pentagon system (HSPS) of type h^n, namely, $n \geq 5$, $n(n-1)h^2 \equiv 0 \pmod 5$, and if h is odd, then n is odd, are also sufficient, except possibly for a handful of cases. Certain types of HSPSs are investigated and applied in the construction of other combinatorial structures, including Steiner pentagon packing designs (SPPDs). Among other things, the spectrum of SPPDs is determined to within a finite number of possible exceptions, of which the largest unknown order is 189.

Keywords: Holey Steiner pentagon systems, Steiner pentagon packing designs.

A Conjecture on ODCs by Paths

155

Volker Leck

An Orthogonal Double Cover (ODC) of the complete graph is a set of graphs such that every two of them share exactly one edge and every edge of the complete graph belongs to exactly two of the graphs.
We consider the case when all graphs are paths. Then group–generated ODCs are equivalent to a 2–sequencing of the generating group. Special solutions consist of translated halfstarters. We introduce a construction of those solutions that we conject to work in all finite fields of order 1 mod 4.

Chromatic Number and Planarity of Domination-Compliance Graphs

156

Faun Doherty* & J. Richard Lundgren
University of Colorado at Denver

Given an n-tournament T, two vertices x and y dominate T if together they beat all other vertices. The domination graph $dom(T)$ of T is the graph on the vertex set of T with edges between vertices which dominate T. Two vertices x and y are a compliant pair if every other vertex in T beats either x or y. The compliance graph $com(T)$ of T is the graph on the vertex set of T with edges between vertices which form a compliant pair. One can see that the relationship between $dom(T)$ and $com(T)$ is $com(T) = dom(\tilde{T})$ where $\tilde{T}$ is the reversal of T. The domination-compliance graph of T is $dom(T) \cup com(T)$. The questions addressed here will be of the planarity and chromatic number of domination-compliance graphs. We shall determine that the chromatic number of most domination-compliance graphs is three, while some have chromatic number two, which is better than the previous conjecture of four. In order to do this, we shall determine all possible edges of domination-compliance graphs given a domination (or compliance) graph. This characterization enables us to investigate whether or not all domination-compliance graphs are planar. We conjecture that they are.

Key Words Tournament, Chromatic Number, Planarity, Domination Graph, Compliance Graph

Thursday, March 11, 1999
8:40 AM

161 **More Extremal Graphs For an Average-Triangles-Per-Edge, Maximum-Joint-Neighborhood Inequality**

Peter Johnson and Ken Roblee*. Dept. of Discrete and Statistical Sciences, Auburn University

It has been shown that if $G = (V, E)$ is a simple graph with n vertices, m edges, an average (per edge) of t triangles occurring on the edges, and $J = \max_{uv\in E} |N(u) \cup N(v)|$, then $4m \leq n(J+t)$. We say that $G \in ET(n, J, t)$ if G is an extremal graph for this inequality. The extremal cases for $J = n$ and $J = n - 1$ have been solved. For $J = n$, the extremal graphs are the Turan graphs with parts of equal size; notice that these are the complements of the strongly regular graphs with $\mu = 0$. For $J = n - 1$, the extremal graphs are the complements of the strongly regular graphs with $\mu = 1$. (The only such graphs known to exist are the Moore graphs of diameter 2.)

Here, we characterize the graphs in $ET(n, n-2, t)$ for $t = 0, 1$, and give some results for $t > 1$.

Key words and phrases: extremal. strongly regular

162 **Hamiltonian cycles in n-factor-critical graphs and n-extendable graphs**

Akira Saito, Nihon University

This is a joint work with K. Ota (Keio University, Japan). Ore's theorem states that every graph G of order $p \geq 3$ with $\sigma_2(G) \geq p$ is hamiltonian. Although this result is sharp in general, a smaller value of $\sigma_2(G)$ may assure the hamiltonicity of G ifwe put additional assumptions on G. Faudree and van den Heuvel (1995) proved that if G has a k-factor, then $\sigma_2(G) \geq p - k$ guarantees the existence of a hamiltonian cycle. In this talk we investigate degree conditions for hamiltonicity of n-extendable graphs and n-factor-critical graphs. We show that a smaller value of $\sigma_2(G)$ assures the hamiltonicity of G unless G belongs to a finite number of exceptional graphs. We also discuss the sharpness of the result. keywords:degree sums, hamiltonian cycles, n-extendable graphs, n-factor-critical graphs

163 BELL NUMBER IDENTITIES

L.K.Tolman, Brigham Young University

Using the current definition of the Bell numbers and the recursion formula for the Stirling numbers of the second kind, a set of polynomials is defined by initial value and recursion formula. A class of three-parameter numbers is also defined by initial value and recursion. This class and the Stirling numbers of the first kind are interlinked with the Bell numbers and the polynomials to obtain collections of identities homogeneous of any order. Analysis of related items is included.

164 **CLONES IN CONVEX FIGURES**

by Alexander Soifer
University of Colorado at Colorado Springs
P. O. Box 7150, Colorado Springs, CO 80933
asoifer@mail.uccs.edu http://www.uccs.edu/~asoifer

Inspired by a still open Paul Erdös's 1932 problem, I formulated in 1995 the *Squares in a Square Conjecture*, which was proven by Richard Stong: Any finite set of squares of the combined area 1 can be packed in a square of area 2.

Given a figure F. Let $s = s(F)$ be the smallest real number such that any finite set of F-clones of the combined area 1 can be packed in an F-clone of area s.

For example, for a square $\square$, $s(\square) = 2$.

Main Open Problem. For any convex figure F, find $s(F)$.

It is easy to show that for any figure F, $s(F) \leq 2t(F)$, where $t(F)$ is the ratio of the smallest area of a F-clone circumscribed about a unit square to the largest area of a F-clone inscribed in a unit square.

This upper bound is very rough. Yet, it implies, for example, that for a disc O, $s(O) \leq 4$; for a regular octagon R, $s(R) \leq 2 + \sqrt{2}$.

In fact, it appears that the *Squares in a Square* result may hold for all convex figures:

Clones in Convex Figures Conjecture. For any convex figure F, $s(F) = 2$.

Thursday, March 11, 1999
9:00 AM

165

Antimatroid Closure Spaces in a Discrete Plane

John L. Pfaltz, University of Virginia

Antimatroid closure comes in many different varieties, but convex closure provides a very accessible variety of antimatroids. Convexity concepts and convex geometries in n-dimensional Euclidean space, E^n have been well studied, but when the underlying space is discrete, as in a digital image or VLSI layout or systems of integer inequalities, nothing is quite the same. A set C is convex if every geodesic between any two points in C lies completely within C. The problem lies in defining what constitutes a *geodesic*. Readily, in the discrete plane there are many shortest paths. Given a rectangle of pixels and the usual city block metric, essentially every path between two diagonally opposite corners is *shortest*.

We define a geodesic between any two points, *e.g.* a line, to be a uniquely generated closure. It is at most one pixel wide; but it is not necessarily "straight" In fact, most geodesic lines have a "bend" that characterizes the line. One of the more interesting properties of this space is the fact that "parallel" is not a transitive relation. Two "straight" lines may be parallel to a third, yet themselves intersect. But, other interesting geometric properties do hold.

HAMILTONIAN COMPLETION NUMBER OF INTERVAL GRAPHS

166 Arundhati Raychaudhuri, College of Staten Island (CUNY)

Hamiltonian completion number of a graph hc(G) is the minimum number of edges to be added to G to guarantee that G has a Hamiltonian path. A related parameter, path cover number pc(G) is the minimum number of vertex disjoint simple paths required to cover the vertices of G. These two numbers are related by the simple formula hc(G) = pc(G) - 1. These problems in general are NP-hard, although polynomial algorithms have been found for trees, unicyclic graphs and cactii. In this paper, we present a polynomial algorithm for computing hc(G), where G is an interval graph.

Keywords: Hamiltonian path, Hamiltonian completion, path cover, algorithm, interval graph enddocument

A bijection between polygon-dissections and certain permutations

167 Balázs Montágh, The University of Memphis

Let $q = (q_1, q_2, ..., q_k) \in S_k$ be a permutation. A permutation $p = (p_1, p_2, ..., p_n) \in S_n$ contains a pattern q if there is a set of indices $1 \leq i_{q_1} < i_{q_2} < ... < i_{q_k} \leq n$ such that $p_{i_1} < p_{i_2} < ... < p_{i_k}$. For example p contains a pattern 132 if there are indices $1 \leq i < j < k \leq n$ such that $p_i < p_k < p_j$.

D. E. Knuth proved that the number of 132-pattern-free permutations of $[n]$ is the Catalan number C_n. Thus they are equinumerous, for example, to the dissections of convex polygons into triangles by noncrossing diagonals.

M. Bóna showed in 1997 that the permutations containing exactly one 132-pattern are equinumerous to dissections of convex polygons into some triangles and exactly one quadrangle. His proof was inductive, showing that the same recursive formula holds for these two sequences.

This result seems to be isolated, that is, non-generalizable, because the complexity of the counting of permutations containing exactly r 132-pattern increases very quickly as r grows.

In this talk we will show that modifying the original question one can get a surprising connection between the dissections and certain permutations. We will give a bijective proof for the fact that the dissections of a convex $n-r+2$-gon into $n-3r$ triangles and r quadrangles by noncrossing diagonals are equinumerous to permutations containing r 132-patterns such that there are no two patterns with common element. More generally, the dissections of a convex $n-r+2$-gon into $n-2r$ smaller polygons by noncrossing diagonals are equinumerous to permutations containing r 132-patterns such that any two of the patterns have at most one common element. Our proof is based on a modification of the concept of stack-sortable permutations.

168

Minimally Path-Saturated Graphs

Sharon G. Boswell, University of Newcastle, Australia
Roger B. Eggleton*, Illinois State University
James A. MacDougall, University of Newcastle, Australia

A graph is Pm-saturated if every possible addition of an edge creates a new m-path, and it is minimally Pm-saturated if no proper spanning subgraph is Pm-saturated. The study of graphs with these properties complements our studies of graphs which are minimally saturated with respect to other specified graphs, such as triangles (3-cycles) or, more generally, m-cliques. Here we explicitly determine those cycles, paths, and unions of paths which are Pm-saturated, and those which are minimally so. We also determine the threshold where certain families of long paths with small attachments become Pm-saturated. Key words: paths, path-saturated graph, triangle-saturated graph. enddocument

169 Join Congruence Relations on Geometric Lattices

Kenneth P. Bogart and *Laura H. Montague

The flats of a matroid form a geometric lattice. This leads one to wonder whether operations performed on a matroid can be attained by performing instead a lattice theoretic operation (in particular, a join congruence relation) on the corresponding lattice. It turns out that some, but not all, standard matroid operations can be achieved using join congruence relations. We discuss which ones are possible and also the effects on geometric lattices of join congruence relations which are not known to correspond to matroid operations.

keywords: matroid theory, lattice, geometric lattice

170 **A lower bound on the number of hamiltonian cycles**

P.Horak*, Kuwait University, Kuwait
L.Stacho, Slovak Academy of Science, Slovakia

What Hamiltonian graphs have a second Hamiltonian cycle? This question has been around for a long time. A result of Smith implies this property for cubic graphs. Thomason extended the Smith's result to graphs whose all vertices have odd degree. Inparticular, it means that all $r-$regular Hamiltonian graphs, where r is odd, have a second Hamiltonian cycle. Recently Thomassen proved the result for every $r-$regular Hamiltonian graph, where $r \geq 300$. We will present a lower bound on the number of Hamiltonian cycles obtained by a refinement of Thomassen's methods, and discuss limits of the approach.

Keywords: Hamiltonian, regular, bounds

171 The higher you go, the odder it gets

Lou Shapiro, Mathematics Department, Howard University, Washington DC, 20059

We consider planar trees, growing upwards, with n edges. The chance that the root has odd degree approaches 5/9 as n gets large. We show this and also that, as you climb the tree, the chance that the updegree of a random nonterminal node is odd increases from 5/9 to 2/3. The number of planar trees with n edges is the Catalan number C(n) and we also look at other events in the Catalan kingdom exhibiting this increasing oddness phenomenon.

172 **W2 Graphs with no 4-cycles**

B.L Hartnell, Saint Mary's University, Canada

A well-covered graph (M.D. Plummer, 1970) is a graph in which every maximal independent set of vertices is maximum. If a well-covered graph has the property that any vertex can be deleted and the resulting graph is still well-covered (with the same independence number), we say it is a W2 graph (also called 1-well-covered). For example, a 4-cycle is well-covered but not in W2 whereas a 5-cycle is both well-covered and in W2. One approach in the attempt to determine the structure of well-covered graphs has been to focus on girth. In this talk a characterization of the W2 graphs with no 4-cycles (but 3-cycles are permitted) will be given. key words: well-covered, maximal independent sets

173 **Rectangle-free Subsets of a Square Grid**

James T. Lewis, University of Rhode Island

We investigate the maximum cardinality of a subset of a square grid with the property that no four points of the subset are the vertices of a rectangle whose sides are parallel to the sides of the grid. Avoidance of other objects, such as right triangles and L's, is also studied.

Maximal sets of hamilton cycles in complete multipartite graphs

174 Mike Daven *, C.A. Rodger, Auburn University

In a connected graph G, a set S of edge-disjoint hamilton cycles is called *maximal* if $G - S$ contains no hamilton cycle. The *spectrum* of maximal sets in G is the set of those numbers m for which a maximal set of size m exists. The spectrum for maximal sets has been studied for complete graphs and, more recently, for complete bipartite graphs. In this note, we explore extensions of the latter result to complete multipartite graphs, using the methods of amalgamations and outline hamilton decompositions.

keywords: hamilton cycle, multipartite, maximal set, amalgamations, outline hamilton decompositions

175 **BIJECTIVE PROOFS OF THE CATALAN AND FINE RECURRENCES**

Paul Peart* and Wen-Jin Woan, Howard University, Washington, D.C.

A Dyck path of length $2n$ is a lattice path from $(0,0)$ to $(2n,0)$ which uses allowable steps $(1,1)$ and $(1,-1)$ and does not go below the x-axis. A peak on a Dyck path is a lattice point that is immediately preceded by a $(1,1)$ step and immediately followed by a $(1,-1)$ step. Let C_n be the set of all Dyck paths of length $2n$, and F_n the set of all dyck paths of length $2n$ with no peaks at height 1. It is well known that $|C_n| = c_n$, and $|F_n| = f_n$, where c_n is the n^{th} Catalan number and f_n is the n^{th} Fine number. In this paper, we use Dyck paths to provide bijective proofs of the recurrences:

$$(n+1)\,c_n = 2(2n-1)\,c_{n-1}; \quad n \geq 1, \quad c_0 = 1$$

and

$$2(n+1)f_n = (7n-5)f_{n-1} + 2(2n-1)f_{n-2}; \quad n \geq 2, \quad f_0 = 1, \quad f_1 = 0.$$

Keywords. Catalan number, Fine number, Dyck path.

On 1-Z_m-well-covered and strongly Z_m-well-covered graphs

Rommel Barbosa, Department of Mathematics
176 Universidade Federal do Mato Grosso,Cuiabá - MT - Brazil

A graph G is Z_m-well-covered if $|I| \equiv |J|$ (mod m), for all I, J maximal independent sets in $V(G)$. A graph G is a 1-Z_m-well-covered graph if G is Z_m-well-covered and $G\backslash v$ is Z_m-well-covered , $\forall v \in V(G)$. A graph G is strongly Z_m-well-covered if G is a Z_m-well-covered graph and $G\backslash e$ is Z_m-well-covered $\forall e \in E(G)$. Here we will prove some results about 1-Z_m-well-covered and strongly Z_m-well-covered graphs.

Thursday, March 11, 1999
11:30 AM

177

Tournaments for Teams with t Players

David R. Berman, Sandra C. McLaurin, Douglas D. Smith*
University of North Carolina at Wilmington

We generalize whist and pitch tournaments by considering tournaments in which v individual players compete as members of teams with t players, subject to the condition that when all tournament rounds are completed each pair of players will have played on the same team t -1 times and on opposing teams t times. It is further required that either all players play in every round, or that exactly one player sits out in each round. We exhibit examples and provide constructions for team tournaments that apply to all sizes t of teams. We consider Z-cyclic and other specializations of team tournaments.

Key words: team tournaments, whist, pitch, Z-cyclic

178

ong paths through specified vertices in a 2-connected graph

Iark V. Barovich, Florida Atlantic University

t is well known that a path containing at least d edges connects any pair of vertices in a !-connected graph with minimum degree d. We consider a generalization of this useful esult. A (u,v)-path is a path with endvertices u and v. A (u,v;d)-path is a (u,v)-path ontaining at least d edges. Does a set of n vertices lying on a common (u,v)-path in a 2-onnected graph with minimum degree d lie on a common (u,v;d)-path? We show that he answer to this question is always affirmative if n<5, and we construct a ounterexample showing that the answer is not always affirmative if n>6. We also nvestigate the situation for other values of n.

eywords: graph, path, 2-connected

179

Random Walks and Catalan Factorization

Omer Egecioglu, University of California, Santa Barbara
Alastair King, Abdus Salam I.C.T.P.,Italy

In the theory of random walks, it is notable that the central binomial coefficients $\binom{2n}{n}$ count the number of walks of three different special types, which may be described as 'balanced', 'non-negative' and 'non-zero'. One of these coincidences is equivalent to the well-known convolution identity

$$\sum_{p+q=n} \binom{2p}{p}\binom{2q}{q} = 2^{2n}.$$

This talk brings together several proofs of this 'ubiquity of central binomial coefficients' proofs based on Catalan factorization. This relatively new technique is also used to provide new direct proofs of the convolution identity.

Keywords: Central binomial coefficient, random walk, Catalan factorization, bijection, convolution.

180

Well-Covered Trees

Vadim E. Levit* and Eugen Mandrescu, Center for Tech. Edu., Israel

A graph is well-covered if every maximal stable set is also maximum. One theorem of Ravindra (1977) asserts that a tree of order at least two is well-covered if and only if it has a perfect matching consisting only of pendant edges. The edge-join of two connected graphs G_1, G_2 is the graph $G_1 * G_2$ obtained by adding an edge joining two vertices belonging to G_1, G_2, respectively. If both of the newly adjacent vertices are of degree ≥ 2 in G_1, G_2, respectively, then $G_1 * G_2$ is an internal edge-join of G_1, G_2. In this paper we show that any well-covered tree, having at least three vertices, can be recursively constructed using the internal edge-join operation. We also characterize well-covered trees in terms of distances between their vertices and their pendant vertices.

key words:-covered graph, very well-covered graph, perfect matching, spider, pendant vertex, distance, diameter.

181

Extensions of some Z-cyclic Whist Tournaments

Norman J. Finizio* and Adele J. Merritt; University of Rhode Island

In this study we extend some of our earlier work and thereby improve upon the scope of knowledge pertaining to the existence of Z-cyclic Whist Tournaments.

Keywords: BIBD, cyclic designs, whist tournaments

SOME GRAPHIC USES OF AN EVEN NUMBER OF ODD NODES

182 Kathie Cameron* and Jack Edmonds

Perhaps the simplest useful theorem of graph theory is that every graph X has an even number of odd (degree) nodes. We give new proofs of several theorems each of which asserts that, for any input G satisfying specified conditions, G has an even (or odd) number of H's satisfying specified conditions. Each proof consists of describing an exchange graph X, quite large compared to G, such that the odd nodes of X are the objects H which we want to show there is an even number of.Each of these theorems is not so easy to prove without seeing the exchange graph. They include as corollaries the results of Andrew Thomason proving the 1965 conjecture of Lin that the union of any two edge-disjoint hamiltonian circuits of any graph G is also the union of two other edge-disjoint hamiltonian circuits of G (and hence two edge-disjoint hamiltonian circuits of G can not be neighbour vertices in the convex hull of the hamiltonian circuits of G). They include Berman's generalization of Thomason's generalization of the famous Smith theorem, that each edge in a cubic graph G is in an even number of hamiltonian circuits of G. Keywords: parity, hamiltonian, exchange graph, path, tree, lollipop, circuit, existentially polytime

183

An Algorithm for n-realizable Partitions of n(n+1)/2

A. Gregory Starling*, University of Arkansas
Joseph B. Klerlein, Western Carolina University

For a positive integer n, let $P = \{p_1, p_2, p_3, \ldots, p_k\}$ be a partition of $n(n+1)/2$ by positive integers, and let $S = \{S_1, S_2, S_3, \ldots, S_k\}$ be a partition of the set of integers $N = \{1, 2, 3, \ldots, n\}$ by subsets $S_i \subseteq N$. The partition S is called an n-realization of the partition P, provided $p_1 = \Sigma S_1$, $p_2 = \Sigma S_2$, $p_3 = \Sigma S_3$, $\ldots$, $p_k = \Sigma S_k$, where ΣS_i is the sum of the integers contained in S_i.
In this paper we present an algorithm which tests the n-realizability of a partition P of $n(n+1)/2$ by constructing the partitions of $\{1, 2, 3, \ldots, n\}$ which are n-realizations of P.
Key words: n-realizable, integer partition, set partition

184

ON THE BASIS NUMBER OF A GRAPH

Ali A. Ali, Dept. of Math., University of Mosul, IRAQ
Salar Y. Alsardary*, Dept. of Math., University of the Sciences in Philadelphia
s.alsard@usip.edu

The basis number of a graph G is defined to be the least integer k such that G has a k-fold cycle basis. We investigate the change in the basis number of a graph under the operations of addition, deletion of edges in certain ways, and contraction of edges.

Thursday, March 11, 1999
12:10 PM

185
The Existence of Pitch Tournament Designs

Norman J. Finizio and Scott J. Lewis*, University of Rhode Island

A pitch game consists of two teams of four opposing each other at a table. A pitch tournament, PTCH(v) for v players is a schedule of rounds of games where each player partners every other player three times and opposes every other player four times. The main result of this paper is that if a PTCH(v) exists for all v < 314,311 , then a PTCH(v) exists for all v, where v = 8n or v = 8n + 1. The proof of this theorem involves the construction of resolvable transversal designs and group divisible designs.

Key Words: tournament, BIBD, transversal design

Ryser's Embedding Problem for Hadamard Matrices

186
T. S. Michael, U. S. Naval Academy

What is the minimum order $H(r,s)$ of a Hadamard matrix that contains an r by s submatrix of all 1's? Newman showed that

$$H^{\sharp}(rs) \leq H(r,s) \leq H^{\sharp}(r)H^{\sharp}(s),$$

where $H^{\sharp}(t)$ denotes the smallest order greater than or equal to t for which a Hadamard matrix exists. It follows that if 4 divides both r and s, and if the Hadamard conjecture holds, then $H(r,s) = rs$. We improve both the upper and lower bounds in Newman's inequalities and thus determine $H(r,s)$ for infinitely many new pairs (r,s).

Keywords: Hadamard matrix, embedding, combinatorial matrix theory

187
Small Hash Families

Mustafa Atici, International Computer Institute, University of Ege, Turkey

An (n,m,w)-*perfect hash family* is a set of functions $\mathcal{F}$ such that $f: \{1,2,...,n\} \rightarrow \{1,2,...,m\}$ for each $f \in \mathcal{F}$, and for any $X \subseteq \{1,2,...,n\}$ such that $|X| = w$, there exist at least one $f \in \mathcal{F}$ such that $f|_X$ is one-to-one. Perfect hash families have been extensively studied by computer scientists for years, mainly due to their application in database management. In this paper, we study lower bound of $|\mathcal{F}|$ for small values of $n, m,$ and w. We also enumerate non-isomorphic families for small values of $n, m,$ and w.

Keywords: perfect hash family, perfect hash function.

188
Complete Bipartite Graph Path Decompositions

D.G. Hoffman, Auburn University and Carol A. Parker*
North Georgia College State University

We investigate partitioning the edges of the complete bipartite graph K(a,b) into paths of length k. We give necessary and sufficient conditions for such a decomposition, and present the techniques used in one of our cases, namely when a, b, and k are odd.

Key Words: Path decompositions, complete bipartite graph

230

A celebrated conjecture of Paul Erdos

Niall Graham, University of Alabama in Huntsville

A celebrated conjecture of Paul Erdos states that any subgraph G of the n-cube Q_n must contain asymptotically half of the edges of Q_n to guarantee the inclusion of a cycle C_4 in G. In pursuit of this conjecture, it is shown that any 2-coloring of the edges of Q_n that avoids a monochromatic C_4 must contain (asymptotically) half of its edges in each color. Other related questions are also considered.

KEYWORDS: hypercube,ramsey theory,Erdos conjecture.

The minimum-weight codewords in the dual binary code of the design $PG_{m,m-1}(F_q)$

Peng Ding

231

February 2, 1999

Recently Calkin, Key and de Resmini proved that the minimum weight of the dual of the binary code of the design of points and r-subspaces of $PG_m(F_q)$ and that of the design of points and r-flats of $AG_m(F_q)$, where q is even, $1 \leq r < m$, $m \geq 2$, is $(q+2)q^{m-r-1}$. We prove here that the minimum-weight words in the dual binary code of the design $PG_{m,m-1}(F_q)$ of points and hyperplanes in projective m-space are the incidence vectors of the hyperovals. Further we prove that the dual binary code of the design $PG_{3,2}(F_{2^t})$ of points and hyperplanes is generated by the incidence vectors of the hyperovals.

keywords: dual code, even type set, oval, hyperoval, ovoid

234 Optimal Packing Designs

Kirsten Mackenzie-Fleming, Central Michigan University

If there exists a nearly resolvable balanced incomplete block design (NRBIBD) with parameter set 2 – (sk + 1, k, k –1) then there exists an optimal 2 – (s(sk + 1), sk + 1, sk + 1) packing design. The proof of this theorem is constructive and will be outlined in the talk.

Keywords: optimal, packing, design.

Some Results On Orthogonal Arrays 189

D. V. Chopra, Wichita State University, Wichita.

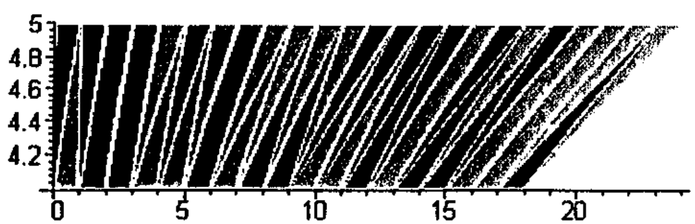
An array T with m rows (constraints), N columns (runs), and with S levels is merely a matrix T of size $(m \times n)$ with S elements (say; $0, 1, 2, .., S-1$). Here we restrict ourselves to arrays with $S = 2$ elements (i.e. 0 and 1). T is called a balanced array (B-array) of strength $t (t \leq m)$ if in every $(t \times N)$ submatrix T^* of T, every vector $\underline{\partial}$ $(t \times 1)$ of weight i $(0 \leq i \leq t$; the weight of a vector is the number of in it) and its permutation $P(\underline{\partial})$ appears with the same frequency μi (say). The vector $\underline{\mu}' = (\mu_0, \mu_1, .., \mu_t)$ is called index set of the array T. If $\mu_i = \mu$ for each i, then T is called an orthogoanal array (O-array). Clearly $N = \mu_{2^t}$ if T is an O-array. In this paper we present same interesting results on O-arrays by using the concept of B-Arrays.

Key words: balanced array, orthogonal array, constraints of an array, strength t array, levels of an array.

170 Tiling Intervals with 3 Element Sets

Aaron Meyerowitz, Florida Atlantic University

Consider a set $A = \{0, x, x+y\} \in \mathbb{R}$. Then there is an interval $[0, t)$ which can be decomposed into disjoint translates of A and its reflection $B = \{0, y, x+y\}$. The following diagram illustrates a tiling of $[0, 16+6\lambda)$ when $A = \{0, 1, 4+\lambda\}$:



A Mathematical Moment in Music

Phyllis Z. Chinn*, Humboldt State University; Wesley C. Chinn and Sami Shumays, Harvard University

191

A canon in music can be described in terms of one melody line (which may be sung or played by an instrument) repeated in a translation, resulting in multiple voices performing simultaneously, creating harmony from the melody. The simplest example of a canon is a round in which a (usually short) melody is repeated several times with new voices starting and ending at different times, but each new voice entering at the same point in the melody line relative to the preceding voice. The translation in a round is usually a unit fraction of the length of the initial line, where the denominator of the fraction equals the number of voices. Pleasing chord structure is achieved by all notes belonging to the same chord structure in the same progression. A more unusual form of canon, called a mensuration canon also exists. In this canon, which can be described as a dilation, each voice carries the same melody line but at a different speed. The musical considerations to decide what "works" harmonically are clearly much more complex for mensuration canons.

This presentation will show scores and play examples of both types of canon to demonstrate the transformations involved Key words: mathematics and music, mathematical recreations.

The Role of Ideal Sets for the Set Partitioning Problem

192

Robert Goldberg, Jacob Shapiro, and Isak Taksa*
Graduate Center, City University of New York

This paper deals with the well-known set partitioning problem which is closely related to load balancing and scheduling problems. This problem is stated as follows: given a set of positive numbers S, partition the set into k subsets such that a measure of the dispersion of elements is minimized under a given norm. The norms considered in the literature are makespan, least squares and in general, power p norms. The quality of the partition is measured by the ratio of the norm of the partition obtained by the algorithm to that of the optimal partition (termed competitive ratio.)

Since this problem is known to be NP-hard, general optimal solutions cannot be obtained in polynomial time. The only input sets for which the norm of the optimal solutions can be easily computable are ideal sets which have a partition such that the sum of the elements of each subset is equal to the average distribution. Therefore, in an attempt to find a tighter upper bound, it is natural to consider ideal sets. However, this paper shows that the competitive ratio for general sets has worse bounds than that of ideal sets. This indicates the difficulty of proving tighter bounds by providing counter- examples to the competitive ratio since only the norms of the optimal partition for ideal sets can be computed in polynomial time.

Friday, March 12, 1999
9:00 AM

Two Cycle Antiautomorphisms of Mendelsohn Triple Systems

193

Neil P. Carnes, McNeese State University

A cyclic triple, (a, b, c), is defined to be the set $\{(a,b),(b,c),(c,a)\}$ of ordered pairs. A Mendelsohn triple system of order v, MTS(v), is a pair (M, β), where M is a set of v points and β is a collection of cyclic triples of pairwise distinct points of M such that any ordered pair of distinct points of M is contained in precisely one cyclic triple of β. An antiautomorphism of a Mendelsohn triple system, (M, β), is a permutation of M which maps β to β^{-1}, where $\beta^{-1} = \{(c,b,a)|(a,b,c) \in \beta\}$. We give necessary conditions for the existence of a Mendelsohn triple system of order v admitting an antiautomorphism consisting of two cycles of different lengths.

Key Words: Mendelsohn triple system, antiautomorphism

Geometric Codes over Fields of Odd Prime Power Order

194

K. L. Clark* and J. D. Key, Clemson University

The geometric codes are the duals of the codes defined by designs associated with finite geometries. We obtain improved bounds for the minimum weight of geometric codes where the designs are from finite geometries over fields of odd prime power order. Some of the results apply also to the dual codes of non-desarguesian planes of odd order.

Key words: dual codes, minimum weight, projective geometries

Lights Out Type Games and Variations

195

Robert Molina, Alma College

In this paper we investigate a game, sometimes referred to as "lights out," which is played on an $m \times n$ grid graph. Each vertex represents a switch that is either in the on or off state. When one of the vertices is activated, that vertex and each of its neighbors change state. The goal of the game is to obtain a graph with all vertices in the off state. We will review some known results, discuss the nature and number of solutions for a given graph, and consider playing the game with various modifications.

On Convergence and Rates of Generalized Radial Bases Function Networks in Nonlinear Estimation and Classification

196

Adam Krzyżak*and Stan Klasa,Concordia University

In the paper we will investigate asymptotic properties of nonparametric regression function estimates and classification based on generalized radial basis function networks. The networks are trained by a sequence of independent, identically distributed random variables and and their performance is measured by the L_1 error. We discuss the choice of radial functions, centers, and the size of hidden layer in order to obtain consistent estimates and rules. The network structure is determined by complexity regularization. We study rates of convergence in the class of convex closures of classes of bases functions.

Friday, March 12, 1999
10:50 AM

197

Doubly nested balanced incomplete block designs

D.A.PREECE* & D.H.REES (Univ. Kent at Canterbury, UK) and J.D.MORGAN (Old Dominion University, USA)

Suppose that the block size for a balanced incomplete block design (BIBD) is a multiple of m, and that each of the b blocks can be partitioned into m sub-blocks in such a way that the mb sub-blocks themselves constitute a BIBD. We then have a nested BIBD, as defined in the statistical literature in 1967. Suppose further that the sub-block size is a multiple of n, and that each of the mb sub-blocks can be partitioned into n sub-sub-blocks in such a way that the mnb sub-sub-blocks constitute yet another BIBD. We then have a doubly nested BIBD. Many doubly nested BIBDs are known to exist and are not hard to construct. Knowledge of the existence of such designs is reviewed, with emphasis on those with no more than 15 treatments and no more than 30 replicates of each treatment.

Keywords: Doubly nested balanced incomplete block designs

198

A sufficient Condition for a Planar Graph to be Class 1

by

Peter Che Bor Lam
Hong Kong Baptist University.

Key words and phrases: Edge chromatic number, planar graph.

In this paper, we first give some upper bounds on the number of edges for two classes of simple planar graphs. Using these results, we obtained some sufficient conditions for a simple planar graph to be Class 1.

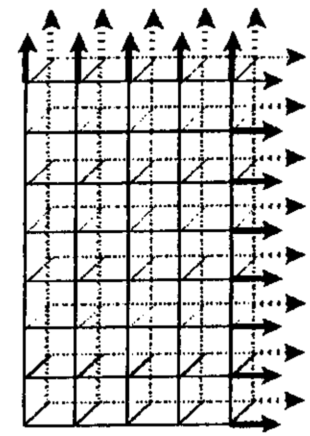
199

Cops and Robbers on Cayley Graphs

Robert Hochberg, NMC

A *pursuit game* on a graph, G, is played as follows: some cops, $C_1, \ldots, C_k$ each select and occupy a vertex of the graph (not necessarily distinct) and then a robber, R, selects and occupies a different vertex.

The cops then move, meaning that each cop chooses to either remain on its current vertex or move to an adjacent vertex; the cops have complete information about the graph and all players' whereabouts, and they may coordinate their movements. Then the robber moves by either staying put or moving to an adjacent vertex. Play then alternates until R is caught.



The question is: On which graphs do k cops suffice to catch a robber? We prove that for Cayley graphs of Abelian groups, of degree d, $(d/2)+1$ cops suffice, settling a conjecture of Hamidoune (1987), who had proved $3d/4$.

200

A Heuristic for a Leaf Constrained Minimum Spanning Tree Problem

P. Micikevicius*, N. Deo; University of Central Florida

Leaf Constrained Minimum Spanning Trees (LCMST) are of practical use in circuit and communication network design. The problem of finding a LCMST is defined as follows: given an undirected, weighted graph G and a positive integer l, find a spanning tree with the smallest weight among all spanning trees of G which contain at least l leaves. This problem is NP-complete. In this paper we consider two polynomial-time heuristics for computing an LCMST: (a) Iterative Refinement, where an unconstrained MST is computed first and then its edges are exchanged until the leaf constraint is satisfied, (b) One-Time-Tree-Construction in which the spanning tree is constructed by selecting edges in a greedy fashion so that the resulting tree is satisfies the leaf constraint. We analyze empirical results for both heuristics.

Key words: Minimum Spanning Tree, Leaf Constrained Minimum Spanning Tree, Approximate Algorithms.

Friday, March 12, 1999
11:10 AM

201

Balanced Nested Designs and their Applications

Ying Miao, University of Tsukuba

The notion of a balanced nested design was first introduced by Fuji-Hara and Kuriki in connection with a balanced array. In this talk, we will generalize this concept and describe the applications and constructions of the generalized balanced nested design.

Key words: Balanced nested design; Balanced array; Balanced n-ary design.

202

Edge-magic Index Sets of (p,p)-grapgs

by

Wai Chee Shiu[1] and Peter Che Bor Lam
Hong Kong Baptist University.

Key words and phrases: Edge-magic, Edge-magic Index, Edge-magic Index set, unicyclic graph.

A graph $G = (V, E)$ with $|V| = p$ and $|E| = q$ is called edge-magic if there is a bijection $f : E \to \{d, d+1, \cdots, d+q-1\}$ such that the induced mapping $f^+ : V \to \mathbb{Z}_p$, where $f^+(u) = \sum_{v \in N(u)} f(uv) \pmod p$, is a constant mapping. The edge-magic index of a graph G is the set of positive integers k for which the k-fold of G is edge-magic. In this paper, we determine the edge-magic index set of graphs whose order and size are equal.

203

The Search for an Honest Man

A.Bargteil*, W.Gasarch; U. of MD at College Park

Legend has it that the greek philosopher Diogenes carried a lantern and searched for an honest man. We rephrase his quest in more rigorous terms. If you have n men, of which at most a answer randomly, and the rest answer honestly, then (1) how many questions do you need to ask to find an honest man? (2) how many questions do you need to classify everyone? We denote (1) by $H(a, n)$ (H for Honest) and (2) by $C(a, n)$ (C for Classify). Each question is Boolean and directed at one of the men. We have obtained some upper and lower bounds on these quantities and some exact answers. The really new concept here is the very DEFINITION of a question, which is needed before one can even define lower bounds.

Keywords: Truth tellers and liars

Parallel Heuristics for the Diameter-Constrained MST Problem

A. Abdalla*, N. Deo, and R. Franceschini, University of Central Florida

204 Diameter-Constrained Minimum Spanning Trees arise in many applications, including in distributed mutual exclusion. It is of particular interest in multiprocessor environments where each pair of processors communicate by message passing. The Diameter-Constrained MST problem can be stated as follows: given an undirected, edge-weighted graph G and a positive integer k, find a spanning tree with the smallest weight among all spanning trees of G which contain no path with more than k edges. This problem is NP-complete, and some approximate heuristics can be found in the literature. We examine the best available heuristics for this problem and present some improvements, new heuristics, and a new parallel implementation. First, we present a reformulation of an existing algorithm that constructs a Diameter-Constrained MST in a modified greedy fashion,

employing a heuristic to select edges to add to the tree at each stage of the construction. Our modification renders the original algorithm faster and more easily parallelizable. Then, we present an entirely new heuristic that computes an unconstrained MST and iteratively refines it by edge replacements until it has no path larger than k. Our method is guaranteed to terminate, and it produces spanning trees with small k– not achievable with the previously published heuristics. We discuss parallel implementation of these heuristics on the MasPar MP-1 – a massively parallel SIMD machine with 8192 processors.

205 **Some results on the $L(j,k)$-labeling of K_n^d**

John P. Georges (presenter), David W. Mauro and Melanie Stein
Trinity College, Hartford, CT

For positive integers j and k, $j \geq k$, an $L(j,k)$-labeling of graph G is an assignment of integers to $V(G)$ such that the difference between the labels of adjacent vertices is at least j, and the difference between the the labels of vertices that are distance two apart is at least k. The λ_k^j-number of G is the minimum span taken over all $L(j,k)$-labelings of G. In this paper we extend the authors' previous work on the λ_k^j-number of products of complete graphs. In particular, we investigate aspects of $\lambda_k^j(K_n^d)$ for $d = 3$ and 4. We show that $\lambda_1^1(K_n^4) = n^2 - 1$ if and only if there exists a pair of mutually orthogonal Latin squares of order n.

Keywords: λ_k^j-labeling, product of complete graphs, Latin square

206 **Algorithm of Edge Labeling to a Magic Graph by Factorization**

Gwong C. Sun*, Yung-Nien Yang, and Yueh-Hsiang Liao
University of Louisville

A graph G=(V,E) is magic if we can find an edge labeling assignment L:E->{1, 2,...} such that the sum of all edge labels incident to each vertex has the same value, and this value is called the magic index w. A magic graph can be factorized by labels into a set of factors which are 1-regular, 2-regular, or mixed (1,2)-regular. The edge in a 2-regular is labeled 1 and edge in a 1-regular is labeled 2 such that every label in G is the sum of the labels of the corresponding edges in all factors. In this paper we present the algorithm to find all the 1-regular, 2-regular, and (1,2)-regular factors of a magic graph. Then, the magic labeling is found.

A characterization of matroids

07 Lorenzo Traldi, Lafayette University

Let M be a matroid on a set E, and consider these two subsets of E: $A = \{S \subseteq E \mid \text{every } s \in S \text{ is a loop in } M/(S - \{s\})\}$ and $B = \{S \subseteq E \mid \text{some } s \in S \text{ is a loop in } M/(S - \{s\})\}$. Then A and B have the same minimal elements, namely, the circuits of M. We prove that this property actually characterizes matroids, i.e., if C is a clutter on E and A and B are defined using C rather than M, then C is a matroid basis clutter if and only if A and B have the same minimal elements.

On an Optimal File Transfer on Path Graph with Step Arc Cost

208 Yoshihiro KANEKO*, Gifu University, Japan

A problem of obtaining an optimal file transfer on a file transmission N is to consider how to distribute, with a minimum total cost, copies of a certain file of information from some source vertices to other vertices on N by the respective vertices' copy demand numbers. The problem is a generalization of a Minimum Spanning Tree problem, which contains a Shortest Path problem. Thus far, we have dealt with N with linear arc cost. In order to apply theory to actual network systems, we should consider the case that each arc cost has a step function. In general, even on 2-step arc cost version, our problem has been shown to be NP-hard. In this talk, we restrict N to a path graph structure and we consider how to obtain an optimal file transfer on such N.

Key words network theory, algorithm

209

An irreducibility test of a polynomial over two element field

Makoto Matsumoto, Keio University

In 1998, the speaker and T. Nishimura proposed a pseudorandom number generator, named Mersenne Twister (MT), with period $2^{19937}-1$ and 623-dimensional equidistribution property. This generator is now widely used in financial engineering and particle physics. MT is based on a linear recurrence over two element field, whose characteristic polynomial is of degree 19937. One key to construct MT is an irreducibility testing algorithm of the characteristic polynomial of some special type of linear recurrence over two element field with computational complexity the square of degree, improving the cube in Berlekamp's test. The same algorithm found an irreducible (and primitive) trinomial $X^{859433}+X^{288477}+1$, whose degree is a new world record (it is the 33rd Mersenne exponent; this is a joint work with T. Kumada, H. Leeb, and Y. Kurita). The speaker explains the irreducibility test algorithm.

Keywords: irreducible polynomial, irreducibility test, finite field, random-number generation, Mersenne prime.

210

Cartesian Products of Edge-Magic and Edge-Graceful graphs

Karl Schaffer, De Anza College; Sin Min Lee*, San Jose State University

In the paper we show a relationship between the edge-gracefulness and the edge-magic of the Cartesian product of edge-graceful graphs. If G is a (p,q) graph in which the edges are labeled 1,2,3,...q so that the vertex sums are distinct, mod p, then G is edge-graceful. If such a labeling may be done so that the vertex sums are identical, mod p, then G is edge-magic. In 1990 the authors showed that the Cartesian product of even-regular, odd order edge-graceful graphs is also edge-graceful. In this paper we show that in many cases the Cartesian product of these graphs is also edge magic. In particular, the product CmXCn of two cycles is always edge-magic.

Key words: edge-graceful, edge-magic, Cartesian product, torus graphs

211

On the conjecture that among all graphs on n nodes and e edges, one with maximum number of spanning trees must be almost-regular

L. Petingi, College of Staten Island City University of New York
J. Rodriguez*, Brooklyn Campus Long Island University

Let $\Gamma(n,e)$ denote the class of all simple graphs on n nodes and e edges. The number of spanning trees of a graph G is denoted by $t(G)$. A graph $G_0 \in \Gamma(n,e)$ is said to be t-optimal if $t(G_0) \geq t(G)$ for all $G \in \Gamma(n,e)$. The problem of characterizing t-optimal graphs for arbitrary n and e is still open. It is conjectured that a necessary condition for a graph to be t-optimal is that the degrees of any two of its vertices should differ by at most 1. (Such graphs are said to be almost-regular).

We show that the conjecture is true "asymptotically" in the following sense: if G_0 is a graph with r nodes of degree $k-1$ and s nodes of degree k and if G_h is the disjoint union of G_0 and h cliques on k nodes, then there is an h_0 such that for $h \geq h_0$ every t-optimal graph in the class $\Gamma(n,e)$, to which the complement of G_h belongs, is almost-regular.

KEYWORDS: Spanning trees, almost-regular graphs, degree = sequences.

212

On the Existence of Convergent Transfer Subgraphs in Labeled Directed Acyclic Graphs

Mohammed Ghriga*, Long Island University, NY and Paul Kabore, NIST, MD

Let L and L' be finite alphabet sets. Let $G=(V,E)$ be a labeled directed acyclic graph, where $V=\{v_0,v_1,v_2,\ldots,v_n\}$, vertex v_0 is a source, and the labels of arcs in the Cartesian product $L\times L'$. It is assumed that $\forall v \in V, \forall (a,b) \in L\times L' : (v,[a,b],v') \in E \wedge (v,[a,b],v'') \in E \rightarrow v' = v''$. For an arbitrary subgraph H of G, we let $R_H(v)$ ($d_H^+(v,a)$, respectively) denote the set of vertices reachable from v in H (the number of outgoing arcs with labels of the form $[a,x] \in L\times L'$ at v for some x, respectively). We let $out_H(v) = \{a \in L | d_H^+(v,a) \neq 0\}$. A subgraph $G'=(V',E')$ of G is said to be a *convergent transfer subgraph* (CTS) for v_k in V if the following conditions hold: 1) $v_0, v_k \in V'$, 2) $\forall v \in V'$: $v \in R_{G'}(v_0) \wedge v_k \in R_{G'}(v)$, 3) $\forall v \in V'$: $|out_{G'}(v)| \leq 1 \wedge (|out_{G'}(v)| = 0 \leftrightarrow v = v_k)$, and 4) $\forall v \in V'$, $\forall a \in L_i$: $a \in out_{G'}(v) \rightarrow d_{G'}^+(v,a) = d_G^+(v,a)$.

An important decision problem, of practical significance to protocol testing, can be stated as follows: does $v \in V$ have an CTS in G? The purpose of such CTSs is to transfer a protocol entity to a specific state. There are no known efficient algorithms and exhaustive enumeration is used. In this paper, we present a *polynomial time* algorithm to the above decision problem when the cardinality of the set $\Omega_v(G) = \{v \in V | \exists a \in L : d_G^+(v,a) \geq 2\}$ is $O(log\ |V|)$.

Keywords: Acyclic Directed Graphs, Graph Traversals, Time Complexity

On the Coefficients of Polynomials in Finite Fields 213

Nelson A. Carella, March 1998.

In this paper we will demonstrate the existence of primitive polynomials with k prescribed consecutive coefficients in the finite field F_q for all sufficiently large q such that gcd(q, k!) = 1, and k + 1 < n / 2, with finitely many exceptions in every case. This result is also extended to primitive normal polynomials.

Key words: Finite fields, coefficients, primitive and primitive normal polynomials.

Mathematics Subject Classifications: 11T06.

An order for order preserving functions on a poset. 214

Hosien S. Moghadam Math Dept. Uni. of Wisconsin, Oshkosh, WI

Let P be a poset on $\{1, 2, ...n\}$ and $f : P \rightarrow R$ an order preserving real valued function. For any filter F of P define $f(F) = \sum_{x \in F} f(x)$.If f and g are two order preserving functions on P we say that $f \leq g$ iff $f(F) \leq g(F)$ for all filters F of P ,and $f(P) = g(P)$.We will talk about some of the known results for special case of P and discuss the case for P being the product of two chains.

215

Crown Graphs

Claudia Marcela Justel, Lilian Markenzon*
Instituto Militar de Engenharia, RJ, Brasil

It is an well known result that, being $G = (V, E)$ a graph of maximum degree m, there exists a graph H which is regular of degree m and that contains G as an induced subgraph. We are interested in the particular case when H and G are members of the same class. If G is a maximal outerplanar graph (*mop*), with $|V| > 3$, we know that H cannot be outerplanar, since the unique regular mop is a triangle.

216

On Optimal Acyclic Orientations of a Graph

Ashok T. Amin, Univ. of Alabama in Huntsville

Let (G,R) denote a directed graph obtained from undirected graph G by an acyclic edge orientation R of edges in G. We consider the problem of determining the orientations of edges in G which maximizes or minimizes the number of nonadjacent pairs of vertices x,y such that there is a directed path from x to y in (G,R). In the general case the problem is NP-Hard. A polynomial time algorithm is presented for the case when G is a tree for the maximization problem, and extremal trees are determined. Keywords: acyclic orientations, graphs

217

Asymptotic Results for the Genus of a Finite Field

Dawn M. Jones, Western Michigan University

In this talk we define what is meant by the genus of a finite field. We give some asymptotic results for some classes of finite fields by using known genus formulae.

Keywords: Finite field, genus

219

Regular Graphs of Given Girth

Linda Eroh, Western Michigan University

Hoffman and Singleton identified those values of r for which there could exist r-regular graphs of girth 5 and order r^2+1, namely, $r = 2, 3, 7$, and possibly 57. It is known that there are no r-regular graphs of girth 5 and order r^2+2. We consider r-regular graphs of girth 5 and order r^2+3. In particular, we will apply eigenvalue methods aided by factorizations in Maple to show the nonexistence of such graphs for $5 \leq r \leq 11$ and to give simpler constructions for the Robinson graph, $r = 4$, and the two graphs when $r = 3$. Furthermore, it is known that no r-regular graphs of girth 7 and order $r^3 - r^2 + r + 1$ exist. We show that no r-regular graphs of girth 7 and order $r^3 - r^2 + r + 2$ exist and consider eigenvalue methods for the next possible order.

220

Finding the same number twice

A. Chan, W. Gasarch, C. Kruskal*, U. of MD at College Park

There are several problems (sorting, element distinction (ED), 2-sum, 2-list-ED) that all have complexity $n \log n + \Theta(n)$; however, a more refined look at the constants may reveal that there complexities can be ordered. We look at two of them: (1) 2-sum: given n numbers, do any *two* of them sum to zero? (2) 2-list-ED: given two lists of $\frac{n}{2}$ elements, is there some element on both lists? We show that if $T(n)$ is the number of threshold graphs on n vertices, then 2-sum requires at least $\log_2 T(n)$ linear comparisons. We also show a lower bound on 2-list-ED that uses Stirling numbers of the second kind and complex analysis.

Keywords: Element Distinctness, Threshold Graphs, Stirling numbers

221

On the genus of finite loops

C. E. Ealy Jr., WESTERN MICHIGAN UNIVERSITY

Informally, a loop is non-empty set, L, with a multiplication with 1 such that the equations ax = b and ya = b have unique solutions in L for any a,b in L. In this paper, we define the genus of a loop and bound the genus of the loop by the genus of some groups associated with the loop. Key words Groups, loops, graphs

224

Pushing the Cycles out of Multipartite Tournaments

Jing Huang, Gary MacGillivray, Kathryn L.B. Wood*
University of Victoria, Canada

Let D be a digraph and $X \subseteq V(D)$. By pushing X we mean reversing each arc of D with exactly one end in X. Klostermeyer proved that it is NP-complete to decide if a given digraph can be made acyclic using the pushing operation. Here, we characterize, in terms of forbidden subdigraphs, the multipartite tournaments which can be made acyclic by pushing. This implies that the problems of deciding if a given multipartite tournament can be made acyclic using the pushing operation and, if so, finding a suitable subset X of vertices to push, are solvable in polynomial time.