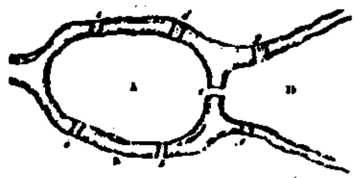
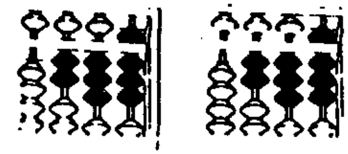
TWENTY-SIXTH SOUTHEASTERN INTERNATIONAL CONFERENCE ON





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COMBINATORICS GRAPH THEORY COMPUTING

Florida Atlantic University

March 6 -10, 1995

CRC PRESS, INC.

INDEX

This is an index of presenters only, accurate as of February 23.

MONDAY, MARCH 6, 1995

REGISTRATION begins at 8:00 A.M. in the downstairs lobby of the University Center, where COFFEE WILL BE SERVED. GCN (left or front) and GCS are the two Halves of the Gold Coast Room. FAU Rooms 202 A and C are reached through the second floor Lounge.

	GCN	GCS	202 A	202 C
9:00 AM	OPENING and WELCOME			
9:30	ASSMUS			
10:30	COFFEE			
10:50	1 LEONARD	2 RAJPAL	3 SINGER	4 WAXMAN
11:10	5 FRANCEL	6 RALL	7 MCCOLM	8 GOLDBERG
11:30	9 RINKER	10 TRALDI	11 FAUDREE	12 JOHN
11:50	13 LEVAN	14 H WU	15 KAYLL	16 WN LI
12:10 PM	17 RAINES	18 MARTIN	19 Ta SZABO	20 J WU
12:30	LUNCH (On your own -- Cafeteria open; there are many nearby restaurants)			
2:00	ASSMUS			
3:00	COFFEE			
3:20	21 Y-H WU	22 SAVAGE	23 COLLINS	24 MONSON
3:40	25 CARPENTER	26 FINIZIO	27 PIKE	28 LUNDGREN
4:00	29 SHOBE	30 CORNEIL	31 R LI	32 DEOGUN
4:20	33 CACERES	34 HURD	35 SCHWEIZER	36 BOYER
4:40	37 KNILL	38 SARASAMMA	39 ACREE	40 HEDETNIEMI
5:00	41 DG HOFFMAN	42 GYARFAS	43 GOULD	44 Z TONG
5:20	45 DOBSON	46 C CHEN	47 JM HARRIS	48 LANGLEY
5:40	49 Q YU	50 LODI	51 KLERLEIN	52 McRAE
6:30	CONFERENCE RECEPTION in the BOARD of REGENTS ROOM on the THIRD floor of the ADMINISTRATION BUILDING.			

There will be Conference transportation back to the motels at 6:05 PM, returning to the reception about 6:35. There will be transportation from the reception back to the motels.

TUESDAY, MARCH 7, 1995

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. Rooms 202 A and C are reached through the second floor Lounge. There will be book exhibits in Room 232 from 9AM-5PM.

	GCN	GCS	202-A	202-C
8:30 AM	53 FC HARRIS	54 SCHAAL	55 HATTINGH	56 HAXELL
8:50	57 GRIMALDI	58 HADFIELD	59 FRICKE	60 GARDNER
9:10	61 TANNY	62 DUTTON	63 MARKUS	64 TRENK
9:30	HEINRICH			
10:30	COFFEE			
10:50	65 GETU	66 KUBICKI	67 LAMPERT	68 VANDERJAGT
11:10	69 SHAPIRO	70 S-H LEE	71 SCHULTZ	72 FLETCHER
11:30	73 SPORTSMAN	74 CHIU	75 SMART	76 JONOSKA
11:50	77 Y-L LAI	78 WILLIAMS	79 HARE	80 JIA
12:10 PM	81 KOUNTANIS	82 J KENNEDY	83 SLATER	84 GAVLAS
12:30	LUNCH BREAK (ON YOUR OWN)			
2:00	HEINRICH			
3:00	COFFEE			
3:20	85 GORDON	86 NAIR	87 MADDELA	88 LATKA
3:40	89 RUSKEY	90 MCNALLY	91 PILLONE	92 G CHEN
4:00	93 COOPER	94 JT LEWIS	95 LASKAR	96 C-M FU
4:20	97 BAILEY	98 S-R KIM	99 GHOSHAL	100 VANAKEN
4:40	CONFERENCE PARTICIPANTS ARE INVITED TO HEAR SIGMA XI NATIONAL LECTURER FAN R. K. CHUNG IN THE GOLD COAST ROOM			
6:00	CONFERENCE PARTY at the home of JACK FREEMAN : 741 AZALEA ST, 395 - 7921.			

CONFERENCE TRANSPORTATION will leave for the motels at 5:50. There will be transportation from the UNIVERSITY CENTER to the party at about 5:55, and from the motels at about 6:30. There will be transportation from the party back to the motels. As always, we urge car-pooling, especially with parking spaces scarce near Freeman's. The main parking for the party, a church parking lot, is several blocks from Freeman home, and a very interesting walk; the walk from the University is a bit longer, but very pleasant, should you be adventurous.

WEDNESDAY, MARCH 8, 1995

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. Rooms 202 A and C are reached through the second floor Lounge. There will be book exhibits in Room 232 from 9AM-5PM.

	GCN	GCS	202-A	202-C
8:30 AM	101 EVANS	102 TAMFIRESCU		
8:50	105 KIRKPATRICK	106 BRIDGLAND	107 JIPSEN	108 WE CLARK
9:10	109 JIANG	110 JAMISON	111 E PHELPS	112 MERZ

9:30 KREHER

10:30 COFFEE

10:50	113 SULLIVAN	114 RASHIDI	115 HIND	116 MCKENNA
11:10	117 KHARAGHAM	118 AMIN	119 TESMAN	120 UNGERER
11:30	121 DIPAOLA	122 WEST	123 S-M LEE	124 PENRICE
11:50	125 PEDERSEN	126 DUNBAR	127 DEFIGUERIDO	128 PARKS

12:15 PM **CONFERENCE PHOTOGRAPH** at the OUTDOOR STAGE. We will lead you from the lobby, if you can't find it on your own, but PLEASE PARTICIPATE!

12:30 LUNCH BREAK (ON YOUR OWN)

2:00 KREHER

3:00 COFFEE

3:20	129 LABAHN	130 ALES	131 W GU	132 KONG
3:40	133 MEHTA	134 CRIBB	135 N DEAN	136 S-M TONG
4:00	137 KRAHN	138 LAWSON	139 MULL	140 H-L FU
4:20	141 MYRVOLD	142 PETERSON	143 NOSTRAND	144 DUFOUR
4:40	145 HAAS	146 SEAGER	147 PAGLI	148 MAURO
5:00	149 SOBEL	150 SCHAPER	151 VALDES	152 SUN

There will be a tea reception honoring women in Combinatorics, and open to participants of any sex, in Room 215 of the Science and Engineering Building. The tea, from 5:20 to 6:30 PM, is organized by Carolyn Johnston of FAU and Kathryn Fraughnaugh of University of Colorado in Denver. The The CONFERENCE BANQUET will be at the Sheraton, just south of Glades Road and west of I-95. Some folks will probably have drinks at the Sheraton s happy hour, which runs until 7:00PM. The banquet will start at 7:15. Conference transportation will be available to the motels at 5:45(a few minutes later from the reception). There will be transportation from the motels and the University Center to the Sheraton at approximately 6:35. There will be transportation back to the motels after the banquet

THURSDAY, MARCH 9, 1995

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. Rooms 202 A and C are reached through the second floor Lounge. There will be book exhibits in Room 232 from 9AM to 5PM.

	GCN	GCS	202-A	202-C
8:30 AM	153			
8:50	157 JL ARKIN	158 S XU	159 YN YEH	160 PFALTZ
9:10	161 GROSSMAN	162 O'DONNELL	163 Ti SXABO	164 K HUANG
9:30	ERDOS			
10:30	COFFEE			
10:50	165 SOIFER	166 SCHIERMEYER	167 SOLTES	168 VANHOOTEN
11:10	169 LAWRENCE	170 FRAUGHNAUGH	171 OSSOWSKI	172 AP SPRAGUE
11:30	173 X YU	174 HARANT	175 LEVIN	176 KLASA
11:50	177 MOLINA	178 SHREVE	179 HARBORTH	180 SALZBERG
12:10 PM	181 L CLARK	182 KUMAR	183 PEKEC	184 TAFT
12:30	LUNCH BREAK (ON YOUR OWN)			
2:00	185 FISHER	186 SANTORO	187 HAVAS	188 OWEN
2:20	189 BRAWLEY	190 RISPOLI	191 MORIN	192 H KIM
2:40	193 TURGEON	194 GROSS	195 ARSHAM	196 PENG
3:00	*COFFEE*			*Ulam Sessions:*
3:20	201 TAPIA	202 GARGANO	203 SHADER	204 COOK
3:40	205 HURLBERT	206 ROELANTS	207 KELMANS	208 KRYZAK
4:00	209 J ARKIN	210 JAGOTA	211 TB SPRAGUE	212 BLASS
4:20				
4:30	PARTICIPANTS ARE INVITED TO ATTEND A SIGMA XI LECTURE BY PHYLLIS CHINN			

There will be an informal CONFERENCE PARTY 6:00-7:30 in the Cafeteria Patio area--to be moved indoors if weather dictates. There will be Conference transportation back to the motels at 5:45 PM and back to the party at 6:15. There will be transportation back to the motels after the party.

FRIDAY, MARCH 11, 1995

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:30 A.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. Rooms 202 A and C are reached through the second floor Lounge. There will be book exhibits in Room 232 from 9:00 to 11:30.

	GCN	GCS	202-A	202-C
8:30 AM				
8:50	217 HOCHBERG	218 PROSKUROWSKI	219 GUAN	
9:10	221 BOHMAN	222 WYELS	223 SCHELP	
9:30	MULLEN			
10:30	COFFEE			
10:50	225 MALOUF	226 QIU	227 DOMKE	
11:10	229 SHARESHIAN	230 KOOSHESH	231 HOLM	
11:30	233 SHERMAN	234 BENEDICT	235 RASMUSSEN	
11:50	237 ASHLOCK	238 SERRAFZADEH	239 JOHNSON	
12:10 PM	241 BEARD	242 DEJTER	243 PACHTER	
12:30	LUNCH (ON YOUR OWN)			
2:00	MULLEN			
3:00	245 ROGERS	246 SUDBOROUGH	247 HA LEWIS	
3:20	249 GOLDWASSER	250 JANWA	251 MEYEROWITZ	
3:40	253 ZHAO	254 IHRIG	255 SHEKHTMAN	
4:00	257 HARTNELL	258 FIALA	259 KABATIANSKI	
4:20	261 SERVATIUS	262 DUNNING	263 CHOPRA	
4:40	265 UEMURA	266 HAGHIGHI	267	

There will be transportation back to the motels following the last talks.

THANKS FOR COMING!!

There will be an informal SURVIVORS PARTY, at the home of Aaron Meyerowitz and Andrea Schuver, 454 NE Third Street, DETAILS TBA. Tell us if you need transportation.

WE'LL SEE YOU IN BATON ROUGE FOR THE TWENTY-SEVENTH SOUTHEASTERN INTERNATIONAL CONFERENCE ON COMBINATORICS, GRAPH THEORY AND COMPUTING,

*** ?? ??-??, 1996 ***

INVITED INSTRUCTIONAL LECTURERS

Monday, March 6, 1995, Professor Edward F. Assmus, Jr., of INRIA and Lehigh University, will speak at 9:30 AM and 2:00 PM, on *Designs and their Codes.*

Tuesday, March 7, 1995, Professor Katherine Heinrich, of Simon Fraser University, will speak at 9:30 AM and 2:00 PM , on *Graph Decompositions: covering 2-paths.*

Wednesday, March 8, 1995, Professor Donald L. Kreher, of Michigan Technological University, will speak at 9:30 AM and 2:00 PM, on *Constructing t-Designs with* $t > 3$.

Thursday, March 9, 1995, Professor Paul Erdos, of the Hungarian Academy of Sciences, will speak at 9:30 AM, on *New Problems in Combinatorics and Graph Theory.*

Friday, March 10, 1995, Professor Gary L. Mullen, of Pennsylvania State University, will speak at 9:30 AM and 2:00 PM, on *Some Open Problems in Finite Fields with Combinatorial Applications.*

OTHER ACTIVITIES OF INTEREST

The FAU **Sigma Xi** Club is presenting two lectures during the week of the Conference. Sigma Xi National Lecturer Fan R. K. Chung of the University of Pennsylvania will speak on Tuesday, and Phyllis Chinn of Humboldt State University will speak on Thursday, as part of A series of lectures, jointly sponsored by FAU s College of Science, on improving the teaching of science.

The **Ulam Conference**, a six-year-old annual conference founded by Piotr Blass to honor the memory, mathematics and public interests of Stan Ulam, is meeting this week. Because of the interest Stan Ulam had in our Conference, we have invited the Ulam conferees to register for our Conference. Three Ulam Lectures appear on our Thursday afternoon program. There will also be Ulam sessions in FAU Room C on Friday, with a schedule to be announced later. The Ulam Conference will also have Saturday sessions, with details to be announced later. Individuals attending the Saturday sessions only will not have to register for our Conference.

Graph decompositions: covering 2-paths

Katherine Heinrich
Simon Fraser University

Most designs can be viewed as graph decompositions. The "base" graph is usually complete and the subgraphs making up the decomposition are usually isomorphic. On the other, hand graph theorists take an arbitrary graph and ask under what conditions it has a certain subgraph or family of subgraphs (for example, when does a graph have a Hamilton cycle). We will begin by looking at families of isomorphic subgraphs of the complete graph so that every path of length 2 is in exactly one subgraph and then look at related graph theory questions (compatible circuits and line-graph decompositions). The presentation will be an overview of results and the methods used to obtain them.

The Revolution in School Mathematics And Its Effects at the University

Phyllis Chinn, Mathematics Profesor Humboldt State U
Co-Director of PROMPT (Professors Rethinking Options in Mathematics for Prospective Teachers)

While much of the emphasis of the call for mathematical reform as reflected in the NCTM Standards and California Math Framework has been directed towards elementary and secondary school mathematics and teachers, many changes recommended for the precollege level would also be useful to implement at the post-secondary level. In fact, once the changes recommended at the precollege level have been implemented, students will graduate with much different preparation and expectations than the studentswhom we have seen in the past. It is time for college math teachers to think about what effect mathematics reform should and will have at these levels.

In particular, K-12 students are to have opportunities to do mathematics: explore, analyze, construct models, collect and represent data, present arguments, and solve problems. These opportunities are equally important for students in post-secondary mathematics courses. This colloquium will give participants an opportunity to experience a math activity in the Framework spirit and then discuss the significance of the proposed reforms at the college level.

Constructing t-Designs with $t > 3$

DONALD L. KREHER
Michigan Technological University

A t–design or t–(v,k,λ) design is a pair $(X,\mathcal{B})$ where: X is a v–element set of ***points***; $\mathcal{B}$ is a family of k–elements subsets of X, called ***blocks*** and every t–element subset $T \subseteq X$ is contained in exactly λ blocks. It is said to be ***simple*** or to have ***no repeated blocks*** if all the members of $\mathcal{B}$ are distinct. When t large and λ is small (say $t > 5$ and $\lambda < 10^6$ for example) very few designs are known. Indeed none, in this range, are known for $t > 7$. Progress on the construction of t-designs will be discussed.

OPEN PROBLEMS IN FINITE FIELDS WITH COMBINATORIAL, GRAPH THEORY AND COMPUTING APPLICATIONS

GARY L. MULLEN
THE PENNSYLVANIA STATE UNIVERSITY

We will discuss a number of problems and conjectures within the theory of finite fields, each of which has an application related to one of the themes of this conference.

Permutation polynomials have applications in various areas of combinatorics including finite geometries, orthogonal latin squares and Costas arrays. In almost any application of finite fields one needs to be able to efficiently do field computations, often in very large fields, and so we will discuss various problems related to the construction of different kinds of bases as well as the distribution of irreducible and primitive polynomials over finite fields. Last but not least, we will discuss the role played by finite fields in the construction of Cayley and Ramanujan graphs.

1 **Some New Z-cyclic Whist Tournaments**

Philip A. Leonard, Arizona State University, Tempe AZ 85287

A method for constructing Z-cyclic whist tournaments has been sought for cases when the number of players is the square of a prime $q = 4n + 3$. The only known example has been one for $q = 7$ due to N. J. Finizio By seeking examples with additional symmetry, the author has arrived at a method that promises to yield numerous examples for each $q > 7$. For $q = 11, 17, 19, 23$, and 31, the first examples were found by searching appropriate arrays by hand. The method, and results of preliminary machine computations, will be discussed in this talk.

2 **On GF(q)-representable paving matroids**

Sanjay Rajpal, Dartmouth College, Hanover, NH 03755.

A paving matroid is a matroid in which no circuit has cardinality less than the rank of the matroid. GF(q)-representable paving matroids having high rank can be used to contruct linear codes with large minimum distance. Using our results about paving matroids representable over GF(4), we show that all (12,6,6) codes over GF(4) have the same weight enumerator. Also, we address the question "Is every excluded-minor for GF(q)-representability a paving matroid?" and show that the answer is in the negative.

3 **Subgraphs and Connectivity in Random Intersection Graphs**

Michał Karoński (Adam Mickiewicz U)
Edward Scheinerman, Karen Singer* (Johns Hopkins U)

We introduce a model for random intersection graphs and describe some thresholds for the model. To obtain a random intersection graph on n vertices, we assign to each vertex a random subset of some universal set and then construct the appropriate intersection graph. We present a theorem about thresholds for the appearance and disappearance of small induced subgraphs in such a random graph. Integral to this analysis is an examination of possible clique covers for the subgraph. The threshold for graph connectivity is also given, with results depending on the relative size of the universal set and the number of vertices. The thresholds may be compared and contrasted with those derived for the Erdős-Rényi random graph model.

key words: random graphs, intersection graphs, connectivity threshold

4 **A Greedy Algorithm Approach to Workload Redistribution Balancing**

Robert R. Goldberg, Jacob Shapiro, Jerry Waxman
City University of New York

We consider the following problem which arises in many applications. Let S be a finite set of real numbers. The sum of all elements in S will be enoted by Sum(S). We are given an ordered partition of S into n sets Si, 0<i<n+1, i.e. U Si = S and Si /\ Sj is empty if i<>j. We are also given two integers m and q. The problem is to find an ordered partition of S into m sets Ti, 0<i<m+1 with the following conditions:

Condition 1. Ti is a subset of the union of the following sets Si-q, Si-q+1, ..., Si, ..., Si+q-1, Si+q where 0<i-q and i+q<n+1.

Condition 2. This partition {Ti} minimizes the "variance", i.e. over all possible partitions satisfying condition 1 the sum of (Sum(Ti) - Sum(S)/m)^2, for 0<i<m+1, is minimal.

It is easy to show that in general this problem is NP-hard. We propose a greedy algorithm which gives approximate solutions in polynomial time and describe the results of simulations which demonstrae that this algorithm gives very good approximations of the optimal solutions.

5 Ternary designs with more than one replication number. II

Margaret Francel*, The Citadel and D. G. Sarvate, University of Charleston

A Balanced Ternary Design BTD(V,B;R;K,Λ) as we know today is an arrangement of V points in B blocks (i.e. multisets) each of size K, such that every point occurs 0,1 or 2 times in a block, every point occurs R times in the design, and every pair of distinct points occurs Λ times in the design. Tocher's original definition of ternary designs does not restrict the replication number R to be a constant. However he showed the replication number R will come between the two numbers Λ(V-1)/(K-1) and Λ(V-1)/(K-2). A ternary design where all replication numbers are used is called a full ternary design (FTD).

One of the results we obtained last year is that the necessary conditions are sufficient for the existence of FTDs with block size 3.

Here we prove that FTDs for block size 3 are unique and FTDs with even block size do not exist. The question of the existence of FTDs with odd block size is completely open. Even for block size 5, no example of full ternary designs is known.

The second problem we deal with is the problem of constructing ternary designs where all but one or two possible replication numbers are not used. We show when and how such designs can be constructed for block size 3 and 4.

Key words: Balanced Ternary Designs

6 **The Ultimate Categorical Independence Ratio of a Graph**

Jason I. Brown & Richard J. Nowakowski, Dalhousie University, Nova Scotia
Douglas Rall*, Furman University, Greenville, South Carolina

Let $\beta(G)$ denote the independence number of a graph G of order n. We introduce $A(G) = \lim_{k\to\infty} \beta(G^k)/n^k$ where G^k denotes the k^{th} categorical power of G. This limit always exists and lies in the range $(0, 1/2] \cup \{1\}$. We show that this limit can assume any such rational number. A useful technique for bounding $A(G)$ is to consider special spanning subgraphs. These bounds allow us to compute $A(G)$ efficiently for many G. We give a condition which if true for G shows that $A(G) > \beta(G)/n$. The question "For which G does $A(G) = \beta(G)/n$?" is partially answered by showing it to be true if G is a Cayley graph of an Abelian group or if G is a connected graph that has an automorphism with a single orbit.

Key Words: Categorical Product, Independence Number

7 **Weak Threshold Functions**

by Gregory McColm, Univ. of S. Florida, Dept. of Math.
mccolm@math.usf.edu

Bollobas and Thomassen [BT] proved that every monotone graph property has a weak threshold; the proof was quite sophisticated and strictly combinatorial. We generalize this result using an easier probabilistic proof. This generalization allows us to go beyond random graphs and get threshold results in other situations.

We can get threshold functions when the underlying distribution is not uniform, e.g., in random graphs, where some edges are more probable than others. We can get threshold results when the underlying and / or resulting structure is infinite, e.g., selecting random sets of points from the unit interval. We can do both together, e.g., choosing random infinite sets of integers where, say, for each n, the probability that n is in the set is p_n: the threshold here would be an asymptotic density (of p_n as $n \to \infty$) much below which a random set would a.s. fail to satisfy the property, while much above the threshold, the random set would a.s. satisfy the property. And we can look at even weirder examples as time permits.

[BT] B. Bollobas & A. G. Thomassen, *Threshold Functions*, Combinatorica 7 (1986) 35-38.

8 **Parameterized Level Graph Searches**

Diego Betancor, Robert Goldberg*, Jacob Shapiro, Jerry Waxman
Baruch College - CUNY Queens College - CUNY

Level graphs were introduced by the authors to model communication networks with the following two charactistics: (1) there is a natural ordering of the network arcs into levels with different bandwidths, and (2) there is an independent distance metric (such as air distance) between the network nodes. Many existing networks have these properties. Algorithms for efficiently generating quasi-shortest paths in them (LGS, ALGS, BALGS) have been studied previously both analytically and via simulation by the authors. In this paper, we explore the parameters controlling path quality and computational complexity in the LGS family of level graph heuristic algorithms. These parameters include the number of levels, weight of each level, cost per arc, and level density. We show that BALGS reduces the run time significantly, with virtually no loss in the quality of the paths produced.

9

Small Multigraph Designs

Susan Rinker*, D.G. Hoffman, Auburn University

We investigate G designs of order n ad index lambda, where G is a loopless multigraph on three vertices.

10

Characterizations of matroid ports

Lorenzo Traldi, Lafayette College

If e is an element of a matroid M then the port of M with respect to e is the clutter {C-e : C is a circuit of M containing e}. Lehman [J. SIAM 12 (1964), 687-725] proved that if M is connected then it is completely determined by any of its ports. The proof involves a way of recovering all of M's circuits from the port; this directly suggests a way of defining "circuits" for general clutters. P. D. Seymour [Quart. J. Math. Oxford (2) 27 (1976), 407-413] gave a family of forbidden minors that characterize the matroid ports among general clutters. We disucss several non-matroidal properties shared by the circuits of these forbidden minors.

11

NUMBER OF CYCLE LENGTHS IN GRAPHS WITH MINIMUM DEGREE AND GIRTH

PAUL ERDŐS, RALPH FAUDREE*, DICK SCHELP
University of Memphis

Let $n(g,k)$ denote the minimnum number of different cycle lengths in any graph of girth at least g and minimum degree at least k. For example, $n(3,k) = k-1$. It will be shown that that there is a constant c such that

$$n(5,k) \geq ck^2, \quad n(7,k) \geq ck^{5/2}, \quad n(9,k) \geq ck^3,$$

and more generally for $t \geq 7$,

$$n(4t-3,k) \geq ck^{t/2}.$$

12

Genetic Approach for finding $IR(Q_n)$

David John, Wake Forest University

Genetic algorithms use a relatively new paradigm for solving problems which require searching a solution space of large size. These probabilistic algorithms are motivated by the natural operations of mating, gene crossover and mutation. In the literature, most often a genetic algorithm is associated with an optimization problem, frequently continuous. Recently, genetic algorithms have been applied to combinatorial problems. In particular, one focus has been a general genetic approach for the solution of $NP-hard$ problems. Given a graph $G = (V, E)$, the upper irredundance number, $IR(G)$, is the size of a maximal subset of vertices that are independent, in the sense of coverings. For this talk all graphs will be Queen's graphs. Exact values of $IR(Q_n)$ are known for certain small sizes of n. Bounds are also known. A genetic algorithm is constructed to find better lower bounds on $IR(Q_n)$, for some values of n. Goal-directed mutation is an important operator for this combinatorial search.

13

Hamming Triple Systems on 15 Points

Mike LeVan*, Kevin T. Phelps, Auburn University

Define a *Hamming triple system* on 15 points, denoted by $HTS(15)$, to be a Steiner triple system on 15 points which is embedded in a non-linear Hamming code of length 15. We shall investigate which Steiner triple systems on 15 points are the words of weight three in a binary perfect code of length 15 which contains the zero vector.

14

The 3-connected Graphs with Exactly Three N-essential Edges

James G. Oxley, Louisiana State University
Haidong Wu*, Southern University

Let G be a simple 3-connected graph. An edge e of G is essential if neither the deletion G\e nor the contraction G/e is both simple and 3-connected. Tutte's Wheels Theorem proves that the only simple 3-connected graphs with no non-essential edges are the wheels. In earlier work, as a corollary of a matroid result, the authors determined all simple 3-connected graphs with at most two non-essential edges. This paper specifies all such graphs with exactly three non-essential edges.

15

"Normal" distributions on matchings in a graph: an overview

Jeff Kahn, Rutgers U and Mark Kayll* U of Montana
kayll@selway.umt.edu

A probability distribution p on the set $\mathcal{M}$ of matchings in a fixed graph G is *normal* if there are edge weights $\lambda : E(G) \to \mathbf{R}$ such that, for each $M \in \mathcal{M}$, the probability $p(M)$ is proportional to $\Pi_{e \in M} \lambda(e)$. Normal (in this sense) distributions have some interesting and useful properties. For example, if p lies in the interior of the matching polytope of G, then edges e, f of G which are in a certain sense "far apart" behave approximately independently; *i.e.*, $\Pr(e, f \in M) \approx \Pr(e \in M)\Pr(f \in M)$ for *fixed* $e, f \in E(G)$ and $M \in \mathcal{M}$ chosen at *random* according to p. This talk will explore this and other properties of normal distributions.

keywords normal probability distributions, graph matchings, matching polytope, near-independence

16

On Scheduling of Two Processors

Wing Ning Li
Department of Computer Science, University of Arkansas

We consider the problem of scheduling tasks with precedence constraint on several processors. Traditionally, completion time has been considered as the only goal for the scheduling. We propose another formulation of the scheduling problem such that both the completion time and the communication cost are considered. We show the simplest subproblem of this scheduling problem, which is scheduling unit tasks with precedence constraint having unit communication cost on two identical processors, is NP-complete. Without considering the unit communication cost, however, this problem can be solved by lower order polynomial time algorithms.

Monday March 6,1995
12:10 p.m.

17 Embedding Partial Extended Triple Systems

Michael Raines* and C. A. Rodger, Auburn University

Let K_n^ℓ be the complete graph with one loop attached to each vertex. An *extended triple* is defined to be a loop, a loop with an edge attached (known as lollipops), or a K_3. A (partial) extended triple system is a set $B(n)$ of extended triples defined on the vertex set $\{1,\ldots,n\}$ which partition (a subset of) the edges of K_n^ℓ. If the extended triple system contains no loops or lollipops, then it is simply a Steiner triple system.
It will be shown that any partial extended triple system of order n can be embedded in an extended triple system of order at most $4n+10$.

Keywords: Triple systems, embeddings.

18

ON THE BETA-INVARIANT FOR GRAPHS

Jessica K. Benashski, Ryan R. Martin*, Justin T. Moore, Lorenzo Traldi
Lafayette College

We discuss Crapo's beta-invariant for matroids, but restrict our focus to graphical matroids. Using Tutte's "wheels and whirls" theorem, we list all the 3-connected graphs with beta-values less than 10. We also characterize the 3-connected graphs for which removal of an edge results in a series- parallel network. In addition, we take up the relationship between the beta-invariant and a measure of network reliability: the expected number of connected components of a network whose edges fail independently with probability p is a polynomial in p, equal to a summation involving the beta-invariants of subnetworks.

Ratio of the local and global average degrees of graphs

19

*Tamas Szabo, Zsolt Tuza

Key words: global and local average degree, groupie vertex

In a simple graph G the *global average degree* of the graph is defined as the arithmetic mean of the degrees of all vertices, and denoted by t_G, while the *local average degree* of a vertex $v \in V(G)$ is defined as the arithmetic mean of the degrees of its neighbors, and is denoted by t_v. Bertram, Erdős, Horák, Širáň and Tuza examined the function

$$f(n) = \max_G \min_{v \in V(G)} \frac{t_v}{t_G}$$

and proved a very strong assymptotical result on $f(n)$. We prove similar results confining ourselves to bipartite graphs and trees only.

20

General Deadlock-Free Routing Algorithm
for m * n Wormhole-Routed Torus Networks

ChunlinYang, Jie Wu*, Florida Atlantic University

The torus network is one of the popularly used interconnection networks. As an extension of the mesh network, it has many desirable properties. In this paper, we present a general deadlock free routing algorithm for an m * n (where m and n are positive integers) wormhole-routed torus network. The proposed algorithm is based on a special trail-based model. The objective is achieved by finding two consecutive Hamiltonian paths in the torus network. As an extension, a general routing algorithm is also given for an m * n * k three-dimensional torus network.

21

The Discrete Logarithm Problem in GL(n,q)

Alfred Menezes and Yi-Hong Wu*, Auburn University

The discrete logarithm problem in a finite group G is the following: given elements a and b in G, find an integer k such that b=a^k, provided that such an integer exists. In this talk, we present a probabilistic polynomial time reduction of the discrete logarithm problem in the general linear group GL(n,q) to the discrete logarithm problem in some small extension fields. Implications of this reduction to public-key cryptography will be discussed.

Keywords: discrete logarithm problem, public-key cryptography,

22

A Recurrence for Counting Graphical Partitions

Tiffany M. Barnes and Carla D. Savage(*), North Carolina State University

In this paper, we give a recurrence to enumerate the set G(n) of partitions of a positive even integer n which are the degree sequences of simple graphs. The recurrence gives rise to an algorithm to compute the number of elements of G(n) in time O(n^4) using space O(n^3). This appears to be the first method for computing |G(n)| in time bounded by a polynomial in n, and it enables us to tabulate |G(n)| for n <= 220.
We have also tabulated the ratio |G(n)|/|P(n)| for even n <= 220, where P(n) is the set of all partitions of n. It is an open question whether the ratio |G(n)|/|P(n)| goes to zero.

Keywords: graphical partitions, degree sequences of graphs, integer partitions, counting

23

COUNTING HAMILTONIAN PATHS IN RECTANGULAR GRIDS

Karen L. Collins*, Wesleyan University and Lucia B. Krompart

Given a rectangular grid with m vertices in each column and n vertices in each row, how many Hamiltonian paths are there from the lower left corner to the upper right corner? For m=1, the answer is 1 for any value of n. For m=2, the answer is 0 if n is even and 1 if n is odd. If m=3 then the answer is surprisingly 2 to the (n-1)st power. We give generating function answers for grids with fixed m=4,5.

24

The Effects of Vertex-deletion and Edge-deletion on the Clique Partition Number of a Graph

Sylvia D. Monson, Queen's University, Kingston, Ontario, Canada

The clique partition number, cp(G), of a graph G is the minimum number of cliques of a graph such that each edge of the graph belongs to exactly one clique. The clique covering number, cc(G), is defined in a similar way except that each edge belongs to at least one clique. R.C. Brigham and R.D. Dutton (1990) examined the effects on the clique covering number of removing a vertex or an edge. We will look at the effects of vertex- and edge-deletion on the clique partition number.

Monday March 6, 1995
3:40 p.m.

25 **On Hadamard matrices from resolvable Steiner designs**

L. L. Carpenter* and J. D. Key, Clemson University

Any resolvable Steiner 2-design $\mathcal{D}$ with parameters $2\text{-}(2k^2-k,k,1)$ defines a class of Hadamard $4k^2 \times 4k^2$ matrices that contains symmetric constant (row- or column-) sum matrices. We examine some designs with these parameters for resolvablity and show that, for any prime p, the p-ary code of any $(4k^2, 2k^2 \pm k, k^2 \pm k)$ design $\mathcal{M}_\rho$ obtained using a resolution ρ of $\mathcal{D}$ must contain the all-one vector. In the case of an oval design from a regular hyperoval in a desarguesian plane of order 2^m, we construct designs with parameters $(2^{2m}, 2^{2m-1} \mp 2^{m-1}, 2^{2m-2} \mp 2^{m-1})$ and having the minimal 2-rank $2m+2$ from codewords of the design $\mathcal{M}_\rho$ arising from the construction using a particular resolution ρ.

KEY WORDS: codes, designs, finite geometries

26 **Some Results Concerning the Number and Distribution of Common Primitive Roots**

Norman J. Finizio* and James T. Lewis
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Let $p_1, \ldots, p_n$ be distinct primes and set $N = p_1 p_2 \cdots p_n$. $W \in Z_N$ is called a <u>common primitive root</u> of $p_1, p_2, \ldots, p_n$ if and only if W is congruent to $w_i \pmod{p_i}$, $1 \le i \le n$, with w_i a primitive root of p_i. It is well known that there are $\phi(\phi(p_i))$ primitive roots of p_i, where ϕ is the Euler function. Via the Chinese Remainder Theorem there are then $\pi[\phi(p_i-1)]$ common primitive roots of $p_1, \ldots, p_n$. In this paper we discuss the distribution of the common primitive roots in Z_N for the special case $N = 3p_1p_2$ where each p_i is a prime of the form $p_i = 2^{b_i}u_i + 1$ with u_i odd, $b_i \ge 2$, $b_1 \ne b_2$.

KEY WORDS : primes, primitive roots, common primitive roots

27 **Hamilton Decompositions of Line Graphs**

David A. Pike, Auburn University

We show how a perfect 1-factorisation of a graph G can be used to obtain a Hamilton decomposition of the line graph, L(G), of G.

28 **Graphs with Interval p-Neighborhood Graphs**

J. Richard Lundgren*, Patricia A. McKenna, Sarah K. Merz
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Craig W. Rasmussen
Naval Postgraduate School, Monterey, CA, 93943

The (p)-neighborhood graph ("p open neighborhood graph") of a graph G, $N_p(G)$, is defined on the same vertex set as G, with $[x,y] \in E(N_p(G))$ if and only if $|N(x) \cap N(y)| \ge p$ in G, where $N(v)$ is the open neighborhood of vertex v. The $[p]$-neighborhood graph ("p closed neighborhood graph"), $N_p[G]$, is defined in the expected fashion. If G is the underlying graph of a symmetric digraph D, then the p-neighborhood graph of G is the p-competition graph of D. The case $p=1$ has been extensively studied by several authors. We consider the general case, asking, "which graphs have interval p-neighborhood graphs", and give specific results for the case $p=2$. We also relate the p-neighborhood graph of G to the square of G, again giving specifics for the case $p=2$.

Key Words: Competition Graph, p-Competition Graph, Neighborhood Graph, p-Neighborhood Graph, Interval Graph, Square of a Graph

29

Some Steiner Systems of Bagchi and Bagchi

Franklin D. Shobe, Clemson University

The designs $BB(p,q,e)$ of Bagchi and Bagchi [1] are 2-$(pq,p,1)$ designs where p and q are odd prime powers and $(p-1)|(q-1)$. When $q=p$, the design $BB(p,p,1)$ is isomorphic to the desarguesian affine plane of order p; we prove this using a result of Dembowski and Ostrom. When $q=p^d$ and $d>1$, the design $BB(p,p^d,1)$ is a 2-$(p^{d+1},p,1)$ design and we will show that it is different from the design of points and lines of the affine geometry $AG_{d+1}(\mathbf{F}_p)$. We prove this by showing that in the $BB(p,p^d,1)$ designs with $d>1$, there are in the span of a triangle two parallel lines and a third line which intersects one of the two parallel lines but not the other.

[1] S. Bagchi and B. Bagchi, "Designs from pairs of finite fields: I. A cyclic unital $U(6)$ and other regular Steiner 2-designs", *J. Combin. Theory, Ser. A*, 52:51–61, 1989.

Keywords: Steiner systems, designs, Bagchi and Bagchi, affine geometry

30

Linear Time Algorithms for Dominating Pairs in AT-free Graphs

Derek Corneil* (Univ. of Toronto), Stephan Olariu (Old Dominion Univ.) and Lorna Stewart (Univ. of Alberta)

An asteroidal triple (AT) is an independent triple of vertices in which each pair is joined by a path that misses the neighbourhood of the third. Graphs that have no ATs generalize interval, permutation, trapezoid and cocomparability graphs and display various aspects of linearity. One such property, when the AT-free graph is connected, is the existence of a dominating pair of vertices (ie a pair such that every path between them dominates the graph). In this paper we present a linear time algorithm (based on Lexicographic Breadth First Search) that computes a dominating pair for a connected asteroidal triple-free graph. It is interesting to note that such an algorithm was not known for cocomparability graphs, a strict subset of AT-free graphs. We also show that if the graph has diameter greater than three, then we can find an implicit representation for all dominating pairs in linear time (even though the number of such pairs may grow quadratically with |V|).

Keywords: algorithms, dominating pairs, asteroidal triple-free graphs, lexicographic breadth first search.

Hamiltonicity of Claw--free Graphs with Large Neighborhood Unions

31

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Let G be a claw--free graph of order n and connectivity k. If there exists some t which is less than or equal to k such that the number of neighbors of every independent set of t vertices is greater than t(n-3)/(t+1), then G is Hamiltonian.

Keywords: claw--free graphs, Hamiltonian.

32

Diametral Path Graphs

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In this paper we introduce a new class of graphs called Diametral Path Graphs. We investigate their structural properties, in particular we study the structure of minimum connected dominating sets in diametral path graphs.

Key Words. Diameter, Diametral Path, Asteroidal Triple-Free Grap, Connected Domination.

33 WEIGHT DISTRIBUTION OF DUALS OF CERTAIN CYCLIC CODES

Alberto Cáceres[1]* and Oscar Moreno[2]

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[2] University of Puerto Rico at Rio Piedras, Rio Piedras, PR 00931

For prime numbers p for which 2 is semiprimitive, i.e., $s = ord_p(2) = (p-1)/2$, we consider duals of cyclic codes of block length $n = 2^s - 1$ and set of zeros $\{\alpha^p\}$ over finite fields $\mathbf{F}_{2^s} = GF(2^s)$. Here α is a primitive element of $\mathbf{F}_{2^s}$. We compute exponential sums $\sum_{x \in \mathbf{F}_{2^s}} (-1)^{Tr(ax^p)}$ for $a \in \mathbf{F}_{2^s}$, to obtain the weight distribution of these codes and find three non trivial weights. Using Davenport-Hasse theorem we extend the results for arbitrary degree field extensions $\mathbf{F}_{2^{sm}}/\mathbf{F}_{2^s}$.

Key words: cyclic codes, duals, exponential and Gaussian sums.

34 **Computer Search For Primitive Roots, Motivated By A Digraph**

by

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We reported previously on a digraph defined by squaring mod p *(see Congressus Numerantium* V. 82, 1991, p. 167-177, and *The Fibonacci Quarterly*, V. 30, 1992, p. 322-334). Vertices are $\{0, 1, 2, \ldots, p-1\}$ and ab is a directed edge if a^2 is congruent to b, modulo p. At that time we used the digraph to ease the proofs of several known results, such as "2 is a primitive root for p if $p = 1 + 4q$ for some odd prime q." We can now say something about the conjecture that "2 is a primitive root for p whenever $p = 1 + 4q^2$ for some odd prime q."

Isomorphism of Hamilton Cycle Decompositions of K_{2n+1} 35

Dan Ashlock, Iowa State and David Schweizer *, Holy Cross

Two Hamilton cycle decompositions of the complete graph on 2k+1 vertices are isomorphic when a permutation of the vertices takes the cycles of one decomposition onto the cycles ff the other. In this paper we investigate the number of isomorphism classes of such cycle decompositions. We also show that checking isomorphism is easy; i.e. requires only polynomial time.

36

Biclique Partitions of the Mycielskian of a Graph

Elizabeth D. Boyer*, University of Wyoming
David Fisher and Patricia McKenna, University of Colorado at Denver

Let G be a graph with vertex set {1,2,...,n}. The Mycielskian of G, u(G), is the graph with vertex set {x1,...xn,y1,...yn,z} and edges {(xi,xj),(xi,yj)|(i,j) is an edge in G} and {(yi,z)| for all i}. We examine the biclique partition number, bp(u(G)), of the Mycielskian of G. In particular, we give an upper and lower bound for bp(u(G)) in terms of the rank of the adjacency matrix for G, the order of G and bp(G) and bp(G+v) where G+v is the graph obtained by adding one new vertex v to G which is adjacent to all vertices of G. In addition, we give some results on how bp(G+v) is related to bp(G).

Key Words: biclique partition, Mycielskian

Monday, March 6, 1995
4:40 p.m.

37 On the Asymptotics of Superimposed Distance Codes

E. Knill, Los Alamos National Laboratory

Let $\mathcal{F}$ be a family of k element subsets of a v element set. $\mathcal{F}$ is a superimposed distance code with parameters p, r, d, k, v (an SDC(p, r, d, k, v) for short) if for every p distinct members $U_1, \ldots, U_p$ of $\mathcal{F}$ there are at most r members $V \in \mathcal{F} \setminus \{U_1, \ldots, U_p\}$ such that $|V \setminus \bigcup_{i=1}^{p} U_i| < d$. Superimposed distace codes were introduced by Dyachkov and Rykov (Problems of Control and Information Theory 12 1–13, 18 237–250). Large SDC's have applications in non-adaptive boolean group testing and are used for efficient screening of clone libraries in support of the human genome project. For group testing, the most interesting aspect of the asymptotic behavior of the maximum size SDC's is determined by

$$D(p, r, \alpha, \beta) = \limsup_{v} \frac{\log(|\mathcal{F}_v|)}{v},$$

where $\mathcal{F}_v$ is a maximum size SDC($p, r, \lfloor \alpha v \rfloor, \lfloor \beta v \rfloor, v$). I will discuss what I know about the properties of D and the corresponding function for the closely related probabilistic SDC's. There are many open problems whose solutions would contribute toward a better understanding of these asymptotics.

Keywords: Superimposed distance codes, r-cover free families of sets, non-adaptive group testing, library screening, packings, error-correcting codes.

38 Minimum Co-operative Guard Problem For 3-spiral Polygons

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A partial solution to the minimum co-operative guard problem in 3-spirals is proposed. The algorithm is based on an efficient scheme to partition a given 3-spiral into two sub-spirals, where each sub-spiral is at most a 2-spiral.

key word: co-operative guards, spiral polygons

39 Hamiltonian Problems and Z_i-free Graphs

Glenn Acree, Wake Forest University

The first Hamiltonian result using the forbidden subgraph method was the following theorem of Goodman and Hedetniemi. *If G is a 2-connected $K_{1,3}$-free, Z_1-free graph, then G is Hamiltonian.* Since that time the forbidden subgraph method has been used extensively, and quite successfully in the area of Hamiltonian problems. Throughout this work the complete bipartite graph $K_{1,3}$, the claw, has played a vital role. In many cases, the family of graphs denoted by Z_i, the graphs obtained by identifying a vertex of K_3 with an end-vertex of P_i, has played its part as well. The purpose of this presentation is to examine the importance of the Z_i-free restriction. For $i = 1$, the restriction is quite strong as has been noted. It will be seen that this is true even when the $K_{1,3}$ restriction is replaced by a similar restriction of $K_{1,r}$ graphs for $r \geq 4$. Several other results are given involving Zi-free graphs in an effort to explore the strength of such restrictions for $i \geq 2$.

40 Stable labellings and covering functions of graphs

G. Fricke, Wright State University,
S.M. Hedetniemi and S.T. Hedetniemi*, Clemson University
A.A. McRae, Appalachian State University

Let G = (V,E) be a graph and let f: V -> {0,1} be a funtion satisfying the condition: for every vertex v in V, f(v) = 1 - min {f(u): u in N(v)}. We say that f is a stable labelling of G. Let |f| = the sum of f(u) for all u in V. The lower stable labelling number sl(G) = min {|f|: f is a stable labelling and SL(G) = max {|f|: f is a stable labeling}. In this preliminary paper we study stable labellings and their fractional counterparts and show that they are closely related to vertex covers and other covering functions of graphs.

Monday, March 6, 1995
5:00 p.m.

41

The Intersection Problem for Star Designs

Elizabeth J. Billington, U. of Queensland,
and D.G. Hoffman*, Auburn U.

An m-star design of order n is a partition of the edges of the complete graph on n vertices into m-stars. We determine all but an exiguous handful of those triples (m,n,i) for which there are two m-star designs on the same n-set having exactly i stars in common.

42

Monochromatic Path Covers

A. Gyárfás

Computer and Automation Institute, Hungary & University of Memphis

How many vertices can be covered by t monochromatic paths in an edge-colored complete graph? What if the paths must have the same color? Problems and results of that flavour are discussed. A new result with P. Erdös is that t paths of the same color covers at least a $\frac{t+1}{t+2}$ fraction of the vertex set of a two-colored complete graph.

43

Forbidden Subgraphs and Hamiltonian Properties

Ronald J. Gould
Emory University

Over the years a variety of families of forbidden subgraphs have been discovered that imply a graph has some hamiltonian-type property. Here we characterize such families when the families are of size two and the properties are that of being traceabe, hamiltonian, pancyclic, cycle extendable and several others. We also consider the problem when the families have size three.

44

Connectivity In A Fussy Graph

Zengxiang Tong*, Otterbein College
Deda Zheng, South Carolina State University

This paper introduces the concepts of a fuzzy graph and the connectivity between each pair of vertices in the graph. An algorithm for finding the connectivity between two vertices is given. The applications to acturial science and other fields are also mentioned in the paper.

Monday, March 6, 1995
5:20 p.m.

45 PACKING ALMOST STARS INTO THE COMPLETE GRAPH

Edward Dobson, Louisiana State University

We verify that the Tree Packing Conjecture is true for every sequence of trees $T_1, \ldots, T_n$ such that there exists $x_i \in V(T_i)$ and $T_i - x_i$ has at l east $i - \sqrt{6(i-1)}/4$ isolated vertices.

Key Words: Packing, Tree, Complete graph.

46 Factors of Graphs with Odd-Cycle Property

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In this paper we state a variant of Hall's condition for the existence of a perfect matching in a graph with the odd-cycle property that any two odd cycles either have a vertex in common or are joined by an edge. As applications, we abtain some results on factors in graphs with the odd-cycle property.

47 Forbidden Triples of Subgraphs and Traceability

Ronald J. Gould and John M. Harris*
Emory University

Given a family $\mathcal{F}$ of connected graphs, a graph G is said to be $\mathcal{F}$-free if G contains no induced subgraph that is isomorphic to a graph in $\mathcal{F}$, and the graphs in such a family are called forbidden subgraphs. A topic considered recently has been the investigation of which sets of subgraphs can be forbidden in order to imply certain graph properties. In particular, the various hamiltonian properties have been considered in these matters. For instance, it is known that there are six different pairs of graphs that imply traceability (the existence of a hamiltonian path) when forbidden, and Faudree and Gould showed that these six pairs constitute all such pairs. Here we extend this same idea to triples of subgraphs. First, several particular families of triples are shown to imply traceability in sufficiently large graphs, and then it is shown that these are the only families that enjoy this property.

48 The Competition Graphs of Interval Digraphs

Larry J. Langley*, J. Richard Lundgren, Sarah K. Merz
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Interval digraphs were introduced by Sen, Das, Roy and West as an object analogous to interval graphs. We examine the competition graphs of interval digraphs. We show that the competition graph of an interval digraph is an interval graph. Furthermore, every interval graph is the competition graph of an interval-point digraph and thus the competition graph of an interval digraph. It has been observed that the competition graphs of digraphs modeling actual food webs are frequently interval. Do these results indicate an explanation for this phenomenon?

49

STAR-FACTORS WITH PRESCRIBED PROPERTIES

Guizhen LIU
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and Simon Fraser University, Burnaby, BC, Canada

A {K(1,1), K(1,2), ..., K(1,n)}-factor or (1, n)-star-factor of a graph G is a spanning subgraph of G each components of which is isomorphic to one of {K(1,1), K(1,2),..., K(1,n)}. If for any subset S of V(G) and |S| = k, G - S has a (1, n)-star-factor, then we say that G is (k, n)-star-critical. An necessary and sufficient condition for a graph to be (k, n)-star-critical is given. The properties of (k, n)-star-critical graphs are discussed. Furthermore, the relationship between star-factors and toughness of graph is studied.

50

Efficient Partition of Planar Graphs into Three Forests

R. Grossi, U Florence, Italy *E. Lodi* *, U Siena, Italy

Given a graph G, the *arboricity* $a(G)$ of G is the minimum number of edge-dsjoint spanning forests into which G can be decomposed. For a *planar* graph G, it is well known that $a(G) \leq 3$ [T. Nishizeki and N. Chiba, *Planar Graphs: Theory and Algorithms*, North-Holland, 1988]. We focus therefore on the problem of partitioning a planar graph G with n vertices into *three* edge-disjoint spanning forests. Such a partition can be useful to compact the $n \times n$ adjacency matrix of G into optimal $O(n)$ space, *still* achieving a constant-time query to check whether or not any two chosen vertices of G are adjacent.
There are several unpublished algorithms that require $O(n)$ time to partition G into $k \geq 4$ forests. However, for the case $k = 3$, the best known solution requires $O(n\sqrt{n}\log n)$ time, and can be obtained by applying the elegant algorithm of Gabow [*ACM STOC1991*], originally designed to work with arbitrary graphs. We show that it is possible to obtain an $O(n \log n)$ time solution for $k = 3$. Indeed, we reduce our problem to the one of labelling the edges of G with $k = 3$ colors, so that no cycle has all edges of the same color. We use simple combinatorial properties of planar graphs, along with graph transformations that preserve the planarity and the coloring on G. Moreover, we need the Union-Find data structure with arbitrary deunions to test whether two vertices are connected by a path of edges with the same color.
Keywords: Algorithms, asymptotic complexity, planar graphs.

51

On Hamiltonian Cycles in Cn Xs,t Cm

Joseph B. Klerlein*, Western Carolina University
A. Gregory Starling, University of Arkansas

In 1978 Trotter and Erdos gave necessary and sufficient conditions for the direct product, CnXCm, of two directed cycles to be hamiltonian. In this paper we give some sufficient and some necessary conditions for hamiltonian cycles in Cn Xs,t Cm, the directed graph obtained from CnXCm by joining vertices which are
s units and t units apart in a single m-cycle.

52

Various Minus Functions for Graphs

Srini Maddela, Clemson University
Alice A. McRae*, Dee A.Parks Appalacan State University

Many graph parameters can be defined in terms of (0,1)-valued functions on the vertices of a graph: a vertex has a value 1 if it is in a set S or a value 0 if it is not in S. By defining set parameters in terms of functions, it is possible to identify new parameters by generalizing the two-valued function definition. In this talk, we expand on the many classes of problems that can be defined in terms of (-1,0,1)-valued functions on the vertices of a graph.

Tuesday March 7, 1995
8:30 a.m.

53 **An Introduction to Steiner Minimal Trees on Grids**

Frederick C. Harris, Jr. fredh@cs.unr.edu, Computer Science, U of Nevada, Reno, Nevada 89557

The Optimization problem is simply stated as follows: Given a set of N vertices, construct a connected network which has minimum length. The problem is simple enough, but the catch is that you are allowed to add junctions in your network. Therefore the problem becomes how many extra junctions should be added, and where should they be placed so as to minimize the overall network length.

This intriguing optimization problem is also known as the Steiner Minimal Tree Problem, where the junctions that are added to the network are called Steiner Points.

We will discuss what is known about the problem, and then focus our attention on grids. The conjectured characterization of the Steiner Minimal Tree (SMT) for a $2 \times m$ grid is generally known to be the Full Steiner Tree on the vertices of the grid, while the only other conjectured characterizations for grids previously known were for square grids. We will present the conjectured characterization of SMT's for $2 \times m$ grids, as well as conjectured characterizations of SMT's for grids up through $8 \times m$.

Keywords: Steiner Minimal Trees, Grids

54 **Some Generalizations of Problems of Rado**

by Daniel Schaal
Clarion University of Pennsylvania

Given a system S of linear equations and inequalities in $x_1, x_2, \ldots, x_m$ with integral coefficients, one can ask the following questions.

(1) Regularity. Is it true that for every positive integer t and every t-coloring of the positive integers there exists a monochromatic solution to the system S?

(2) Zero-sum. Given a coloring $f:\{1,2,\ldots\} \to Z_m$ where Z_m is the cyclic group of residues modulo m, can we always find a solution $x_1, x_2, \ldots, x_m$ to the system S such that $f(x_1)+f(x_2)+\ldots+f(x_m)=0$?

In this talk we shall present some additional problems of this nature, some new results concerning these problems with particular systems, and the results of some computer experiments which give further insight into these problems.

55 **On weak domination in graphs**

J. H. Hattingh*, Rand Afrikaans University, Johannesburg, South Africa
R. C. Laskar, Clemson University, Clemson, S. C.

Sampathkumar and Pushpa Latha conjectured that the independent domination number, $i(T)$, of a tree T is less than or equal to its weak domination number, $\gamma_w(T)$. We show that this conjecture is true, prove that $\gamma_w(T) \leq \beta(T)$ for a tree T, exhibit an infinite class of trees in which the differences $\gamma_w - i$ and $\beta - \gamma_w$ can be made arbitrarily large, and show that the decision problem corresponding to the computation of $\gamma_w(G)$ is NP-complete, even for bipartite graphs. Lastly, we provide a linear algorithm to compute $\gamma_w(T)$ for a tree T.

* Presenter.

AN EXTENSION OF HALL'S THEOREM 56 FOR HYPERGRAPHS

P.E. Haxell, University of Waterloo

Let V be the disjoint union of finite sets A and X, and let $r \geq 2$ be an integer. Let $\mathcal{H}$ be a hypergraph such that each edge of $\mathcal{H}$ is a subset of V of size at most r that contains exactly one element of A. For example, an r-partite r-uniform hypergraph has this property. We give a condition which ensures that $\mathcal{H}$ contains a matching from A to X, that is, a set of pairwise disjoint edges of size $|A|$.

keywords: hypergraphs, matchings

57

Generating Sets and the Fibonacci Numbers

by
Ralph P. Grimaldi

Rose-Hulman Institute of Technology

Key words: Generating set, Fibonacci numbers

For $n \in Z^+$ let $[n] = \{1, 2, 3, \ldots, n\}$. If $\phi \neq S \subseteq [n]$, then $S + 1 = \{s + 1 | s \in S\}$. We say that $S(\subseteq [n])$ generates $[n + 1]$ if $S \cup (S+1) = [n+1]$. For $n \geq 1$ let g_n count the number of such generating sets for $[n + 1]$. Then $g_n = F_n$, the n -th Fibonacci number, where $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. In this paper we examine two related sets of numbers: (1) g(n, k), the number of subsets of [n] of size k that generate [n + 1]; and, (2) h(n, k), the number of occurrences of k among the g_n subsets of [n] that generate [n + 1].

58

USE OF GRAPH THEORY IN A PARALLEL MULTIFRONTAL METHOD FOR SEQUENCES OF UNSYMMETRIC PATTERN SPARSE MATRICES

Steven M. Hadfield *, USAF Academy and Timothy A. Davis, University of Florida

Multifrontal matrix factorization methods used for solving large, sparse systems of linear equations decompose sparse matrices into overlapping dense submatrices which can be represented by vertices with relationships between submatrices shown via various types of edges. This presentation describes the use of graph theory in a new parallel, distributed memory multifrontal method for the LU factorization of sequences of matrices with an identical, unsymmetric pattern. The directed acyclic graphs (DAGs) formed by these vertices and the various edge sets are used to structure the computations, schedule the parallel factorization, and provide a robust capability to dynamically change the pivot ordering to maintain numerical stability. Pivot reordering determines necessary permutations based on a path analysis of two component edge sets. The path properties represented by these edge sets define the impacts of these permutations on the structures of the submatrices and the number of nonzeros in the matrix factors. Transitive reductions of these edge sets provide the communications paths needed for parallel implementation.

KEYWORDS: Sparse matrices, parallel algorithms, directed acyclic graphs, applications

59

Fractional Parameters of Graphs

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Henda C. Swart, University of Natal, Durban, South Africa

Let G be a graph with vertex set V. A real-valued function $f : V \rightarrow [0,1]$ said to be dominating if $f(N[v]) = \sum_{x \in N[v]} f(x) \geq 1$ for all $v \in V$. A real-valued function $f : V \rightarrow [0,1]$ is irredundant if for every vertex $v \in V$ with $f(v) > 0$ there exists a closed neighborhood N[x] such that $v \in N[x]$ and $f(N[x]) = 1$. We introduce several related fractional parameters, among them fractional independence and present some preliminary results.

60

Decompositions of the Complete Digraph into Orientations of Cycles

Robert Gardner, Dept. of Math, East Tennessee State University

Let g be a subgraph of G. A *decomposition* of G into copies of g is a collection of isomorphic copies of g, $\{g_1, g_2, \ldots, g_N\}$ such that $\bigcup_{i=1}^{N} g_i = G$, with decompositions of digraphs similarly defined. An $n-$*cycle system* is a decomposition of the complete digraph into copies of an $n-$cycle. An analogous decomposition of digraphs is a decomposition of the complete digraph into copies of a digraph d, where d is some given orientation of a cycle. We explore such decompositions by presenting direct constructions in certain cases.

Keywords: graph decompositions, cycle systems, directed triple systems, Mendelsohn triple systems

61

Periodicities of Solutions of the Generalized Lyness Recursion and certain of its Relatives involving the Maximum Function

Ed Barbeau and Stephen Tanny * , U of Toronto

The mapping $T_c(x,y) = \left(y, \frac{y+c}{x}\right)$ of the real plane into itself is used to examine the periodicity of the solutions of the generalized Lyness recursion $x_{n+1} = \frac{x_n+c}{x_{n-1}}$. When c is fixed, we show that this mapping is periodic with given period independent of (x, y), if and only if $c = 0$ or $c = 1$. Certain cubic curves remain invariant under T_c, and some of these are loci of periodic points for different periods. For the related recursion $y_{n+1} = \frac{\max(A,y_n)}{y_{n-1}}$, we define the analogous map, which we use to identify the periodicities of $\{y_n\}$ and some of its generalizations.

Key Words: Lyness recursion; periodic solution.

62

Deleted Subgraph Isomorphism

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A variant of the Partitioned Graph Isomorphism Problem (as given by Knisely, Wallis, and Domke) is defined, for either vertices or edges, as follows: For a given graph G and integer k, does G possess two disjoint subsets, A and B, each of m =84 k vertices (edges) such that G-A is isomorphic to G-B? We establish bounds on valid values of k, i.e., values in which the answer can be 'yes,' and exact values for several common classes of graphs.

63

Graphs with exactly two sizes of minimal dominating set

Jean Dunbar, Converse College
Lisa Markus*, Furman University
Douglas Rall, Furman University

Let γ denote the size of the smallest minimal dominating set of a graph and let Γ denote the size of the largest minimal dominating set of a graph. Graphs with $\gamma = \Gamma$, called *well-dominated*, were investigated by Finbow, Hartnell and Nowakowski. Here, we investigate graphs with $\gamma < \Gamma$ where every minimal dominating set has size γ or Γ. Such graphs are said to be in the class $\mathcal{D}_2$. We characterise the trees with this property and exhibit a neighbourhood union bound for the split graphs in $\mathcal{D}_2$. We also show that any r-regular bipartite graph in $\mathcal{D}_2$ has at most $4r$ vertices.

Key word: dominating set.

Bounded Bitolerance Digraphs

64

Kenneth P. Bogart, Dartmouth College
Ann N. Trenk*, Wellesley College

Tolerance graphs arise as a generalization of interval graphs in which some overlap of intervals is tolerated. A graph G is a *bounded bitolerance graph* if each vertex x corresponds to a real interval I_x with a left–tolerant region and a right–tolerant region so that $x \sim y$ in G if and only if $|I(x) \cap I(y)|$ does *not* lie entirely within tolerant regions of both. When an adjacency occurs between vertices x and y, it can be caused by y exceeding x's tolerance, by x exceeding y's tolerance, or both and thus we are motivated to represent tolerance graphs as directed graphs.

Our results include characterizing the classes of unit, proper and bounded bitolerance digraphs in the cases where the underlying graph is a tree or a cycle, and proving a correspondence between the class of bounded bitolerance digraphs and digraphs whose arc set is the intersection of the "less than or incomparable to" relations of two interval orders.

Key words: tolerance orders and graphs, interval orders and graphs, dimension of an ordered set.

Tuesday March 7, 1995
10:50 a.m.

65

Lattice Paths and Bessel Functions

Seyoum Getu*and Louis W. Shapiro, Howard University

We take a large class of sequences that arise in combinatorial problems and express them in terms of Bessel functions. An infinite lower triangular matrix M is called Riordan if the generating function for the k^{th}column is $g(z)\,(f(z))^k$, $k = 0, 1, 2, \ldots$ with $g(z) = 1 + \sum_{n>0} g_n z^n$ and $f(z) = z + \sum_{n\geq 2} f_n z^n$. An **RT** or an $\mathbf{RT}(b, \lambda, \varepsilon, \delta)$ matrix $L = (l_{n,k})_{n,k\geq 0}$ is a Riordan matrix with $l_{n+k,k} = l_{n,k+1} + bl_{n,k} + \lambda l_{n,k+1}, k \geq 1$ while $l_{n+1,0} = (b+\varepsilon)l_{n,0} + (\lambda+\delta)l_{n,1}$ and $l_{0,0} = 1$, $l_{n,k} = 0$ for $n > k$ where $b, \lambda, \varepsilon, \delta$ are constants.

It is shown that an RT matrix L can be factored into three simpler matrices; these being of Pascal, Catalan and Fibonacci type. From this factorization L is expressed in terms of the modified Bessel functions. For example $C(z) = \sum_{n\geq 0} C_n \frac{z^{2n}}{(2n)!} = I_0(2z) - I_2(2z)$ where C_n is the sequence of Catalan numbers 1,1,2,5,14,42,132,... and I_n denotes the n^{th} modified Bessel function.

Key words: Riordan matrix, RT matrix, Bessel functions, Catalan numbers, Fibonacci numbers.

66

PATH SPECTRUM OF A GRAPH

Michael S. Jacobson, Andre E. Kezdy, Ewa Kubicka,
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Department of Mathematics, University of Louisville

A path of a graph is maximal if it is not a proper subgraph of any other path of the graph. The path spectrum of a graph is the set of lengths of all maximal paths in the graph. A finite set S of positive integers is called graphic if there is a finite, connected, simple graph having S as its path spectrum.
We discuss which finite subsets of positive integersare graphic and which of them are realizable by trees.

Keywords: maximal paths

The Acquisition Number of a Graph

D. E. Lampert*and P. J. Slater - University of Alabama in Huntsville 67

The operation "consolidation" is defined on a graph G for which each vertex has a nonnegative integer value associated with the vertex. In a consolidation a vertex transfers some or all of its value to an adjacent vertex of equal or greater value. If all consolidations in a sequence reduce the value of one participating vertex to zero, and the starting value of each vertex is one, this is an acquisition sequence, and the operations are acquisitions. Various properties of acquisition are developed. In particular, the parameter a(G), defined to be the minimum number of vertices with a nonzero value at the conclusion of such an acquisition series, is the main focus of this paper.

Also, a brief discussion of similarities to clustering and broadcast problems is provided.

Keywords: acquisition, broadcasting, clustering, independent sets.

Achievement and Avoidance of a Strong Orientation of a Graph

68

Gary Chartrand, Western Michigan University
Frank Harary, New Mexico State University
Michelle Schultz, Western Michigan University
Donald W. VanderJagt*, Grand Valley State University

An orientation of a graph is strong if it results in a strong digraph. It is known that a graph has a strong orientation if and only if it is 2-edge connected. Two players A and B play a game on a 2-edge connected graph G. They alternate turns, with A going first. On a given turn, the player assigns an orientation to an edge of G not already oriented. The goal for A is to achieve a strong orientation, while the goal for B is to avoid a strong orientation. We determine graphs on which A wins and others on which B wins. The game is then generalized as follows. Let G be a 2-edge-connected graph of size m and let $s_1, s_2, \ldots, s_m$ be a sequence, where $s_i \in \{A, B\}$ for $1 \leq i \leq m$. This sequence describes the order in which A and B take turns; namely, s_i takes the ith turn. For various graphs, we determine those sequences where A wins and those where B wins.

Keywords: Strong digraph, edge connectivity

69

The Combinatorics of $B' = B^3$

Louis W. Shapiro, Mathematics Department, Howard University

In this note we give combinatorial interpretations of several differential equations including the one in the title. The combinatorial settings involve tennis pairings, trees, random walks, and Stirling permutations. This leads quickly to the related differential equation $B' = xB^3$. The key bijection is that showing that the number of permutations in S_{2n} which can be written as a disjoint product of cycles all of even length is the same as the number which can be written as a disjoint product of cycles all of odd length. As one application we give combinatorial interpretations as generating functions for

$$\sqrt{\frac{1}{3-2e^z}}, \frac{1}{\sqrt{1-2z-z^2}}, \text{ and } \sqrt{\frac{1-z}{1-3z}}.$$

70

On Magicness of Sabidussi's Sum of Graphs

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A graph G is magic if we can define an edge labeling assignment, L : E -> {1, 2, ...}, such that the sum of all edge labels incident to each vertex has the same value. We investigate under what conditions graphs which are representable as Sabidussi's sum of graphs are magic.

71

EFFICIENT OPEN DOMINATION IN GRAPHS

Heather Gavlas, *Kelly Schultz, Western Michigan University,
Peter Slater, University of Alabama in Huntsville

A set S of vertices of a graph G is called an efficient open domination set for G if the neighborhoods N(v), v in S, form a partition of V(G). A graph is an efficient open domination graph if it contains an efficient open domination set. We show that determining if a graph is an efficient open domination graph is an NP-complete problem. A linear algorithm for determining the maximum number of vertices of a tree that can be efficiently open dominated is presented. Also, a recursive characterization is given for efficient open domination trees.

Key Words: Efficient, Open, Domination

72

GREATEST MIDPOINT TOURNAMENTS

Raymond R. Fletcher III, University of Texas of the Permian Basin

A digraph G is said to have the *unique greatest midpoint (UGM) property* if for any ordered pair (a,b) of vertices of G, the set M of all midpoints of directed paths of length 2 from a to b contains a unique point (ab) which has an arc into it from each point of M. A (looped) tournament with the UGM property is called a *greatest midpoint tournament (GMT)*.

For each odd integer n there is a certain rotational tournament A_n of order n which serves as our principle example of a GMT. We construct GMT's of every even order except 2,4,8 by determining all 1-point extensions of A_n. We also show how a large class of odd order GMT's can be constructed by inverting certain subsets of arcs in A_n.

An *irreducible* point x in a GMT T has the property that if $a,b \in T - \{x\}$ then $ab \neq x$. We show that irreducible points may be *replaced* by arbitrary GMT's, and use this result to show that given any two GMT's T_1 and T_2, there exists a GMT which contains both T_1 and T_2 as vertex disjoint subtournaments.

Key words: tournament; amalgamation; rotational tournament.

Tuesday March 7, 1995
11:30 a.m.

Automorphism Groups of Power Graphs of Trees

73

Scott Sportsman Western Carolina University

The rth power graph of a graph G, denoted by G(r), is defined by V(G(r)) = V(G) and E(G(r)) = {vw : d(v,w) <= r}, where d(v,w) is the distance between v and w in G. For a tree, T, we consider the question for what powers is the automorphism group of the power graph of T the same as that of T. We introduce a parameter X for trees so that Aut(T) = Aut(T(r)) for r [illegible] X and Aut(T) is a proper subset of Aut(T(r)) for r > X. Previously we have shown that the groups Aut(T), Aut(T(2)) , Aut(T(3)) , [illegible] , Aut(T(d)) form a totally ordered set with respect to set inclusion where d is the diameter of the tree. The number of distinct groups in the chain is called the length of the chain. Given a fixed diameter, we also characterize the trees that have chains of minimal length.

On Pseudogracefulness of Unions of Cycle and Path

74

Sin-Min Lee and Teresa Chiu*, Math and CS
San Jose State University, San Jose, CA 95192

A (p, p-1) graph is called pseudograceful if there exists a labeling on vertices V by 0, 1, 2, ..., p-2, p, and the induced edge labels are 1, 2, ..., p-1. Recently, R. W. Frucht showed that K1,n is not pseudograceful for n >= 3, and combs are pseudograceful. He showed that union of Cn and Pm is pseudograceful for n = 3, m <> 3, and for n = 4, m >= 2. We consider the same problem for n >= 5.

On the complexity of some problems in efficient domination and closed neighborhood order domination

75

Chris Smart* and Peter J. Slater

For a graph G, F(G) is the maximum number of vertices that can be dominated so that no vertex is dominated twice. This problem can be formulated as a 0-1 integer program. By allowing fractional values between 0 and 1, we have F_f, or fractional efficient domination, which can be formulated as a linear program. The integer version of the dual of this linear program gives rise to another graph parameter, namely, closed neighborhood order domination (CLOD), in which it is sought to minimize the sum of the vertex weights subject to the constraint that, for any vertex v, the sum of the weights in the closed neighborhood of v must equal or exceed the order of v's closed neighborhood. We denote this minimum value by W(G). In this paper, complexity questions involving F and W are considered. It is known that deciding if W(G) < k is NP-complete,as is deciding if all the vertices of a graph can be dominated efficiently (i.e., does F(G)=|V(G)|?). In this paper we show that deciding if W(G)<|V(G)| and if F(G)=W(G) are NP-complete, even for bipartite graphs.

keywords: domination, efficient domination, linear programming.

The Road Coloring Problem and Monocyclic Decomposable Digraphs

76

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A strongly connected digraph is said to be aperiodic if the gcd of the cycles is one. An edge-colored digraph is synchronizing if for every vertex v there is a sequence of colors such that following that sequence of colors in the graph, regardless of the starting vertex, we always end at the vertex v. We say that the coloring is deterministic if for every vertex the outgoing edges have distinct colors.
The road coloring problem asks whether every k-out aperiodic digraph can be deterministically colored with k colors in a synchronizing manner. A complete solution of this problem is not known.
A k-out digraph G=(V,E) has monocyclic decomposition if the set of edges E can be factored into k disjoint subsets E(1),...,E(k) such that the subgraph G(i)=(V,E(i)) is 1-out digraph with exactly one cycle c(i). We show that if k-out aperiodic digraph has a monocyclic decomposition such that the gcd of the cycles c(i) is one, then the graph has a synchronizing coloring. We present a necessary and sufficient condition for existence of such decomposition. This condition also gives an algorithm that determines whether a k-out aperiodic digraph has a monocyclic decomposition.

Keywords: the road coloring problem, digraphs, edge coloring, synchronization.

Tuesday March 7, 1995
11:50 a.m.

Dependency Graphs and Automation of Program Parallelism

77

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The objective of this paper is to automate the parallelization of a program with n statements. The programs considered are written in a very simple language that contains the bare minimum number of statements. The programs are normalized and they are represented by dependency graphs. It is proven that the problems related to the program parallelism are NP-complete if we consider general graphs. Exploiting the properties of dependency graphs all these problems become polynomially solvable. Specifically, an $O(n^3)$ algorithm computes the minimum number of steps needed to execute a program when enough processors are available, when there is a restriction on the number of processors, the minimum number of steps is also computed with time complexity $O(n^3)$. Finally, an $O(n^4 \log n)$ algorithm is presented that assigns program statements to the minimum number of processors so that the overall processor utilization is maximized.

keywords: *parallelism, dependency graph, program partition, NP-complete.*

On bandwidth of the strong product of paths and cycles

78

Y.L. Lai and *K.L. Williams, Western Michigan University

The *strong product* of graphs G_1 and G_2, denoted $G_1(S_P)G_2$, is $G = (V(G_1) \times V(G_2), E)$ where $((x_1, y_1), (x_2, y_2)) \in E$ if $(x_1, x_2) \in E(G_1)$ and $(y_1, y_2) \in E(G_2)$, or if $x_1 = x_2$ and $(y_1, y_2) \in E(G_2)$, or if $y_1 = y_2$ and $(x_1, x_2) \in E(G_1)$. For graphs in general it is well known that the decision problem associated with finding bandwidth is NP-complete. Let $G = G_1(S_P)G_2$ for any of the following cases: each of G_1 and G_2 is a path; either G_1 or G_2 is a path and the other is a cycle; or each of G_1 and G_2 is a cycle. For each of these cases we provide a linear time algorithm to number the vertices of G with a bandwidth numbering. Also, for each of these cases we provide the exact value of a bandwidth numbering.

Keywords: strong product, bandwidth, algorithm, NP-complete.

Utilization of Cut Vertices in Finding the Domination Number of a Graph

79

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When a graph G=(V,E) contains one or more cut vertices, the calculation of the domination number may be improved. Let S be a subset of V. Set S is a beatable dominating set if there exists a set of vertices S' such that (1) |S'| < |S| and N[S'] ⊇ N[S] or (2) |S'| = |S| and N[S'] properly contains N[S]; otherwise, S is unbeatable. We employ the concepts of beatable sets to develop an algorithm for finding the domination number of a graph containing a cut vertex. We then extend this algorithm to graphs containing more than one cut vertex. Finally, we note that these algorithms should be extendable to other parameters for which beatable sets can be defined.

key words: domination, cut vertex

Extremal Cayley Digraphs of Finite Cyclic Groups

80

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For any positive integer k and any positive real number d, let m(d,k) (resp. (r,k)) denote the largest positive integer m such that the Cayley digraph Cay(m,A) of the cyclic group Z/(m) of integers modulo m has diameter (resp. average distance) at most d for some k-element set A of integers. It has been proved that these functions have appilcations in the construction of interconnection networks. A brief survey will be presented on the study of these and other related extremal functions.

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Tuesday March 7, 1995
12:10 p.m.

A balanced approach to the rectilinear Steiner tree problem

81

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We present a new balanced approach for the construction of a Rectiline ar Steiner Tree (RST) on the oriented plane (rectilinear) for a finite set S of points. A sequence of propositions leads to the construction of the RST from a balanced connection of local optimal subtrees. The local subtrees result through the use of a Divide and Conquer strategy on S. The set of the original points is decomposed into successive "envelopes" starting from the outside points and moving inwards. Spanning trees along with the Steiner points of the envelopes are used as a heuristic to determine which subtrees to connect. Two processes known as Steinerlization and Upper/Lower Overlap Maximization are used to minimize the length of the Steiner tree topology. The envelope process has been impleme nted and compared with other RST published approaches. The comparison is done in terms of time complexity of the algorithms as well as experimentally.

Keywords: Steiner Tree, Minimization, Rectilinear Space, Spanning Tree, Divide and Conquer.

Minimum Coverings of K_n with Hexagons

82

Janie Ailor Kennedy, Auburn University

A *covering* of K_n with hexagons is a pair (S,C), where S is the vertex set of the complete undirected graph K_n and C is a collection of edge-disjoint hexagons which partition $E(K_n) \cup P$, where $P \subseteq E(\lambda K_n)$. The collection of edges belonging to P is called the *padding*, and n is called the *order* of the covering. A covering is a *minimum covering* if $|P|$ is as small as possible. In some cases a minimum covering has several different paddings. We give constructions for minimum coverings of K_n with hexagons for each n and each λ, with all possible paddings.

Paired Domination

83

T. W. Haynes (ETSU) and P. J. Slater* (UAH)

In a graph G=(V,E) if we think of each vertex s as the possible location for a guard capable of protecting each vertex in its closed neighborhood N[s], then "domination" requires every vertex to be protected. Thus, $S \subseteq V(G)$ is a dominating set if $\bigcup_{s \in S}$ N[s]=V(G). For total domination each guard location must in turn be protected, so we would want $\bigcup_{s \in S}$ N(v)=V(G).

For paired-domination each guard is assigned another adjacent one, and they are designated as backups for each other. A paired-dominating set is a dominating set whose induced subgraph contains at least one perfect matching. We will summarize results on the paired-domination number γ_p (G) and introduce the paired-domatic number d_p (G).

Keywords: domination, paired-domination, (paired-)domatic number.

Anticonnected Digraphs

84

Heather Gavlas, Western Michigan University

A semipath in a digraph containing no directed path of length 2 is called an antipath. A digraph D is anticonnected if D contains a u-v antipath for each pair u, v of vertices of D. A digraph is antihamiltonian if it contains a hamiltonian anticycle. A digraph D is antihamiltonian extendable if every antipath can be extended to an antihamiltonian cycle and is antihamiltonian-connected if D has a hamiltonian u-v antipath for every pair u, v of vertices of D. Some results concerning antihamiltonian extendable digraphs and antihamiltonian-connected digraphs are presented. The antidistance between two vertices is the length of a shortest antipath between them. The eccentricity of a vertex v is the maximum antidistance between v and another vertex of D. The minimum eccentricity is the antiradius rad D of D and the maximum eccectricity is its antidiameter diam D. The subdigraph of D induced by those vertices with eccentricity rad D is called the anticenter of D, while the subdigraph induced by those vertices with eccentricity diam D is its antiperiphery. All digraphs are determined that can be the anticenter or antiperiphery of some anticonnected digraph. It is shown that if G is a graph of order $n \geq 3$ such that $\deg v \geq (3n-1)/4$ for every vertex v of G, then every orientation of G is anticonnected and that this bound is sharp.

Tuesday March 7, 1995
3:20 p.m.

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85

Catepillars, paths and the Tutte polynomial

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We give an elementary procedure based on simple generating functions for constructing n (for any n =84 2) pairwise non-isomorphic trees, all of which have the same degree sequence and the same number of paths of length k for all k =84 1. The motivation for these counterexamples comes from an investigation of Tutte polynomials for trees. I will also talk (briefly) about the NSF-REU setting in which this research took place.

86

Construction of Self-complementary Graphs

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An algorithm to generate a class of self-complementary graphs is presented. Given a graph H with 2p vertices, we present an algorithm that generates a class of sel-complementary graphs SC(H) of 4p vertices such that H is a subgraph of every member of SC(H).

key word: self-complementary graphs.

87

Total efficient domination in permutation graphs

S.T. Hedetniemi, R.C. Laskar and
*Srinivasarao Maddela, Clemson University

A vertex set S in a graph G=(V,E) is a total efficient dominating set if for every vertex v in V, $|N(v) \cap S| = 1$. The total efficient domination number γ_t(G) is the minimum cardinality of a total efficient dominating set in G, if such a set exists. In this paper we present a polynomial time sequential algorithm for computing $\gamma_t(G)$ for permutation graphs. We also investigate the parallel complexity of this problem.

88

Squaring the Tournament - an Open Problem

Brenda J. Latka*, Lafayette College
Nate Dean, AT&T Bell Labs

Let T be a tournament with edge relation E. We define the directed graph T^2 on the set of vertices of T as follows:
T^2 has the edge relation E^2 such that for all vertices x and y, if E(x,y) then E^2(x,y) and if E(x,y) and there is a vertex z making xyz a 3 cycle in T then E^2(y, x).
Dean has conjectured that if every vertex in a tournament T has out degree at least one then there is some vertex whose out degree in T is at least doubled in T^2. We prove the conjecture for several special cases.

KEYWORDS: tournament, digraph

Tuesday March 7, 1995
3:40 p.m.

89 **A Gray Code for the Ideals of Cycle-free Posets (or Enumerating Spider Squishings)**

Gang Li and *Frank Ruskey, CS, University of Victoria

A poset is cycle-free if its Hasse diagram, regaded as an undirected graph, is a tree. Let P be a cycle-free poset (spider) on a ground set of size n. We show that the ideals of P (ways of squishing) can be listed in a Gray code manner; successive ideals differ by only one element. Furthermore, this Gray code can be generated in time O(n) per ideal and space O(n) in total. For some classes of posets the amortized running time can be reduced to O(1) per ideal.

Cataloging Self-complementary Graphs of Order Thirteen

Myles F. Mcnally* and Robert Molina, Alma College

90

A self-complementary graph G of odd order has a unique decomposition into edge disjoint subgraphs, one of which is a bipartite self-complementary graph of order |G|-1. In this paper we report the generation of the 5600 self-complementary graphs of order thirteen based on the enumeration of their components and their composition from them. After reviewing previous work in the cataloging of self-complementary graphs we describe the bipartite decomposition method. We then turn to the algorithmic solution employed, its complexity, and resulting computer programs. Programmed on personal computer, total running time was less than two hours.

Keywords : self- complementary, bipartite, decomposition

Connected Domination and C-Irredundance

91

J. Ghoshal, R. Laskar, D. Pillone*
Clemson University

Abstract

For a connected graph $G = (V, E)$, a set $S \subseteq V(G)$ is a connected dominating set if S is a dominating set and the induced subgraph $< S >$ is connected. A minimal connected dominating set is a connected dominating set for which if $s \in S$ then $N[s] - N[S - s] \neq \emptyset$ or s is a cut vertex of $< S >$. In this paper we define the following concept: if G is a graph and $S \subseteq V(G)$ then S is **c-irredundant** if for each $s \in S$ either $N[s] - N[S - s] \neq \emptyset$ or s is a cut vertex of $< S >$, the graph induced by S. The upper and lower c-irredundant numbers are defined for any graph G and some properties of c-irredundant sets are investigated.

92

Vertex Disjoint Cycles and Longest Cycles in Star-free Graphs

G. Chen*, L. Markus, R.H. Schelp
North Dakota State U, Furman U, U of Memphis

Let k be a positive integer. In this talk we will give some sufficient conditions for a graph to be hamiltonian and the sufficient conditions for a graph containing m vertex-disjoint cycles.

Key words: cycles, gree, hamiltonian cycles

93 COUNTING TILINGS FLAWS AND ANTI-COLORINGS

Joshua Cooper, Massachusetts Academy of Math & Science at WPI

Given a rectangular subset of the (labelled) square lattice, how many ways can o ne tile it with polyominoes? Placing a vertex in each lattice element and drawi ng edges between vertices corresponding to adjacent unit squares creates an unde rlying graph G. Define a *flaw* to be an edge α in some subgraph G' such that

i) $\alpha \notin G'$
ii) $\alpha \in G$
iii) The endvertices of α are connected in G'

Then we need only count those subgraphs of G (on the same set of verties) whi ch are *unflawed*, i.e., contain no flaws. Various methods for doing this, as well as some results concerning the number of such subgraphs, wil be discuss ed - including the use of *anticolorings*, assignments of colors to the vertices of G such that the derived subgraph of each color is connected, and a corresponding "antichromatic polynomial".

key words: polyominoe, chromatic polynomial, tilings

94 A Collatz Type Difference Equation

Dean S. Clark and James T. Lewis*
University of Rhode Island

Let x_1, x_2 be given integers. For $n = 3, 4, \ldots$ set

$$x_n = \begin{cases} (x_{n-1} + x_{n-2})/2 & \text{if } x_{n-1} + x_{n-2} \equiv 0 (\text{mod } 2) \\ x_{n-1} - x_{n-2} & \text{otherwise} . \end{cases}$$

In contrast to the notorious $3x + 1$ problem, the limit behavior of the solution of this difference equation can be analyzed.

95 Minimal Rankings

J. Ghoshal, R. Laskar*, D. Pillone
Clemson University

Abstract

A k-ranking, f, for a graph G is a function $f : V(G) \to \{1, 2, \ldots, k\}$ such that if $u, v \in V(G)$ and $f(u) = f(v)$, then every $u - v$ path contains a vertex w such that $f(w) > f(u)$. In this paper we define minimal rankings of graphs. Properties of minimal rankings are established and then used to determine χ_r, the minimum ranking number, and ψ_r, the maximum ranking number over all minimal rankings, for complete n-partite graphs and for split graphs.

96 A NEW CONSTRUCTION FOR A CRITICAL SET IN SPECIAL LATIN SQUARES

Chin-Mei Fu*#, Hung-Lin Fu** and Wen-Bin Liao#
Mathematics, Tamkang U, Tamsui, Taipei Shien, Taiwan, R.O.C.
** Applied Math, Chiao-Tung U, Hsin-Chu, Taiwan, R.O.C.

A critical set is a partial latin square which is uniquely completable to a latin square and omitting any entry of the partial latin square destroys this property. The size of a critical set is the number of entries in the partial latin square. In this paper, w give the different proof to prove that the lower bound of the sizes of critical sets in a latin square of order n is at least n + 1, and we give a construction to construct a critical set of special latin squares of order 2 to the vth power - 1.

Tuesday March 7, 1995
4:20 p.m.

Unique Domino Tiling

97 Craig K Bailey*, T S Michael, US Naval Academy

Several papers have counted the number of ways various shapes can be tiled with dominoes. This paper explores which convex shapes (subboards of the standard checkerboard) can be uniquely tiled by dominoes. The main theorem characterizes those shapes with one straight side.

KEYWORDS: domino, tiling

Some results on the double competition number of a triangle-free graph

98 Suh-Ryung Kim, Math., Kyung Hee U

Let D be an acyclic digraph. The competition graph of D has the same set of vertices as D and an edge between vertices u and v iff there is a vertex x in D such that (u, x) and (v, x) are arcs of D. The competition-common enemy graph of D has the same set of vertices as D and an edge between vertices u and v iff there are vertices w and x in D such that (w, u), (w, v), (u, x), and (v, x) are arcs of D. The competition number (The double competition number) of a graph G, denoted by $k(G)$ $(dk(G))$, is the smallest number k such that G together with k isolated vertices is a competition graph (competition-common enemy graph) of an acyclic digraph.

In this paper, we show that $dk(G) \leq k(G)$ for any triangle-free graph G with $k(G) \geq 2$. We also find graph G with $dk(G) > k(G) = 2$. Finally, we give a new upper bound for the double comptition number of a bipartite graph with n vertices.

Key words. competition numbers, double competition numbers, triangle-free graphs, bipartite graphs

Strong Bondage and Strong Reinforcement Numbers of Graphs

99

J. Ghoshal(*), R. Laskar, D. Pillone
Clemson University, Clemson SC
C. Wallis
Gainesville College, Gainesville GA

Abstract

A Strong dominating set (sd-set) in a graph $G = (V, E)$ is a set $D \subset V(G)$ in which for every vertex u in $V(G) - D$ there exists at least one vertex v in D such that $\deg(v) \geq \deg(u)$ and $(u, v) \in E(G)$. Define γ_s to be the smallest cardinality of a sd-set in G. This concept was introduced by Sampathkumar and P. Latha. In this paper, we initiate the study of strong bondage and strong reinforcement numbers in a graph. The strong bondage number b_s for a graph G is the cardinality of the smallest number of edges $F \subset E(G)$ such that $\gamma_s(G-F) > \gamma_s(G)$. The strong reinforcement number r_s for a graph G is the cardinality of the smallest number of edges $F \subset E(\bar{G})$ such that $\gamma_s(G+F) < \gamma_s(G)$. In this paper, we find exact values of b_s and r_s are found for some classes of graphs and sharp bounds for trees. Some general bounds are also given.

100 CHARACTER THEORY AND IDEMPOTENTS IN AFFINE DIFFERENCE SETS

John L. Hayden, Math & Stat, Bowling Green State U, Bowling Green, OH 43403
Troy D. VanAken* Math, University of Evansville, Evansville, Indiana 47722

In this paper we provide new proofs of Hall's Multiplier Theorem and a theorem of Wilbrink in the context of affine difference sets. The proofs employ the idempotents of the associated group algebra.

AMS classification number: 05B10 (primary), 05B20 (secondary)
Keywords: affine difference set, multiplier, relative difference set

101

Mutually Orthogonal Sets of Latin Squares Based on Groups

Anthony B. Evans, Wright State University

Given a group G, how large can a set of mutually orthogonal Latin squares based on G be? The exact answer to this question is known for elementary abelian groups, groups with cyclic Sylow 2-subgroups, and groups of order 15 or less.
But for other classes of groups, with few exceptions, only the most obvious bounds are known.
We will discuss progress toward answering this question.

102

CONNECTEDNESS OF LINE GRAPHS AND REPRESENTATION OF TRANSFORMED GRAPHS

Christina Zamfirescu* and Eric Klein, City University of New York

The connectivity numbers of a graph are related to each other. The same is true for the line connectivity numbers. Another important related notion is the cyclic connectedness, and we establish here a strong relationship between the cyclic connectivity numbers of a graph and its line graph. Moreover, we introduce a related new notion and undertake a similar investigation.

A unified representation of the line graph, total graph, subdivision graph, and other transformations of a graph will be given for the directed case, using the concept of intersection digraph.

Keywords: connectivity and line connectivity numbers, cyclical n-connectedness, cliquewise n-connectedness, transformation digraphs, intersection digraphs.

Wednesday, March 8, 1995
8:50 a.m.

105 **Small G Designs of Arbitrary Index**

Kimberly Kirkpatrick*, D.G. Hoffman, Auburn University

We investigate G designs of order n and index lambda, for various simple graphs G.

106 **The Spanning Trees Forced by the Path and the Star**

Michael F. Bridgland*, Jennifer S. Zito, Supercomputing Research Center, Bowie, MD Robert E. Jamison, Clemson University, Clemson, SC

Say that a set S of trees of order n *forces* a tree T if every graph having each tree in S as a spanning tree must also have T as a spanning tree. Say that S is a *spanning tree forcing set* for order n if S forces every tree of order n. It is easy to see that, for each order $n > 1$, the star belongs to every spanning tree forcing set. We show that, for each order $n \notin \{1,6,7,8\}$, the path belongs to every spanning tree forcing set. We give a computationally tractable characterization of the trees forced by the path and the star, thereby constructing many trees that do not belong to any minimal spanning tree forcing set. We show that the following related decision problem is NP-complete: An instance is a pair (T, G) consisting of a tree T and a graph G of order $|G| = |T|$ having both the path and the star as spanning trees, and the problem is to determine whether T is a spanning tree of G.

Key words: spanning tree, panarboreal graph, closure system, subset sum.

107 **Vertex colorings of total digraphs**

Peter Jipsen

A (pseudo) digraph is *total* if for all $x, y \in V(G)$ either $\vec{xy}$ or $\vec{yx}$ is an edge of G (so in particular, all loops $\vec{xx}$ are edges of G). For a set S of vertices the *outgoing neighbours of S* are denoted by $N^+(S)$, and the incoming neighbours by $N^-(S)$. A partition of $V(G)$ into color classes $C_1, \ldots, C_n$ is called an *admissible (vertex) coloring* of G if $N^+(C_i)$ and $N^-(C_i)$ are unions of color classes. A vertex coloring is *proper* if there are at least two color classes and at least two vertices of the same color. In collaboration with R. Kramer and R. Maddux we have proved that there are exactly five finite total digraphs that have no proper admissable coloring. In an algebraic setting, this result implies that there are exactly four finitely generated minimal varieties in the lattice of varieties generated by total tense algebras. A similar classification of minimal varieties in the lattice of relation algebra varieties is still incomplete, but may be more accessible when translated into the corresponding coloring problem for directed 3-hypergraphs.

108 **DOMINATION NUMBERS OF KNESER GRAPHS AND THEIR Q-ANALOGUES**

W. Edwin Clark, Department of Mathematics, University of South Florida, Tampa, FL 33620-5700 email: eclark@math.usf.edu

For integers $1 \le k < n$ the *Kneser graph* K(n,k) is the graph whose vertices are the k-subsets of a fixed n-set and whose edges are the pairs {A,B} where A and B are disjoint. The q-analogue of K(n,k) is the graph $K_q(n,k)$ whose vertices are all k-dimensional subspaces of a fixed n-dimensional vector space over GF(q) and whose edges are the pairs {U,W} where $U \cap W = \{0\}$. We attempt to determine the (total) domination numbers for these graphs. It is trivial that the (total) domination number of K(n,k) and $K_q(n,k)$ is k+1 if $n \ge k(k+1)$. When $n < k(k+1)$ we obtain various upper and lower bounds -- which are exact in some cases. An especially good upper bound is obtained for the (total) domination number of $K_q(n,k)$ using properties of gaussian polynomials and an apparently new general upper bound for the (total) domination number of a graph. These concepts may also be phrased in terms of blocking sets.

KEYWORDS: Kneser graph, domination number, total domination number, gaussian

109

A Recursive Construction for Rotational Steiner Triple Systems

C. J. Colbourn and Z. Jiang*
University of Waterloo

A Steiner triple system of order v, denoted $S(v)$, is called k-rotational if it admits an automorphism consisting of one fixed point and k cycles of length $(v-1)/k$. The existence problem for k-rotational $S(v)$ was solved for $k = 1, 2, 6$ by Phelps and Rosa and for $k = 3, 4$ by Cho in the 1980's. In general, the existence problem for k-rotational $S(v)$ with $k > 4$ can be reduced to that for p-rotational $S(v)$ for p an odd prime and orders $v \equiv 1, 18p+1 \pmod{24p}$ when $p \equiv 1 \pmod 4$ or $v \equiv 1, 6p+1 \pmod{24p}$ when $p \equiv 3 \pmod 4$. We develop a recursive construction for k-rotational $S(v)$ (the recursion is on k); With this recursive construction we establish the existence of a p-rotational $S(v)$ of the above orders, and therefore completely determine the spectrum of k-rotational $S(v)$ for any integer k.

110

ON THE CLOSURE OF THE PERFECT TREES

Robert E. Jamison Math Sci, Clemson U, rejam@clemson.clemson.edu

Let F be a family of (isomorphism types of) trees of order n. Then F forces a tree T iff every graph which has spanning subtrees isomorphic to all the trees in F also necessarily has a spanning subtree isomorphic to T. The family F is closed if it contains all the trees that it forces. It is an easy observation that the trees of order n without a perfect matching are closed. Curiously, the complementary question of whether the trees WITH a perfect matching are closed seems to be quite hard.

This talk will examine this issue from several perspectives. I will discuss several general closed families of trees and give a structure theorem for graphs all of whose spanning subtrees are perfectly matched. Finally, I will give what little is known about the closure of the perfect trees.

111

Using Graph Coloring to Find Boolean Rank

Eric Phelps, University of Colorado at Denver

Given an $n \times m$ $(0,1)$ matrix A, its boolean rank is the minimum number such that $A = B \otimes C$ where B and C are $(0,1)$ matrices of dimensions $n \times$ and $r \times m$ respectively, and where $\otimes$ is normal matrix multiplication wit the exception $1+1=1$. Boolean rank has also been studied as the minimu covering in bicliques of bipartite graphs. Finding this rank is known to b NP-Complete. A construction for a graph G depending on the matrix A wi be given. The chromatic number of G will be shown equal to the boolea rank of A and a method for obtaining B and C from a coloring of G will als be given. This may seem like trading one NP-Complete problem for anothe but a great deal more is knownabout graph coloring and this knowledge ca now be applied to the boolean rank problem. For example, conditions on th matrix A can be found which will lead to polynomial rank decompositio algorithms.

Keywords: Boolean Rank, Biclique Covering.

112

The Domination and Competition Graphs of a Tournament

David C. Fisher, J. Richard Lundgren, Sarah K. Merz*
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California State University, San Marcos, CA 92096

Vertices x and y *dominate* a tournament T if for all vertices $z \neq x, y$, either x beats z or y beats z. Let dom(T) be the graph on the vertices of T with edges between pairs of vertices that dominate T. We show dom(T) is either an odd cycle with possible pendant vertices or a forest of caterpillars. Since dom(T) is the complement of the competition graph of the tournament formed by reversing the arcs of T, complementary results are obtained for the competition graph of a tournament.

113 **Steiner Triple Systems with Many Affine Hyperplanes**

J. D. Key and F. E. Sullivan*, Clemson University

Steiner triple systems on 3^d points whose ternary codes have dimension one more than that of the affine geometry design of points and lines of $AG_d(F_3)$ are examined. It is shown how they can be constructed, what their ternary codes consist of, and how they can always be extended to 3-designs using codewords of weight-4 of the ternary code.

KEY WORDS: codes, designs

114 **Stratified Graphs**

Gary Chartrand, Linda Eroh, Reza Rashidi*, Michelle Schults, and Naveed Sherwani, Western Michigan University

A graph whose vertex set is partitioned into classes is called a stratified graph. These graphs emanate from oblems in VLSI design. There are numerous concepts and problems in graph theory that have analogs in stratified graphs. Several of these will be discussed.

115 **Extending Vertex-Colourings to Total Colourings**

Hugh Hind, University of Waterloo

In 1971 M. Rosenfeld and N. Vijayaditya proved that a graph with maximum degree three could be totally coloured using at most five colours. It remains of interest to know whether any fixed vertex-colouring (using at most five colours) of a graph of maximum degree three can be extended to a total colouring using the same five colours. Some positive and negative results will be discussed.

Keywords: total colouring, vertex-colouring

116 **p-Competition Graphs of Symmetric Digraphs and p-Neighborhood Graphs**

J.Richard Lundgren, Patricia A. McKenna*, Sarah K. Merz
University of Colorado at Denver, Denver, CO, 80217-3364

Craig W. Rasmussen
Naval Postgraduate School, Monterey, CA, 93943

The p-competition graph G of a digraph D is a graph on the same vertex set as D, with $[x,y]$ in the edge set of G if and only if $|Out(x) \cap Out(y)| \geq p$ in D. For the case $p=1$, G is called the competition graph of D. If D is a symmetric digraph, the competition graph of D is the neighborhood graph of the underlying graph of D, a topic on which a considerable amount of research has been done. Similarly, for symmetric D the p-competition graph of D is the p-neighborhood graph of the graph which underlies D. Here we investigate the properties of p-neighborhood graphs and identify familiar classes of graphs as 2-neighborhood graphs.

Key Words: Competition Graph, p-Competition Graph, Neighborhood Graph, p-Neighborhood Graph

117

A D-OPTIMAL DESIGN OF ORDER 150

W. H. Holzmann and H. Kharaghani*
University of Lethbridge

A construction for a D-optimal design of order 150 is given. The design is not circular, but is block circular of block size 15. An attempt is made to explore the existence of symmetric D-optimal designs of order greater than 10. As a consequence, we get a complex D-optimal design of order 75 and a symmetric complex D-optimal design of order 2.

KEYWORDS: D-optimal designs, complex D-optimal designs, symetric matrix

118

Realizability of a Tree with a given Distance Distribution

H. Wang and A. T. Amin*
The University of Alabama in Huntsville

Let Di denote the number of pairs of vertices in a graph G at distance i. Then distance distribution of G, denoted dd(G), is dd(G) = (D1,D2,...,Dn-1). We consider the problem of realization of sequence of positive itegers (A1,A2,...,Ai), i<n, as a partial distance distribution of a tree when i = 2, and 3. In particular a polynomial time algorithm is given to determine realizability of a sequence (A1,A2) as a partial distance distribution of a tree.

Keywords: tree, distance distribution.

119

3-Choosability of K(m,n)

Anil Shende, Bucknell University
Barry Tesman*, Dickinson College

Let S(x) be a list of colors assigned to vertex x of a graph G. G is S-list colorable if there is an ordinary vertex coloring f of G such that f(x) is in S(x), for every x in V(G). If G can be S-list colored whenever all lists S(x) have cardinality k, then G is k-choosable. In this talk, we present some recent results on the k-choosability of bipartite graphs. By proving that K(5,q) is 3-choosable for q<=12, we finish, with a result of O'Donnell, the classification of 3-choosable complete bipartite graphs.

Keywords: Broadcasting, Gossiping, Trees

120

Minus *k*-subdomination in graphs II

Johannes H. Hattingh and Elna Ungerer*, Rand Afrikaans University, Johannesburg

Let $G = (V, E)$ be a graph and $k \in \mathbf{Z}^+$ such that $1 \le k \le |V|$. A k-subdominating function (kSF) to $\{-1,0,1\}$ is a function $f : V \to \{-1,0,1\}$ such that the closed neighborhood sum $f(N[v]) \ge 1$ for at least k vertices of G. The weight of a kSF f is $f(V) = \sum_{v \in V} f(v)$. The k-subdomination number to $\{-1,0,1\}$ of a graph G, denoted by $\gamma_{ks}^{-101}(G)$, equals the minimum weight of a kSF of G. We give a sharp lower bound for γ_{ks}^{-101} for trees and calculate γ_{ks}^{-101} for an arbitrary cycle.

* Presenter.

Wednesday, March 8, 1995
11:30 a.m.

Some maximally related pairs of triple systems

121 Jane W. Di Paola, FTICA

We seek pairs of Steiner triple systems, i.e., two systems on 6t+1 and 6t+3 points respectively, which have a large number of triples in common. For t=2 we find two such systems which have 18 triples in common and for t=3, there are two systems which have 45 triples in common.

A general method to obtain these results is noted although the ingredients for its application may not exist for all values of t. Serendipitously we obtain for v=21 a partial triple system with 68 triples which cannot be extended to a full Steiner triple system on 21 points.

When the method applies, the common starter set for v=6t+1 and v=6t+3 will consist of $6t^2-3t$ triples.

The bar visibility number of a graph 122

Douglas B. West, University of Illinois, Urbana

(joint work with Y.-W. Chang, J. Fink, J. Lehel, M.S. Jacobson, and A.E. Kézdy)

Suppose each vertex of a graph G is assigned a union of horizontal intervals ("bars") in the plane. This assignment is a *bar visibility representation* of G if u,v form an edge precisely when some bar for v is visible from some bar for u via a vertical segment not intersecting another bar. Technically, each bar contains its left endpoint but not its right, so intervals with a common endpoint can block vision without seeing each other. The *bar visibility number* $b(G)$ is the minimum t such that G has a bar visibility representation assigning at most t bars to each vertex. If G has n vertices and e edges, then $b(G) \geq (e+6)/(3n)$. Hence $b(K_n) \geq \lceil n/6 \rceil$. We prove also that $b(K_n) \leq \lceil n/6 \rceil + 1$. We believe that K_n is the n-vertex graph with maximum bar visibility number, and we prove that $b(G) \leq \lceil n/6 \rceil + 2$ for every n-vertex graph G, using the theorem of Lovász that every graph with m vertices can be decomposed into $\lfloor m/2 \rfloor$ paths and cycles. Analogous results hold for digraphs, where bars form a visibility representation of D if $u \to v$ when some bar assigned to v lies above some bar for u.

123

On Vertex Assignment Problem

Sin-Min Lee, Math and Computer Science

San Jose State University, San Jose, CA 95192

Given a connected graph G = (V,E) with a non-negative integer-valued mapping d : V -> N, we want to find the minimum number k such that, for each v in V, we can associate a subset f(v) of {0, 1, ..., k-1} with :

i) |f(v)| = d(v), and

ii) if (u,v) in E, then f(u) and f(v) are disjoint.

A. Tavern solved the case when G is a cycle. We generalized his result to some general classes of graphs.

124 **Clique-Like Dominating Sets in Perfect Graphs**

Stephen G. Penrice, DIMACS & SUNY at Cortland

Continuing work by Bácso and Tuza, Cozzens and Kelleher, and the author, we investigate dominating sets which induce subgraphs with small clique covering number. We focus here are dominating sets in connected perfect graphs. We show that for certain kinds of perfect graphs (comparability graphs, cocomparability graphs, chordal graphs, and cochordal graphs), we may be assured of finding a dominating sets with "small" clique covering number provided we forbid three simple induced subgraphs. It would be interesting to extend these results to the entire class of perfect graphs; we prove a special case of this.

125 **Lifting local dynamics to global in computations on graphs**
John Pedersen, U of South Florida,Tampa, FL 33620 jfp@math.usf.edu

An arbitrary finite graph is used as the interconnection network for a distributed computation. At each vertex there is a copy of the same finite state automaton, which has some local dynamic. It is used to determine the state of a vertex, depending on its current state and the states of neighbors. At each discrete step of a computation, states are only updated for a random small subset of vertices. This model generalizes cellular automata by allowing an irregular graph for the communications network and having asynchronous state updating. These assumptions are believed to be more realistic for many biological systems. The talk presents results on how to make the global behavior of the network imitate the behavior of the individual automata, starting from random initial states. This is a generalization of the problem of producing a synchronous pulse in the global state from an initial random state.

126 **Parallelogram Graphs**

Jean Dunbar, Converse College *
Joydeep Ghoshal, Clemson U Ralph Grimaldi,
Rose-Hullman Inst.

We introduce *parallelogram graphs* as follows: for positive integers $m \leq n$, assume there are two horizontal axes derived from the closed intervals [0, m] and [0, n]. If unit intervals with integer endpoints, [i, i+1] and [j, j+1], are selected from [0, m] and [0, n], respectively, then the parallelogram created by the two intervals yields a vertex in the graph. Two vertices are adjacent whenever their associated parallelograms intersect. We exhibit preerties of this class of graphs and relationships to other families of intersection graphs. A parallelogram graph is *complete* if it contains mn vertices. A number of parameters for complete parallelogram graphs are calculated.

keywords: intersection graphs, interval graphs, permutation graphs

127 A GREEDY METHOD FOR EDGE-COLOURING ODD MAXIMUM DEGREE DOUBLY CHORDAL GRAPHS
Celina M. H. de Figueiredo(*) Inst. Mat. Univ Federal do Rio de Janeiro
Joao Meidanis, Celia P. de Mello, Ciencia da Computacao U Estadual de Campina

We describe a greedy vertex colouring method which can be used to colour optimally the edge set of certain chordal graphs. This new heuristic yields an exact edge-colouring algorithm for odd maximum degree doubly chordal graphs. This class includes interval graphs and strongly chordal graphs. This method shows that any such graph G can be edge-coloured with maximum degree Delta(G) colours, i.e., all these graphs are Class 1. In addition, this method gives a simple Delta(G) + 1 edge-colouring for any doubly chordal graph.

key words: edge-colouring, chordal graphs, indifference graphs.

128 Iterated Domination Colorings of Graphs
Alice A. McRae, Dee A. Parks* Appalachian State University

Given a graph G=(V,E), a function f:V -> {0,1,2,...} is an iterated domination coloring if

(1) For every vertex v in V, if f(v) > 1, then for all i such that 1 <= i < f(v), there is a vertex w in the neighborhood of v with f(w) = i.
(2) If f(v)=f(w)=k for adjacent vertices v and w, then there exist two vertices y in N(v) and z in N(w), such that f(y) > k and f(z) > k, and neither y nor z is adjacent to another vertex assigned k.

Hedetniemi, Proskurowski, and Telle were the first to study iterated domination numbers. In this talk, we present some algorithmic and complexity results, along with some open questions.

Keywords: Domination, Coloring.

PERIODIC GOSSIPING IN BACK-TO-BACK TREES

129

Roger *Labahn* (*), U Rostock (Germany)
André *Raspaud*, U Bordeaux (France)

For a proper coloring of the edges of a graph G with $c \geq \chi'(G)$ colors, we consider periodic gossiping, i.e. full-duplex all-to-all broadcasting in the 1-port model where communication is made on a link of color i at ay time (round) $\equiv i$ mod c. This procedure is a locally determined way of exchanging information in a network, and if continued infinitely it eventually transmits information generated in any vertex at any time to any other vertex.
Previous work investigated this model on paths, cycles, twodimensional grids, and trees. Here we deal with the d-ary back-to-back tree of height k, BBT_d^k. It consists of two complete d-ary trees of height k the leaves of which are identified.
For a proper coloring of the edges of BBT_d^k with $c \geq d+1$ colors, we present a coloring for which the process of conveying information from any vertex to any other vertex finishes within k periods, where a period is the collection of c consecutive rounds. Finally, we prove that this is optimal for $c = d+1$.
Keywords: Broadcasting, Gossiping, Trees

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On graphs with domination number equal to 2-packing number

Janez Aleš, Department of Mathematics and Statistics, Simon Fraser U

Graph parameters $\eta(G)$ and $\gamma(G)$ are studied in this paper. Let $\eta(G)$ denote the maximum cardinality of a 2-packing in a graph G and let $\gamma(G)$ denote the cardinality of a minimum dominating set of a graph G. Best possible lower bounds for $\eta(G)$ and $\gamma(G)$ in terms of (G) are derived. A tight upper (lower) bound for $\eta(G)$ $(\gamma(G))$ in terms of minimum (maximum) degree of a graph is proven.
A class of η-perfect graphs is introduced in a way similar to the definition of α-perfect graphs. A graph G is η-perfect if for all $U \subseteq V(G)$, $\eta(G_U) = \gamma(G_U)$. Strongly chordal graphs, interval graphs, trees, line graphs of trees, total graphs of trees, the paths P_n, the cycles C_{3n}, and the trampolines T_{2n} are proven to be η-perfect for any integer $n \geq 1$. A class of η-critical graphs is defined.
keywords: Domination, Perfect Graphs, Strongly Chordal Graphs.

On Embedding of a Graph as kth Annulus

131

Sheng Chen and Weizhen Gu*
Southwest Texas State University, Texas, USA

In this paper, we prove that, for any connected graph G and a nonnegative integer k, G can be embedded into a connected graph H so that all vertices of G have eccentricities of $rad(H) + k$ in H if and only if $k \leq \frac{rad(G)}{2}$. This includes embedding G as center ($k = 0$), or periphery ($rad(H) + k = diam(H)$), or annulus ($rad(H) < rad(H) + k < Diam(H)$) as its special cases. A generalization of this result is also discussed.

132

Counting the Geodetic Number of Bipartite Graph

Aaron L. Douthat M.C.Kong*
Department of EECS, University of Kansas

The geodetic cover of a graph G = (V,E) is a set of vertices C in V such that any vertex not in C is on some shortest path between two vertices of C. A minimum cardinality geodetic cover is called a geodetic basis, and the cardinality of a geodetic basis is the geodetic number of the graph.
The problem of finding the geodetic number of a graph is known to be in the class of NP-Complete problems. In this paper, we prove a stronger result that the problem of finding the geodetic number of a graph remains NP-Complete even
when restricted to bipartite graphs. Complexity results for other classes of graphs are also presented.

Keywords: Geodetic number, algorithm, complexity, NP-Completeness.

133 **Interblock Optimal Paths with Applications**

Ashish Mehta (CALC), Polytechnic University email: amehta@calc.poly.edu

The problem of efficiently finding optimal paths (optimizing a given weighting function) between vertices of a directed graph is considered for weighted graphs which have a special structure. These graphs are those which are combinations of blocks which have relatively few vertices adjacent to edges connecting different blocks. An example is the combination of several Object-Oriented Databases (OODBs) into an Interoperable Multiple Object-Oriented Database (IM-OODB), where there are relatively many connections between classes within the same OODB, but relatively few between classes in different OODBs. We assume that all locally optimal paths for all pairs of vertices in each block are precomputed by a version of Dijkstra's algorithm, and that their weights are stored. A relatively small auxiliary graph is constructed based on the original graph. This graph is preprocessed and then affords an efficient method of finding an optimal path between any two vertices of *different* blocks. A variation of this procedure also enables us to calculate *globally* optimal paths between two vertices in the same block. Since optimal paths between vertices in different blocks are assumed to be needed less frequently than those between vertices in the same block, this method provides a satisfactory compromise between storage needs and response time. We investigate general properties of weighting functions for which the procedures developed here are valid.

keywords: directed graph, shortest path, optimal path, path weighting function

134 **Stability of i-Connectivity Parameters Under Edge Deletion**

D.W. Cribb*, U.S. Air Force Academy
J.W. Boland, East Tennessee State University
R.D. Ringeisen, Old Dominion University

The inclusive edge (vertex, mixed) connectivity of a vertex v is the minimum number of edges (vertices, graph elements) whose removal yields a subgraph in which v is a cutvertex. Stability under edge deletion, in which the value of the parameter remains unchanged with the deletion of any edge, is investigated. In particular we examine relationships between stability of the inclusive vertex connectivity parameter and stability of the inclusive mixed connectivity parameter with comparison to established results for stability under edge addition.

Keywords: Inclusive connectivity, stability, edge deletion

135 **Three Dimensional Tutte Embedding**

K. Chilakamarri (Central State University),
N. Dean* (AT&T Bell Laboratories) and
M. Littman (Brown University).

Conditions are given for for a graph to have a convex representation in three dimensions. This extends Tutte's barycentric embedding.

Keywords: embedding, layout, drawing, dimension, convex

136 **On Generalized Diffy Game**

Sin-Min Lee, Shu-Hwa Lee, and Siu-Ming Tong*, Math and CS
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For n >= 3, and u = (a1, ..., an) in N*N*...*N, we define a map f : N*N*...*N -> N*N*...*N by setting f(u) = (|a1-a2|, |a2-a3|, ..., |an-an-1|, |an-a1|). If n = 4, the game (N*N*N*N, f) is called "Diffy" and it is known that for any u in N*N*N*N, there exists an integer st(u) such that f**st(u) (n) = (0,0,0,0). Thus, {(0,0,0,0)} is the "black hole" for n = 4. We completely determine the structure of the black holes for n <= 16, and also give a bound for st(u).

Wednesday, March 8, 1995
4:00 p.m.

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COMBING SEQUENCES AND A NEW COMPLEXITY

Gary Krahn* (U.S. Mil. Acad.)
and Harold Fredricksen (Naval Postgrad. School)

A binary *de Bruijn cycle* (or binary *de Bruijn sequence*) of length 2^n has the property that every n-tuple appears exactly once on a given period. For some applications it might not be necessary that the n bits of interest lie consecutively along the sequence or even preferred they do not. We invetigate sequences called *complete cycles* or *non-classical de Bruijn cycles.* A complete cycle of length 2^n has the property that each of the possible 2^n binary n-tuples lies along a fixed pattern or "comb". The analysis of these complete cycles given here is primarily concerned with combs where $n-1$ of the bits of interest lie consecutively along the sequence. These complete sequences construct 2-cycles, walks along connected graphs that visit every edge exactly twice. We define a *measure* on a 2-cycle as the sum, over all edges of the graph, of the positive difference of the visitation times for each edge. This analysis links the complexity and measure of specific sequences to the discrete log problem.

Key words - Eulerian cycles, 2-cycles, de Bruijn sequences, sequence complexity, Good - de Bruijn digraph, discrete logarithm problem.

138

i-Connectivity in the Join of Two Graphs

James W. Boland, Linda M. Lawson*, East Tennessee State U
Richard D. Ringeisen, Old Dominion U

Inclusive connectivity parameters are local measures of graph vulnerability which are natural restrictions of standard graph connectivity and edge connectivity. For each vertex in the graph, three invariants are defined. These are the inclusive edge connectivity of v, $\lambda_i(v)$, the inclusive vertex connectivity of v, $\kappa_i(v)$, and the inclusive mixed connectivity of v, $\mu_i(v)$. We determine these parameter values in the join of two graphs in terms of invariants of those two graphs. Further, the explicit nature of the graph element sets determining these parameters are obtained for $\kappa_i(v)$. A conjecture is raised regarding the nature of these sets for $\lambda_i(v)$.

Keywords: inclusive connectivity, local vunerability, join.

139

A CLASSIFICATION HIERARCHY FOR 2-CELL IMBEDDINGS OF BRIDGELESS GRAPHS

Bruce P. Mull*, Lake Michigan College,
Dionysios Kountanis and Reza Rashidi, Western Michigan University

Two types of cellular imbeddings have been studied in the past--open and closed 2-cell imbeddings. We introduce a third class of cellular imbedding, the strong 2-cell imbeddings. When restricted to bridgeless graphs, the strong 2-cell imbeddings, together with the closed and open 2-cell imbeddings, form a hierarchy that is both natural and exhaustive. We use strong 2-cell imbeddings to generalize a result of J. Gross anf T. Tucker and give sharp characterizations of cellular imbeddings based only on the maximum region size of the imbedding.

140

Some Results on Regular Graphs with Given Girth Pair

Hung-Lin Fu* and Cheng-Wei Ma, Department of Applied Mathematics, National Chiao Tung University, Hsin Chu, Taiwan, Republic of China

Abstract The length of the shortest odd (even) cycle in G is called the odd (even) girth of G. Let g be the girth of G which is the smaller of the odd girth and the even girth and let h be the larger one. Then (g,h) is called the girth pair of G. A regular graph with valency k and girth pair (g,h) is called a $(k;g,h)$-graph. In this paper we study the $(k;g,h)$-graphs and we construct a class of infinitely many such graphs which have smaller order than previous known results. Also, some smallest $(k;g,h)$-graphs are obtained.

* The speaker is currently visiting Auburn University.

Wednesday, March 8, 1995
4:20 p.m.

Practical Toroidality Testing

141

Eugene Neufeld, and Wendy Myrvold*, Univ. of Victoria

A *torus* is a sphere with one handle. a *toroidal graph* is one that can be embedded on the torus with no overlapping edges. Although various theoretically efficient algorithms for testing toroidality of graphs have been published, none is claimed to be practical. We develop an exponential toroidality tester that is practical enough to compute the minor order obstructions on up to ten vertices and which gives indication that there are several thousands of these obstructions altogether.

Keywords: graph embeddings, torus, planarity, practical algorithms

142

Gridline Indifference Graphs

Dale Peterson, Rutgers University .

An indifference graph is a graph that can be realized on the line with vertices adjacent iff they are within a given distance. This is extended to a grid: A gridline indifference graph (GIG) is a graph that can be realized in the plane with vertices adjacent iff they are within a given distance and are on a common vertical or horizontal line. GIG's are characterized when they are finite and triangulated. They are extended to higher dimensions and characterized. Their relationship with arba graphs is examined.

Key words: indifference graph, arba graph

143

Geometry of Cubic Graphs

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A cubic graph is a regular graph of degree three. The simplest cubic graph is the 1-skeleton of a 3-simplex. Other cubic graphs are seen to form the 1-skeletons of a variety of classical geometric figures. A uniform polytope is a geometric figure which exhibits regularity at vertices, but which is not required to have regular facets. While much is known about regular polytopes and their 1-skeletons, much less is known about other uniform polytopes. Although interesting uniform structures arise in classical Euclidean space, many more purely combinatorial structures are possible. Abstract polytopes are partially ordered structures which generalize the notion of polyhedra in a combinatorial sense. We use our notion of abstract polytopes to realize larger classes of polytopes having n-regular 1-skeletons.

keywords Regular Graphs, Uniform Polytopes, Abstract Polytopes, Chirality, Projective Linear Groups, Hyperbolic Honeycombs

144

A Theorem on Construction of Gracefull Trees

Matthieu Dufour, University of Montreal

A tree A of order n is said to be gracefull if there is a one-to-one function f to {0,1,...,n} such that the induced valuation on the edges determined by the absolute value of x-y, where x and y are the endpoints, is also a one-to-one function from E(A) to {1,2,...,n}.

If A and b are two trees of order n, we define the function r(A,B) as the order of T divided by n, where T is a maximal subtree common to A and B.

We shall prove that, for every tree A, there exists a gracefull tree B such that r(A,B) is at least 1/6.

keywords: gracefull trees, valuations

Wednesday, March 8, 1995
4:40 p.m.

145

Some Results on Clamping a Polygon

Michael Albertson, Ruth Haas*, Joseph O'Rurke
Smith College

When can a polygon be held with parallel grippers of any positive length? Souvaine and Van Wyk introduced a notion of "clamping a polygon," and conjectured that every polygon may be clamped with grippers of any positive length. We establish some special cases of their conjecture.

Key words: computational geometry, clamping

146

DECOMPOSITIONS OF CYCLIC NICHE GRAPHS

Suzanne Seager, Mount Saint Vincent University
Halifax, Nova Scotia, Canada B3M 2J6

A graph G = (V,E) is a loop niche graph if there is a digraph D =(V,A) such that xy is in E iff there exists z is in V such that either xz and yz is in A or zx and zy is in A. If D has no loops, G is a cyclic niche graph, and if D is acyclic, G is a niche graph. Niche graphs have been studied since 1986; loop and cyclic niche graphs are more recent. Here we consider a decomposition which characterizes certain cyclic niche graphs, and apply this decomposition to graphs such as grids.

147

Introducing Spatial Graphs

Fabrizio Luccio and Linda Pagli*, Informatica, Uni di Pisa, Corso Italia 40, 56125 Pisa, Italy

We introduce spatial graphs through a natural extension of the concept of planarity in three dimensions. For a graph $G = (V, E)$, a face is a simple cycle of edges, and a complete set of faces is such that each edge belonging to a cycle in E is on at least one face in the set. G is spatial if admits a three-dimensionalrepresentation R with a complete set of faces consisting of simple surfaces, such that no two faces intersect except along the common edge. In particular G is called k-spatial if R includes k cells (spatial regions); and is called p-spatial if all the faces in R are plane. We prove that all graphs are 1-spatial, but not all of them are $(k > 1)$-spatial or p-spatial. In particular, K_n is $(n-2)$-spatial and p-spatial. We derive some basic properties of spatial graphs as natural three-dimensional extensions of the properties of planar graphs.

Key words. Planarity, Three-dimensional representation, k-spatial graph, Polyhedra, Euler's theorem.

148

Generalized λ-Labelings with a Condition at Distance Two.

John P. Georges and David W. Mauro* — Trinity College, Hartford, CT 06106

An $L(2,1)$-labeling of graph G is an integer labeling of the vertices of G such that adjacent vertices receive labels which differ by at least 2, and labels which are two apart receive labels which differ by at least 1. The λ-number of G is the minimum span of the $L(2,1)$-labelings of G. In this paper, we investigate the analogously-defined $L(j,k)$-labelings of G, for positive integers $j \geq k$. We consider the algebra of λ_k^j-numbers, and we obtain exact expressions for the λ_k^j-number of cycles, paths, and complete multipartite graphs. We also investigate trees, products of paths, and products of complete graphs. Keywords: $L(2,1)$-labeling, λ-labeling, λ-number.

Wednesday, March 8, 1995
5:00 p.m.

149

Multinomial and Hypergeometric Pseudopercolation in 2 & 3 D

by Milton Sobel, Department of Stat. & Applied Prob., UCSB, Santa Barbara, CA 93106

A finite rectangular lattice of known size $(m + 1$ by $n + 1)$ is considered and the total set of all $2mn + (m + n)$ unit segments are divided into two groups: the $2(m + n)$ boundary segments and the $N = 2mn - (m + n)$ internal segments. The latter group is numbered 1 through N and indistinguishable balls marked with these numbers are put into an urn and drawn out one-at-a-time with either of two different sampling schemes: *i)* with replacement (Multinomial Case) and *ii)* without replacement (Hypergeometric Case). The $2(m + n)$ boundary segments of the lattice are "free" to be used but any internal segment can be used (to form a path) only after we observe its number on a ball taken from the urn. The stopping rule (for this boundary-connected problem) is to continue sampling until we have obtained for each of the $(m-1)$ $(n-1)$ internal vertices at least one path to any boundary point. The paths can be separate or have common segments. The number of observations needed, called waiting time (WT), is investigated by finding its expectation $E\{WT\}$, variance $\sigma^2(WT)$ and distribution in the form $P\{WT \geq n + 1\}$; this is done for both of the above sampling schemes. The main tools used are recent results dealing with Dirichlet Integrals. A three-dimensional example (the 3 by 3 by 3 lattice) included in the paper presents no further difficulties for the Dirichlet Integral method. Asymptotic results are needed but have not yet been obtained; hopefully they will be useful even for moderate sized lattices.

KeyWords: Percolation-type, Dirichlet Integrals, Waiting Time, Boundary-Connected.

150

Double Domination of Trees and Block Graphs

Nirmala Anantharaman and Greg Schaper*
Department of Computer and Information Sciences
Teresa Haynes, Department of Mathematics
East Tennessee State University

A vertex of a graph G=(V,E) dominates itself and all adjacent vertices. A subset S of V is a double dominating set of graph G if each vertex of V is dominated by at least two verices of S. Finding a minimum double dominating set of any arbitrary graph is NP-Complete. We present linear runtime sequential algorithms for finding a minimum double dominating set of tree and block graphs. In addition, we show that dominating and double dominating sets can be computed for tree T with diameter diam(T) and number of leaves e in runtime equal O(diam(T)) using O(e) processors.

151

TRIANGULAR TOROIDAL EMBEDDINGS OF CAYLEY GRAPHS

Linda Valdes, San Jose State University

All groups with Cayley graphs that triangulate the torus are given. The embeddings are described as strongly symmetric, weakly symmetric or nonsymmetric.

Keywords: Cayley graphs, symmetric and nonsymmetric embeddings

152

Factorization by Labels of Magic Graphs

Gwong C. Sun, Jian Guan and Thomas L. Holloman, University of Louisville

A graph G=(V,E) is magic if we can find an edge labeling assignment L:E->{1, 2,...} such that the sum of all edge labels incident to each vertex has the same value, and this value is called the magic index w. A magic graph can be factorized by labels into a set of factors which are r-regular or mixed r-regular such that every label in G is the sum of the labels of the corresponding edges in all factors. In a magic graph, the label assignment L with magic index w can be reduced to another label assignment L' with a smaller magic index w' through the factorization process. In this paper, we present definitions and related theorems on the concept of factorization by labels.

157 A PRIVATELY TAPED INTERVIEW WITH PAUL ERDOS

[From the personal collection of Joseph Arkin, transcribed from the tape by Judith L. Arkin and Joseph Arkin, December, 1994.]

The following privately taped interview by David Eidensohn took place in the home of Judith L. Arkin and Joseph Arkin in April, 1976.

Edited by Judith L. Arkin(*), Joseph Arkin and David C. Arney.
USMA, West Point, New York 10996 MADN-A.

In this interview, Paul Erdos discusses the general relevancy of mathematics to mankind. Among many other beautiful thoughts he explains that a mathematician can afford to be careless occasionally and make mistakes because it's so easy to correct them if he thinks it over later.

In all, it is a very beautiful philosophical interview with our creative genius, Paul Erdos.

158 On weighted signed graphs

Shaoji Xu(*)
RUTCOR, Rutgers

A weighted signed graph is a graph with a weight on each edge. Such a graph is called balanced if every cycle has an even number of negative weights. The line index is the minimum sum of weights of edges whose deletion leads to a balanced weighted signed graph. In this paper, we give a bound for the line index; we use Hadlock's method to give an algorithm to calculate the line index for planar weighted signed graphs; we also discuss some other related problems.

Key words: weighted signed graphs, line index, balance, 0-1 programming, maximum cut, planar graphs.

159 FROM TERNARY STRINGS TO WIENER INDICES OF BENZENOID CHAINS

Y. N. Yeh*, W. C. Huang & B. Y. Yang, Institute of Mathematics, Academia Sinica

An explicit, non-recursive formula for the Wiener index of any given benzenoid chain is derived, greatly speeding up calculations and rendering it manually manageable, through a novel envisioning of chains as ternary strings. Previous results are encompassed and two completely new and useful ones are obtained, a formula to determine Wiener Indices of benzenoid chains in periodic patterns, and a formula to estimate errors in the Wiener index induced by errors or indeterminate links in the graph. Keywords: Wiener Index, benzenoid chain

160 **Evaluating the Binary Partition Function when $n = 2^k$.**

John L. Pfaltz, University of Virginia

The binary partition function, $b(n)$, denotes the number of ways that n can be expressed as the sum of powers of 2, that is $n = \sum_i a_i \cdot 2^i = a_0 + a_1 \cdot 2 + a_2 \cdot 2^2 + \cdots$. For example, < 8, 8, 2, 0, 0 > is a binary partition of 32, as are < 32, 0, 0, 0, 0 > and < 0, 0, 0, 0, 1 >. Such binary partitions are important in the study of closure spaces, in which case n must also be a power of 2.

In this talk, we describe a new algorithm that calculates $b(2^k)$ which takes advantage of patterns occurring in these binary partitions when they are lexicographically ordered. The method is significantly faster than the best known algorithm, even though the behavior of both is exponential.

161

ON A PORTION OF THE WELL-KNOWN COLLABORATION GRAPH

Jerrold W. Grossman*, Oakland U, Patrick Ion, Mathematical Reviews

The definition of *Erdős number* is well known: Paul Erdős has Erdős number 0, his co-authors have Erdős number 1, the co-authors of his co-authors who are themselves not his co-authors have Erdős number 2, and so on. It has been conjectured that most current, publishing mathematicians have a finite, and fairly small, Erdős number. As a service to the community, we have compiled a list of the (currently) approximately 5000 people with Erdős number less than or equal to 2 (available ia anonymous ftp to `vela.acs.oakland.edu`, in directory `pub/math/erdos`). This talk will describe the process of compiling such data and discuss some of the difficulty of identification of authors. (Corrections and updates to our data are most eagerly sought.) We will also share some interesting facts about this part of the collaboration graph. For example, at least four people besides Paul Erdős have more than 100 co-authors, and there are some fairly large cliques among persons with Erdős number 1. One moral of this study seems to be that collaboration in the mathematical sciences is flourishing, to the benefit of all.

KEY WORDS: Erdős number, collaboration, cliques

High girth 4-chromatic unit distance graphs in the plane

Paul O'Donnell, Rutgers University

162

We will show the construction/existence of two classes of fixed girth (9 and 12), arbitrary chromatic number graphs. A 4-chromatic graph from each class is embeddable as a unit distance graph in the plane.

Keywords: unit distance graph, girth, chromatic number, Faltings' theorem, polynomial Szemeredi theorem.

163

Intersection properties of subsets of integers

Tibor Szabó, Ohio State and Eötvös Loránd U, Budapest

Let N_k be the maximal integer such that there exist subsets $A_1, \ldots, A_{N_k} \subseteq \{1, 2, \ldots, n\}$ for which $A_i \cap A_j$ is an arithmetic progression of length at least k for every $1 \le i < j \le N_k$. In 1980 R. L. Graham, M. Simonovits and V. T. Sós gave the exact value of N_0 together with the only maximal system. In the following year, for $k \ge 2$, Simonovits and T. Sós determined the asymptotic behavior of N_k. The case of N_1 was open since that time.
In this paper we prove a conjecture of Simonovits and T. Sós concerning the asymptotic value of N_1. We show that

$$N_1 = \frac{n^2}{2} + O(n^{\frac{5}{3}} \log^3 n).$$

Moreover, we slightly improve the best known construction, thus disproving their conjecture on the exact extremal system.

164

Efficiently Generating the Ideals of a Poset

Matthew B. Squire, C.S., North Carolina State U

An *ideal* of a poset is a subset I of the poset elements such that $x \in I$ and $y \ge x$ implies $y \in I$. Generating the ideals of a poset has applications in several scheduling, reliability, and line-balancing problems. Let n be the number of elements in a poset $\mathcal{P}$. The fastest algorithms for generatng the ideals of $\mathcal{P}$ require an amortized time of $O(n)$ per ideal in the worst case. We describe a new algorithm for generating the ideals of a poset, and show that it requires only $O(\log n)$ time per ideal in the worst case.

Thursday, March 9, 1995
10:50 a.m.

165

SQUARES IN A SQUARE

by
Paul Erdős and *Alexander Soifer**
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I. Inscribe in a unit square r squares which have no interior points in common. Denote by $f(r)$ the maximum of the sum of the side lengths of the r squares. The problem is to evaluate the function $f(r)$.

It is easy to show that $f(k^2)=k$. In 1932 the first author conjectured that $f(k^2+1)=k$. It is true for $k=1$.

The function $f(r)$ is increasing; moreover, $f(r+2)>f(r)$. It is easy to show that $\left[\sqrt{r}\right]\le f(r)\le\sqrt{r}$. We conjecture that $f(k^2+2)=k+\frac{1}{2k}$ and $f(k^2+3)=k+\frac{1}{k}$, as well as $f(k^2-1)=k-\frac{1}{k}$.

II. A similar problem can be posed to find the maximum $g(r)$ of the diameters of r non-overlapping discs contained in a disc of unit diameter.

III. It may be interesting to find the set R of all r such that if any r non-overlapping squares are contained in a unit square, then the r discs inscribed in the r squares can be packed in the disc of unit diameter. It is easy to see that $1,2\in R$. On the other hand, $k^2\notin R$ for any non-zero integer k.

166

FORBIDDEN SUBGRAPHS AND PANCYCLICITY

Ralph Faudree, U Memphis, Memphis, TN 38152 Ronald Gould, Emory U, Atlanta, GA 30322 Zdeněk Ryjáček, U West Bohemia, 30614 Pilsen, Csech Republic, Ingo Schiermeyer*, T H Aachen,D-52056 Aachen, Germany

We prove that every 2–connected $K_{1,3}$-free and Z_3–free graph is hamiltonian except for two graphs. Furthermore, we give a complete characterization of all 2–connected, $K_{1,3}$-free graphs, which are not pancyclic, and which are Z_3-free or B–free or W–free or HP_7–free.

167

Good Locations for Trees in Ramsey Theory

Guantao Chen, North Dakota State University, Fargo
Richard H. Schelp, The University of Memphis
Ľubomír Šoltés *, The University of Memphis

For a positive integer k, a set of $k+1$ vertices in a graph is a k-cluster if the difference between degrees of any two of its vertices is at most k -1. Given any tree T with more than k^3 vertices, we show that for each graph G with sufficiently many vertices, either G or its complement contains a copy of tree T such that some vertices in the copy form a k-cluster in G. The same conclusion holds for any tree with maximum degree exceeding k.

168

Algorithms for a Self Delimiting Representation of Trees

James A. Foster, Paul W. Oman, Karen Van Houten*
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We present algorithms which manipulate a near-optimal representation of tree structures. The algorithms demonstrate the advantages of a self delimiting bitstring representation of arbitrary trees. We give examples of these algorithms for several computing applications such as Prefix code dictionary compression, join trees, and for producing random walks and random boolean circuits of arbitrary structure over arbitrary bases. These examples illustrate how the bitstring representation leads to an elegant data structure which would be difficult to achieve so compactly otherwise. The paper also demonstrates how standard tree operations of ancestry and descendant determination, and subtree insertion and deletion, can be performed directly on the bitstring representations, without intermediate decoding operations. For a full version of these and other related results and algorithms, please see James A. Foster, Paul W. Oman, Karen Van Houten, and Weiguo Zhu, *Using Self Delimiting Strings to Represent Trees*, U. of Idaho Technical Report CS 92-06, October 1992.

Keywords: trees, algorithms, data structures, bitstrings, random walks, boolean circuits, Prefix codes, join trees

169

Title: **A combinatorial characterization of (t, m, s)-nets in base b**

K. Mark Lawrence, University of Wisconsin-Madison

We consider (t, m, s)-nets in base b which were introduced by Niederreiter in 1987. These nets are highly uniform point distributions in s-dimensional unit cubes and have applications in the theory of numerical integration and pseudorandom number generation. A central question in their study is the determination of the parameter values for which these nets exist. Niederreiter has given several methods for their construction, all of which are based on a general construction principle from his 1987 paper.

We define a new family of combinatorial objects, the so-called "generalised orthogonal arrays", and then discuss a combinatorial characterization of (t, m, s)-nets in base b in terms of these generalised orthogonal arrays. Using this characterization, we describe a new method for constructing (t, m, s)-nets in base b which is not based on the aforementioned construction principle. This method gives rise to some very general conditions on the parameters (involving a link with the theory of orthogonal arrays) which are sufficient to ensure the existence of a (t, m, s)-net in base b. In this way we construct many nets which are apparently new.

P3-CONNECTED GRAPHS OF MINIMUM SIZE 170

Kathryn Fraughnaugh*, David C. Fisher, and Larry Langley
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A graph is "C4-free" if it doesot contain any 4-cycles. A C4-free graph is "C4-saturated" if adding an edge creates a 4-cycle. At the 3rd Southeastern Conference, Ollmann proved that any n-node C4-saturated graph has at least {3(n-2)/2} edges where {x} is the ceiling function. He also described the set of all n-node C4-saturated graphs with {3(n-2)/2} edges. In 1989, Tuza gave a shorter proof of these results. A graph is "P3-connected" if each pair of nonadjacent nodes is connected by a path with exactly 3 edges. A C4-saturated graph is P3-connected, but not vice versa. We generalize Ollmann's results by proving that any n-node P3-connected graph has at least {3(n-2)/2} edges. We also describe the set of all n-node P3-conected graphs with {3(n-2)/2} edges. This is a superset of Ollmann's set as there are n-node P3-connected graphs with {3(n-2)/2} edges which are not C4-saturated (for example, the graph below).

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171

On Ramsey Numbers R(4,k)

Jacek Ossowski, Courant Institute, New York University

The Ramsey number $R(k, l)$ is the smallest integer n such that in any 2-coloring of a complete graph on n vertices K_n by red and blue, there is either a red K_k (complete subgraph on k vertices, all of whose edges are colored red), or there is a blue K_l. The asymptotics for $R(k, l)$ have been a subject of intensive studies which started in the early 60's. Recently (1994), J. H. Kim completed the 3-decade spanning effort and showed that

$$R(3, k) > c\frac{k^2}{ln(k)} \qquad (**)$$

which matches the upper bound of Ajtai, Komlos and Szemeredi. It seems reasonable at present, that the approach used to obtain (**) could improve the best currently known lower bound for $R(4, k)$

$$R(4, k) > c(\frac{k}{ln(k)})^{\frac{5}{2}}$$

(J. H. Spencer) by replacing the absolute constant c with a function $c(k) \to \infty$ as $k \to \infty$. In the approach, a random dynamic algorithm A creating a K_4-free graphs is investigated. Key properties of the algorithm are approximated by a continuous branching process B. Asymptotic behavior of an ordinary differential equation associated with B is directly related to the possibility of the above improvement. Preliminary report.

Key words: Random Graphs and Algorithms, Branching Processes

Recognition Of Bipartite Permutation Graphs

172 Alan P. Sprague, UAB, Birmingham AL

We present an O(m+n) algorithm to recognize bipartite permutation graphs (where n is the number of vertices and m the number of edges). While not more efficient than the recognition algorithm of Spinrad, Brandstaedt and Stewart [Discr. Appl. Math 18 (1987) 279-292], it appears to be simpler. It is based on Breadth First Search: it performs two Breadth First Searches, plus an O(n) epilogue. The algorim is very similar to a recognition algorithm for unit interval graphs, and leads to a correspondence (not one-to-one) between unit interval graphs and bipartite permutation graphs on the same number of vertices.

Keywords: Graph algorithms, permutation graphs.

173

Subdivisions in planar graphs

Xingxing Yu, Georgia Tech

Given four distinct vertices in a 4-connected planar graph G we give a necessary and sufficient condition when G contains a subdivision of K_4 on these four vertices. As consequences we prove a conjecture of Robertson and Thomas as well as the existence of a K_5-subdivision in a 5-connected apex graph. This research is also related to an old conjecture of Dirac which is equivalent to the following conjecture of Seymour and Kelmans: every 5-connected non-planar graph cotains a K_5-subdivision.

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ON SEPARATING 3-CYCLES IN MAXIMAL PLANAR GRAPHS

Jochen Harant, Technical University of Ilmenau, Germany

By a theorem of H. Whitney, a maximal planar graph without separating 3-cycles is hamiltonian. Here the problem is discussed how many separating 3-cycles such a graph must have to be non-hamiltonian. The case of exactly three separating 3-cycles is considered especially.

A FRACTIONAL RAMSEY THEORM

175

Michael Jacobson, U of Louisville,
Gregory Levin*, Edward Scheinerman, Johns Hopkins

The notation $n \rightarrow (k,l)$ stands for the statement "Every graph G on n verticies has $\alpha(G) \geq k$ or $\omega(G) \geq l$." The Ramsey number $r(k,l)$ is the least n for which $n \rightarrow (k,l)$ is true. This definition may be generalized by replacing α and ω with their fractional analogues, giving the *Fractional Ramsey Number* $r_f(x,y)$. An exact formula for $r_f(x,y)$ is derived for all positive reals x and y, and we see that r_f grows polynomially, not exponentially, in x and y. As with the traditional Ramsey number, the notion of r_f may be extended to consider p-edge colorings of complete graphs, giving $r_f(x_1,\ldots,x_p)$. Some specific results and bounds are presented for this case.

key words: Ramsey number, fractional clique

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Line Tree Classifiers and Applications

Stan Klasa, Hongfeng Yin, C.S. , Concordia U.

Several linear tree classifiers and their applications are discussed in this paper. A supervised tree classifier is constructed using Fisher's linear discriminant for samples with multiple classes. Also, an unsupervised Perceptron algorithm is given and proven convergent for normally distributed samples in this paper. Based on the unsupervised Perceptron, an unsupervised linear tree classifier is proposed for samples with large training sets. Some experimental results are given in digital character recognition and wave form recognition.

177 FIXED AND MOBILE EDGES OF A GRAPH

Robert Molina, Alma College

An edge x of a graph G is a fixed edge if G-x+y isomorphic to G implies x=y. An edge of a graph G is a mobile edge if it is not fixed. A fixed graph is one where all edges are fixed, and a mobile graph is one where all edges are mobile. In this paper we give some sufficient conditions for an edge of a graph to be mobile. We describe various families of fixed and mobile graphs, and characterize mobile trees. The relationships between mobile edges, pseudo-similar edges and the edge reconstruction problem is discussed.

Key words: pseudo-similar, reconstruction

178 **(2 mod 4)-Cycles**

X. Cai, W. Shreve*North Dakota State U

The following conjecture of Chen, Dean, and Shreve, has been proven. Let G be a 2-connected graph with at least six vertices. If the minimum degree is at least three, then G contains a cycle of length 2 (mod 4).

Keywords: CYCLES, CYCLE STRUCTURE

179 **Rado numbers for homogeneous second order linear recurrences –degree of partition regularity**

Heiko Harborth, Techn. Univ. Braunschweig, Germany

In every 2-coloring of $1, 2, \ldots, 17$ there exists a monochromatic sequence of four numbers f_1, f_2, f_3, f_4 which fulfill the Fibonacci recurrence $f_{n-2} + f_{n-1} - f_n = 0$, and 17 is the smallest number having this property. More general, we consider s-term sequences $a_1, \ldots, a_s$ for $s \geq 3$ which fulfill $\sigma a_{n-2} + \tau a_{n-1} + \mu a_n = 0$ with integers $\sigma, \tau, \mu \neq 0$. – The smallest number $N = Ra(s; \sigma, \tau, \mu)$ so that every k-coloring of $1, \ldots, N$ contains a monochromatic sequence $a_1, \ldots, a_s$ is called Rado number. – The largest umber $k = k_0(s; \sigma, \tau, \mu)$ for which a Rado number exists is called degree of partition regularity. – Here we determine upper bounds for $k = k_0(s; \sigma, \tau, \mu)$. – (Common work with Silke Maasberg)

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AN ALGORITHM FOR RECONSTRUCTING A BINARY IMAGE FROM A FEW PROJECTIONS.

Pablo M. Salzberg, University of Puerto Rico, Department of Mathematics and Computer Sciences, P.O. Box 23355, Río Piedras, Puerto Rico 00931

A Binary Tomography problem deals with the reconstruction of a black and white image from the knowledge of parallel projections along a *minimal* set of directions. Pioneer results on this problem were given by P.C. Fishburn et al., 1991. Recently, an important application was proposed by Peter Schwander in connection with the study of the atomic structure of crystal lattices. Using High Resolution Electron Microscopy, Schwander developed a technique that counts the number of atoms in each atomic column along different zone axes of a crystal. In case that there is only one type of atom, this could be thought as a three dimensional array consisting only of 0's and 1's, of which it is possible to measure projections in two to four directions. The goal is to reconstruct the atomic position in the crystal. Since uniqueness can be guaranteed only for a certain number of directions (higher than the number of directions available in this case), we would be looking for approximate solutions. Practical applications involves a cubic lattice of order around 400.

In this contribution the author presents an algorithm to find an approximate solution, based on a continuous extension of the Radon transform on vector spaces over finite fields (Lect. App. Math., AMS, **30**, '94). The algorithm uses the following heuristic: our model can reconstruct any density distribution assigned to vertices of any 3-D array of order *n* whenever we are allowed to use projections along *n+1* directions. In this case, only a smaller number of directions are available, and we use our reconstruction transform limited to the available projections. This furnishes a 3-D array of values. Each of these values -measure on a scale from 0.0 to 1.0 (0 stands for improbable and 1 for highly probable)- gives the likelihood of the presence of an atom (a '1') in that position. By ordering the entries in the reconstruction array, we are able to find a family of cutting planes approaching the true pattern. We shall exhibit reconstructions of several phantoms proposed by Schwander.

Thursday, March 9, 1995
12:10 p.m.

Enumeration of Labelled Multipartite Multigraphs by Degree Parities

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Lane Clark
Southern Illinois University at Carbondale

Counting formulae are obtained for the number of labelled (n, m)-multigraphs of strength s with fixed vertex partition and given number of vertices of even degree.

182

Minimum-Length Fundamental Cycle Set Problem: New Heuristics and an Empirical Study

Narsingh Deo, Nishit Kumar* and James Parsons, UCF, Orlando

The following problem is considered: Given a finite, connected, simple, undirected, unweighted graph, find a spanning tree for which the fundamental set of cycles has total shortest length. The problem finds application in the generation of minimal perfect hash functions. It was shown to be NP-complete in 1982. The two best-known strategies to find suboptimal solutions, namely -- Heuristics UE (1982) and Heuristics UV (1993), are based on variations of the breadth-first search (BFS) technique and deviate from BFS only in their criteria to select a new vertex to explore from. Both the heuristics (i.e., UE and UV) select a new vertex employing (priority) functions which are based on degrees of vertices.

We propose two new heuristics for the problem. One of them (called UV2) is derived by incorporating a lookahead in the UV heuristic i.e., prioritize vertices by the count of vertices up to distane two, instead of looking at the degree. The other (called NT) selects a new vertex based on its propensity to induce fundamental cycles. An empirical study reveals that Heuristics NT outperforms on regular graphs and (extremely) sparse random graphs. In contrary to our expectations, we also found that UV2 (inspite of being a more complex heuristics) does not perform better than UV on random graphs.

A Winning Strategy for the Ramsey Graph Game

183

Aleksandar Pekeč Rutgers University

Two players, Maker (red) and Breaker (blue) alternately color the edges of K_N. Maker is first to play, and the players color one edge per move. Maker winthe game if there is a red K_n. Breaker wins if there is no red K_n after all the $N(N-1)/2$ edges have been colored. Beck showed that Maker has a winning strategy whenever $N > (2+\epsilon)^n$ for n sufficiently large. However, in order to claim a victory, Maker needs to play all $N(N-1)/2$ moves. In this talk we will present a winning strategy for Maker requiring at most $(n-3)2^{n-1}+n+1$ moves. Maker can apply this strategy whenever $N \geq (n-1)2^{n-1}+2$. There are indications that this is the best possible strategy. We will also demonstrate how the ideas presented can be used to develop winning strategies for some related combinatorial games.

Keywords: Combinatorial Games, Algorithms on Graphs, Ramsey Theory

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Linearly recursive tableaux

Earl J. Taft, Rutgers University

We consider tableaux $f = (f_{i_1 i_2 \ldots i_n})$ for $i_1, i_2, \ldots i_n \geq 0$, $f_{i_1 i_2 \ldots i_n}$ in a field k. We say f is linearly recursive if for each $1 \leq j \leq n$, each row parallel to the j-th axis (i.e., the sequence $\{f_{i_1 \ldots i_{j-1} t i_{j+1} \ldots i_n} \mid t \geq 0\}$ for fixed $i_1, \ldots, i_{j-1}, i_{j+1}, \ldots, i_n$) satisfies a linearly recursive relation (independent of the choice of $i_1, \ldots, i_{j-1}, i_{j+1}, \ldots, i_n$). We show that f is (Hadamard, pointwise) invertible, i.e., the tableau $f^{-1} = \left[\frac{1}{f_{i_1 i_2 \ldots i_n}}\right]$ is linearly recursive if and only if each $f_{i_1 i_2 \ldots i_n} \neq 0$ and each row is eventually an interlacing of geometric sequences $(ar^i \mid i \geq 0)$. Moreover, the procedure is effective, i.e., a given f can be tested for invertibility by a finite algorithm. More precisely, one has only to check a finite number of $f_{i_1 \ldots i_2 \ldots i_n}$ for being non-zero, and only a finite number of rows for being an interlacing of geometric sequences. Each of these finite number of rows can be so tested by a finite algorithm (i.e., the case $n = 1$), given by R. Larson and the author [Israel J. Math 72 (1990), 118–132] and by the author [Proc. 23rd Southeastern Conference on Combinatorics, Graph Theory and Computing 1992, Congress Numerantium 89 (1992), 107–110].

Thursday, March 9, 1995
2:00 p.m.

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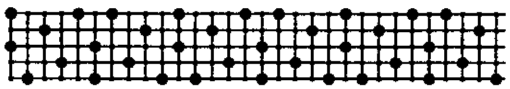
Periodicities in Discrete Systems

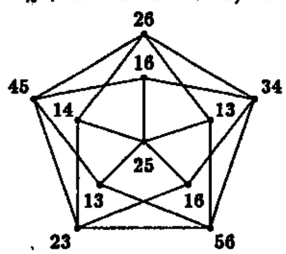
David C. Fisher – University of Colorado at Denver

1. A store uses n chickens for their "Designer Chicken" line. The "Dark Lover" has 2 wings and 2 legs, and costs \$3. The "Party Pack" has 4 wings and 4 breasts, and costs \$8. The "Eleganta" contains 8 legs and 2 breasts, and costs \$14. Let a_n be the maximum sales. Then $a_{n+5} = a_n + \$33$ for all $n > 27$, but $a_{27} = \$173$ and $a_{32} = \$208$.

2. Let b_n be the domination number of a $5 \times n$ grid. Chang and Clark showed $b_{n+5} = b_n + 6$ for all $n > 7$, but $b_7 = 9$ and $b_{12} = 16$.



3. Let c_n be the n-coloring number of Grötzch's graph (below is a 2-coloring with 6 colors). Stahl showed $c_{n+10} = c_n + 29$ for all $n > 1$, but $c_1 = 4$ and $c_{12} = 32$.



For these systems, $d_{n+p} = d_n + q$ for some p, q, and n_0 and for all $n > n_0$. *When does this occur? How do you find p, q, and a minimal n_0? How large can p and n_0 be?*

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ASSOCIATIVE LABELINGS, CONTRACTIONS AND SENSE OF DIRECTION IN DISTRIBUTED SYSTEMS

Paola Flocchini, Dipartimento di Scienze dell'Informazione
Universita' di Milano, Milano, Italy
Nicola Santoro*, School of Computer Science
Carleton University, Ottawa, Canada

In a distributed system, every processing entity x has a distinct "port number" associated to each entity y with whom it can communicate directly. Thus, the system can be viewed as a graph where a node x has a distinct label associated to each of its neighbours (e.g., in a ring the two neighbours are labeled "left" and "right"). If the labels satisfy a specific set of properties, the labeling is called Sense of Direction and it is well known to have a significant impact on the communication complexity of distributed problems. In this paper, we study the impact that associativity has on the properties of labelings. In particular we prove that a restricted set of properties (called Weak Sense of Direction) suffices for an associative labeling to be Sense of Direction. We also show that the traditional labelings of most interconnection networks are all associative instances of "contracted" labelings.

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Extended gcd calculation

George Havas(*) and Bohdan S. Majewski
University of Queensland

Extended gcd calculation has a long history and plays an important role in computational number theory and linear algra. Recent results have shown that finding optimal multipliers in extended gcd calculations is difficult. We study algorithms for finding good multipliers. We present and analyze new algorithms with improved performance.

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Designing an implementation environment for network optimization

Jonathan Owen* Collette Coullard — Northwestern U
David Dilworth — U Michigan

We present a software environment for network optimization, GIDEN: Graphical Implementation Development Environment for Networks. GIDEN features graphical display and basic algorithm animation for network optimization problems. The environment has proven to be useful for teaching, implementing, and developing network optimization algorithms. At the heart of the environment is an object-oriented library of network data structures and a toolkit of special purpose solvers. An overview of the system is presented and design issues are discussed.

Key words: object-oriented, network optimization

Thursday, March 9, 1995
2:20 p.m.

189 A Generalized"Probleme des Rencontres"

Joel Brawley*and Doug Shier, Clemson University

The classical "probleme des rencontres" asks for the probability that a random permutation of the numbers 1, 2, ..., n places no element in its own position. A writer to the "Ask Marilyn" column in the August 21, 1994 issue of Parade Magazine posed a probability question whose solution (which was not given by Marilyn) requires a generalization of the rencontre problem. In this talk we present an analytical solution to the generalized problem and we discuss some of its computational aspects.

190 ON THE DIAMETER OF NETWORK OPTIMIZATION POLYHEDRA

Fred J. Rispoli, Dowling College

Investigations of the diameter of convex polyhedra are motivated by an effort to understand the complexity of polyhedral edge-following algorithms such as the simplex method. These studies are especially relevant for network optimization problems since the network simplex method is often used when solving these problems. In this talk a survey is given on the diameter of the polyhedra that arise in well known network optimization problems.

Key Words: diameter, polyhedra, simplex method

Solution of an Open Problem Posed by David Gale

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T. L. Morin, N. Prabhu, and S. Ikeler

School of Industrial Engineering, Program in Computational Combinatorics and Department of Mathematics

Purdue University
West Lafayette, IN

A long-standing open problem posed by David Gale is solved. Specifically, we consider a Jeep that can carry enough fuel to go a distance *d*, but must cross a desert of width greater than *d*. Therefore, fuel depots must be established along the route, and the problem is to cross the desert using the minimum amount of fuel. When fuel is available on one side only, the Jeep problem is well-solved and the maximum distance the Jeep can cover with a specified integral number of tankfuls of fuel is given by the odd harmonic series. In 1971, Gale posed the problem of a Jeep undertaking round-trip across a desert with fuel available at both ends of the desert. Specifically, the problem is to determine the maximum width desert that can be crossed by a Jeep in a round-trip if it is allowed to purchase fuel at both ends of the desert. This problem is solved and limiting results are obtained.

Optimal Parallel Matching on Bipartite Permutation Graphs

Hiryoung Kim* and Alan P. Sprague, UAB, Birmingham, AL

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We present a cost optimal parallel algorithm for a maximum matching problem on a bipartite permutation graph on EREW PRAMs. Previously, Chen and Yesha [Networks 22 (1993) 29-39] have dealt with this problem. Their solution relies on Dekel and Sahni's [J. Par. Dist. Comput. 1 (1984) 185-205] matching algorithm for convex bipartite graphs, which runs in O(log^2 n) time with O(n) processors. Our method starts with an easily understood greedy algorithm. To achieve cost optimality, We define nontrivial binary operation which is associative and equivalent to the greedy algorithm. Thus, parallel prefix can be applied to the problem.

Keywos: Parallel graph algorithms, Bipartite permutation graphs

Thursday, March 9, 1995
2:40 p.m.

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ADDITIVE PERMUTATIONS WITH REPEATED ELEMENTS

Jaromir Abrham, U. of Toronto, Canada M5S 1A4
Jean M. Turgeon*, Math., U. of Montreal, Canada H3C 3J7

We extend the well-known concepts of additive permutations and bases of additive permutations to the case where repeated elements are permitted. Then the basis, an ordered set, becomes an ordered multiset. If such a basis has at least two positive elements, then it also has at least two negative elements (the number of negative elements does not have to be equal to the number of positive ones). We prove by a counterexample that a higher lower bound for the number of negative elements cannot be obtained. We study in detail the case of symmetric bases and list all bases with repeated elements up to cardinality six.

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A COHERENT RELIABILITY MODEL FOR MULTIPROCESSOR NETWORKS

D. Gross - Seton Hall Univ.*, F. Boesch and C. Suffel - Stevens Institute

There is extensive literature devoted to developing a mathematical reliability model which is applied to the design of many large networks. This model assumes that a graph G has perfectly reliable nodes, but each edge of has a probability of operating. The reliability R is defined as the probability that there is an operating connected spanning subgraph. Reliability analysis problems concern the determination of efficient algorithms to find R. It was also shown that it remains #P-Complete even when G is a simple undirected planar graph and each edge has the same probability ; however there are classes of graphs for which R can be found by a polynomial algorithm. Reliability synthesis problems concerns finding graphs that are, according to some definition, the most reliable in the class of all graphs having a given number of edges and nodes. There are many synthesis results corresponding to the different definitions In those applications such as multiprocessor networks where the edges are perfectly reliable but the nodes operate with some known probabilities, the obvious analog is to define the residual node reliability as the probability that there exits a spanning connected subgraph on the operating nodes. However this definition suffers by not having a desired property called coherence. Namely it is possible that operating subnetworks generated by certain operating nodes become nonoperational when the same nodes operate but some additional nodes also operate. In this work we propose a new definition of a reliability function to handle the case of node failures. We show that it is coherent, and we determine many of its properties. Several analysis and synthesis results are presented.

Key Words : Multiprocessor Network Reliability, Coherent Node Failure Reliability Models

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STABILITY OF THE OPTIMAL STRATEGY IN TOURNAMENT GAMES

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We consider a tournament game which is a two-player zero-sum game with a rectangular payoff matrix T=[T(i,j)], i=1,2,..3,n, and j=1, 2, ...m played in a directed graph having max{n,m} nodes, and n.m arcs each with a fixed length T(i,j). Players independently pick a node, however strategies for player I and II are restricted to their set nodes {i=1, 2,...n} and {j=1, 2, ...,m} respectively. Every time players pick a node, we say a play is made. The game is played many times. The player whose node is at the head of arc connecting the two nodes receives a payoff T(i,j) from the other player. If both pick the same node player I receives T(i,i). We present the optimal strategy for each players and its stability analysis with respect to changes in any elements of T.

Key Words and Phrases: Fractional graph parameters, Graph domination, Two-person Zero-sum games, Minmax theorem.

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Efficient Horizontal Parallelism

C.-S. Peng*, Creighton University
T.-Y. Juang and K.-M. Yu, Chung-Hua Polytechnic Institute

Key words: parallel computation, horizontal parallelism, DO loop.

Much of research and development has been devoted to parallel computation. Codes written for traditional single processor machine were reprogrammed in order to take advantage of the advanced multi-user and multi-process environment. However it is still very impractical to rewrite certain codes. For example FORTRAN programs usually have tens of thousands of lines and it is not only uneconomical but also difficult to guarantee the correctness of the rewritten version. As a result, considerable amount of efforts have been invested on restructuring compiler, processor allocation for horizontal and vertical parallelism and so on.

Given a FORTRAN DO loop, horizontal parallelism simultaneously executes different iterations on at least two processors. If each iteration is assigned a processor, then optimally all processors can start at the same time and finish at the same time. However, due to certain data dependency relations, such as data dependency, data anti-dependency, and data output dependency, this optimum result is usually not achievable.

In this paper, we study the relation between the total execution time and the number of processors assigned to a FORTRAN DO loop. It is generally believed that more processors yields shorter execution time. However this paper shows that two processors are enough to achieve the optimum execution time in many instances.

Thursday, March 9, 1995
3:20 p.m.

A connection between the Veronese map and linear codes

201

H. Tapia (U. Autónoma Metropolitana-I, México)*
C. Rentería (I. Politécnico Nacional, México)

(Keywords: Veronese map, linear codes, error detecting-correcting codes)

It is shown that some of the classical codes such as the Reed-Solomon and the Reed-Muller of order 1 are obtained by using the Veronese map defined on a projective space over a finite field. Also, some linear codes are obtained on the Veronese surface over the finite field with four elements.

NEAR OPTIMAL SOLUTIONS TO SPANNING TREE PROBLEMS USING A GENETIC ALGORITHM APPROACH

202

MICHAEL L. GARGANO (*), RICHARD MUNDE, LOUIS V. QUINTAS
PACE UNIVERSITY

Near optimal/optimal solutions to two spanning tree problems are developed using a genetic algorithm approach. Genetic algorithms use concepts such as survival of the fittest, mating, and mutation to search for solutions to difficult problems. The first problem is to find a spanning tree with optimal sequencing and the second is to find a capacitated minimal spanning tree.

The Moore-Penrose Inverse of the Laplacian of a Graph

203

Steve Kirkland, University of Regina
Miki Neumann, Uiversity of Connecticut
Bryan Shader*, University of Wyoming

We present an explicit formula and a combinatorial interpretation for the Moore-Penrose inverse of the Laplacian of a graph G. We use this to develop lower and upper bounds on the algebraic connectivity of a graph.

Keywords: Laplacians, matrix-tree theorem, algebraic connectivity

Mathematical Modeling and Data Mining

204

Victor Cook, Palm Beach Atlantic College, West Palm Beach

Today it would be inconceivable to trade securities without the assistance of the computer. We will present an overview of how the computer is used to make decisions in this field.

On the linear complexity of periodic sequences and the existence of Perfect Factors. 205

Glenn Hurlbert, Arizona State University

The linear complexity of a periodic sequence over a finite field is defined to be the degree of its minimal generating polynomial. A Perfect Factor is a collection of periodic arrays of period axb in which every k-ary mxn matrix appears exactly once within the period The construction of Perfect Factors has proved to be a crucial step in the construction of DeBruijn Tori, which are Perfect Factors involving just one array. A new technique of Paterson uses linear complexity as a control on the lengths of periodic sequences. Together with techniques of Hurlbert and Isaak, new classes of Perfect Factors are constructed.

Constant Amortized Time Generation of Permutations With Minimal Length Increasing Subsequences 206

Dominique Roelants van Baronaigien
Computer Science, University of Victoria

A constant amortized time algorithm is presented for listing all permutations of the integers 1, 2, ..., n that contain a subsequence of length k where all of the elements in the subsequence are in increasng order.

LAPLACIAN POLYNOMIALS AND THE NUMBER OF SPANNING TREES OF A GRAPH

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A. K. Kelmans

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Department of Mathematics, University of Puerto Rico, San Juan, PR 00931

Let $\mathcal{G}_n^m$ denote the set of simple graphs with n vertices and m edges, $t(G)$ the number of spanning trees of a graph G, and $L(l,G)$ the Laplacian polynomial of G. We give some operations Q on graphs such that if $G \in \mathcal{G}_n^m$ then $Q(G) \in \mathcal{G}_n^m$ and $L(l,G) \le L(l,Q(G))$ for $l \ge n$. Because of relation $t(K_s \setminus E(G_n) = s^{s-n-2}L(s,G_n)$ these operations also increase the number of spanning trees of the corresponding complement graphs: $t(K_s \setminus E(G) \le t(K_s \setminus E(Q(G))$. The approach developed here can be used to find some other graph operations with the same property. We use these operations to give a complete description of those graphs with n vertices and m edges that have the maximum number of spanning trees among all graphs in $\mathcal{G}_n^m$ subject to $n(n-1)/2 - n \le m \le n(n-1)/2$.

Application of Radial Basis Function Networks in Nonparametric Estimation and Classification

208 Adam Krzyzak, Concordia University, Montreal

We will discuss application of neural networks to regularization and approximation of nonlinear functions. We will present convergence results for radial basis function nets in approximation of functions in which VC dimension and metric entropy play important role.

Thursday, March 9, 1995
4:00 p.m.

LATIN K-CUBES WITH SPECIAL PROPERTIES

Joseph Arkin* and David C. Arney, United States Military Academy
West Point, New York 10996 MADN-A

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In this paper using the concepts of the Arkin-Straus Latin k-Cubes, we construct reverse digit pairs in k-cubes of even and doubly-even order. We exhibit an orthogonal, perfect, magic and doubly magic Latin 4-cube of order 8, with the property that each row contains reverse digit pairs.

It is interesting to note that while constructing our reverse digit pairs we were able to show that our Lyamzin constructions are also reverse digit constructions.

As an historic example of its application, we briefly mention the use of the Arkin-Straus Latin k-Cubes in the development of a formulation for a new polymer material.

This paper is dedicated to the memory of Professor Paul Smith, University of Victoria, who died in 1993.

Algorthms For Maximal Induced Subgraphs With Bounded Degree

Arun Jagota*, Giri Narasimhan, U of Memphis, Memphis TN 38152

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We call an induced subgraph H of a graph G a degree-k subgraph if the maximum degree of H is at most k and a degree-k-maximal subgraph if, additionally, every vertex outside H has degree at least k+1 in H. The degree-k-maximal property generalizes the maximal independent set property, which holds for k=0. The maximum independent set problem (MIS) is a well-known and important NP-hard problem. In this paper, we study analogous computational issues for our generalization. Our results can be summarized as follows. We note that the maximum degree-k subgraph problem is NP-hard, even to approximate within some polynomial factor. We design an algorithm, called A, for finding a degree-1-maximal subgraph, but have difficulty generalizing it to k > 1. We present another algorithm, called B, which finds a degree-k-maximal subgraph for arbitrary k >= 0. We show that B runs in time O((k+1) s),where s is the cardinality of the degree-k-maximal subgraph it finds, and that this time bound is tight. We obtain the following approximation results for the maximum degree-k subgraph problem: Algorithm B gives a performance ratio of 2(k+1) on almost every graph; and Algorithm B, combined with a certain approximation algorithm for MIS, gives a worst case performance ratio of O((k+1)n/log^2 n), which is in a sense tight for constant k. Algorithm B minimizes an energy function, which plays a crucial role in analyses of its running time and approximation performance ratio.

Keywords: Approximation Algorithms, NP-hard problems, Independent Sets

Sharp Bounds on the Competitor Distance

Thomas B. Sprague, Alma College

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Let G=(V,E) be a connected graph and S a subset of V. The S-distance (Johns, 1987) is the minimum number of edges in a u-v walk W covering S. Appropriate subsequences of W induce (partial) orderings of S, which links scheduling and routing problems in a natural way. However, realistic scheduling problems often involve constraints beyond the structure of G. Certain tasks must be performed before others, or penalties must be paid for tasks that are performed in a sub-optimal order.

We may tie scheduling and routing problems more tightly together by generalizing S-distance in a way that takes these "competitive" relationships into account. The resulting competitor distance and competitor graphs may be studied from several perspectives. In this paper we briefly review some S-distance results, and compare with the corresponding statements for the competitor distance. We comment on competitor distance calculations, and compute sharp bounds on this distance for various classes of competitor graphs.

ULAM Movement, ULAM University, ULAM Quarterly - Past, Present, Future

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Piotr Blass, Blass Corporation and Ulam University, Palm Beach

We will discuss history and development of the Ulam movement and future plans for the Ulam University.

Friday March 10,1995
8:50 a.m.

Discrepancy of (Quasi-)Arithmetic Progressions

217 Robert Hochberg, University of Connecticut

In the 1930's, Erdos started asking if the natural numbers could be 2-colored so that every finite arithmetic progression starting at contains, within an additive constant, the same number of each color. van der Waerden's Theorem explains why the starting at 0" restriction is present. For this paper, we expanded the class of arithmetic progressions to include quasi-arithmetic progressions, and found large discrepancy there (on the order of the fourth root of log n.) This talk will outline the proof, which is a modification of Roth's method.

Keywords: Arithmetic Progression, Discrepancy, Kulcs.

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Characterizations of Strong k-Trees

Andrzej Proskurowski* and GuoqingWeng
CIS, Univ of Oregon, Eugene, OR 97403

We give several independent characterizations of k-trees that are strongly chordal. These characterizations are based on a forbidden induced subgraph, a recursive construction procedure, and the structure of the union of minimal separators.

219 **CYCLIC POWER OF GRAPHS**

Puhua Guan, Math., U of Puerto Rico

Let G=(V,E) to be a graph. We define the cyclic power of G of order m to be the graph G(m) with vertices $(v_1, v_2, ..., v_m | v_i \in V$, any pair of vertices ; $e = ((u_0, u_1...u_{m-1})(w_0, w_1, ..w_{m-1}))$ is an edge of G(m) if and only if $u_i = w_i$ for $i = 2, 3...n-1$ and $(u_1 w_1) \in E$, or $u_i = w_{i+1}$ for all i. Indices are computed in Z_n For each integer $k \geq 5$, we can obtain a class of graph G(m) with degree k and diameter $clog_{k-1}|G(m)|$, where c is a real number less than 2.

Friday March 10,1995
9:10 a.m.

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THE CONWAY-GUY SEQUENCE

Tom Bohman Rutgers University

A set S of positive integers has distinct subset sums if the set $\{\sum X : X \subset S\}$ has $2^{|S|}$ distinct elements. For example, the sets {1,2,4,8} and {3,5,6,7} have distinct subset sums. How small can the largest element of such a set be? In other words, what is

$$f(n) = \min\{\max\{S\} : |S| = n \text{ and S has distinct subset sums}\}?$$

Paul Erdős conjectures that $f(n) \geq c2^n$ for some constant c. In 1967 John Conway and Richard Guy constructed an interesting sequence of sets of integers. They conjectured not only that these sets have distinct subset sums but also that they are the best possible (with respect to largest element). This presentation consists of a description of the Conway-Guy sequence, a brief outline of the main idea used to prove that these sets have distinct subset sums, and a generalization othis construction which yields other sets that also have distinct subset sums.

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The Hermite invariant as graph invariant

Cynthia J. Wyels, U.S. Military Academy

The ability to determine permutation equivalence or permutation congruence within sets of matices is exactly what is needed to resolve isomorphism questions for designs and graphs. Matrices which are permutation equivalent, and thus permutation congruent, must have identical Hermite invariants. The effectiveness of this invariant has been demonstrated through its settling the isomorphism status of several previously intractable sets of statistical designs.

We focus on the use of the Hermite invariant as a graph invariant. Promising experimental results have been obtained on sets of graphs generally conceded to be difficult cases for which to resolve isomorphism. Analysis of the Hermite invariant via A.K. Dewdney's theoretical measure, its power, also suggests that the Hermite invariant is among the most hihly effective graph invariants.

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Graphs with Given Odd Sets

G. Chen[1], R.H. Schelp[2*], L. Soltes[2]
[1]North Dakota State University, [2]University of Memphis

Given a graph G, its odd set is the set of all integers k such that G has an odd number of vertices of degree k. We show if two graphs G and H of the same order have the same odd sets, then they can be obtained from each other by successive application of the following two operations: (1) add or remove an edge joining two vertices of the same degree, (2) replace independent edges u_1u_2, v_1v_2 by nonadjacencies u_1v_1, u_2v_2. If both graphs are regular or both are forests, it is sufficient to use only the first operation, while in general both operations are necessary.

Friday March 10,1995
10:50 a.m.

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Product of Integers Made a Power

P. Erdos, Hungarian Academy of Sciences; J.L. Malouf*, UNLV; J.L. Selfridge, NIU; E. Szekeres, Turra Murra, Australia.

In a paper so entitled, two of us (EP, JLS) established that the product of consecutive integers is never a power. We consider related problems in which we start with n, not a kth power, and select integers larger than n whose product with n is a kth power. For squares, the solutions are complete modulo some reasonable conjectures.

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Parallel Prefix on the Star and Pancake Network

Ke Qiu, Acadia University

Keywords: Parallel algorithm, interconnection network

We use a routing algorithm found earlier to develop a parallel algorithm that computes prefix of N elements on a star and pancake interconnection networks with P processors in O(N/P + log P) time. This algorithm is optimal in view of the \Omega (N/P + log P) lower bound. The result is interesting considering the fact that both networks have sub-logarithmic degree in terms of the total number of nodes.

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The Rank of the Neighborhood Matrix of a Graph

Gayla S. Domke and Valerie A. Miller*
Georgia State University, Atlanta, GA 30303-3083

The neighborhood matrix of a graph G with vertex set $V = \{v_1, \ldots, v_n\}$, denoted N(G), is an nxn symmetric 0-1 matrix where $n_{ij} = 1$ if and only if v_j is in the closed neighborhood of v_i (i.e. $v_j = v_i$ or v_j is adjacent to v_i) and $v_{ij} = 0$ otherwise. It is easy to see that $N(G) = A(G) + I_n$, where A(G) is the adjacency matrix of G. In this talk we will present results for the rank of the neighborhood matrix of complete graphs, cycles, and paths as well as the cartesian products of these graphs. We will also develop results on the rank of the neighborhood matrix of trees.

key words: rank, neighborhood matrix, cartesian product

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On the Möbius number of the subgroup lattice of S_n

John Shareshian Rutgers University

Stanley has asked whether the equality $\mu(1, S_n) = (-1)^{n-1}\frac{n!}{2}$ holds for all $n \neq 6$, where μ is the Möbius function on the lattice of subgroups of the symmetric group S_n. I will give a formula for $\mu(1, S_n)$, involving certain transitive subgroups, which holds for all $n \in \mathbb{N}$. I will then give a refined version of this formula, involving certain primitive subgroups, which holds for infinitely many values of n. Using the refined formula, the O'Nan-Scott Theorem, and the classificaion of finite simple groups, I will give a class of examples for which the equality mentioned above does not hold, thereby giving a negative answer to Stanley's question.

Keywords: Möbius function, symmetric group

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Replacing the Edges of a Minimum Spanning Tree

Robert R. Crawford and A. A. Kooshesh*
Western Kentucky University

In a network, where the connectivity of the nodes form a tree, it is often necessary to have a replacement for every link between two connected nodes. In this configuration, usually the tree is the minimum spanning tree of the graph that is induced by all possible pair of nodes that can be connected. In this talk, we We preset an algorithm to find a replacement for every edge of a minimum spanning tree in O(max(C, |V|log|V|)) time, where C is the cost of computing the minimum spanning tree of the graph.

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TREES ARE TOLERANCE SPHERE OF INFLUENCE GRAPHS.

Tara S. Holm(*) and Kenneth P. Bogart, Dartmouth College

We introduce the concept of a tolerance sphere-of-influence graph, a generalization of the sphere-of-influence graph defined by G.T. Toussaint. To define a sphere-of-influence graph G(S) on a finite set S of vertices we first assign to each vertex in S a point in the Euclidean plane, and an open ball centered at that point with radius equal to the smallest distance between that point and any other point representing a vertex in S. Two vertices in \$S\$ are adjacent if and only if their balls overlap. In the case of a tolerance sphere-of-influence graph, we also assign each vertex a real number called a tolerance and define two vertices to be adjacent if their balls overlap my more than this tolerance. (We measure overlap of balls along the line joining their centers.) This is one version of the usual way of using tolerances to extend the definition of a class of intersection graphs. Our main theorem states that every tree is a tolerance sphere-of-influence graph. In contrast, not all trees are sphere of influence graphs.

Friday March 10,1995
11:30 a.m.

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Counting Products of Triples in Finite Groups

Curtis Mitchell, Carleton College
Gary Sherman^*, Rose-Hulman Institute of Technology

Let G be a finite group and let p_k(G) denote the proportion of triples of elements (x,y,z) from G for which the cardinality of {xyz,xzy,yxz,zxy,yzx,zyx} is k. Questions and results concerning the distribution of k are discussed. In particular, we show that;

i) the mean value of k is either 1 or at least 53/32,
ii) p_2=0 implies that p_3=p_4=p_5=p_6=0,
iii) p_3=0 implies that p_4=p_5=0.

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CONSTRUCTING 5-REGULAR PLANAR CONNECTED GRAPHS OF EVERY POSSIBLE ORDER

James M. Benedict* and Gerald Thompson   Augusta College

The construction of all possible r-regular, planar, connected graphs from a canonical representative, has been accomplished by various authors for r = 1, 2, 3 and 4. A similar construction for the remaining case r = 5 has not yet appeared in the literature. This note surveys the known results and constructs 5-regular, planar, connected graphs of every possible order. It is shown that for a positive integer p, there exists a 5-regular, planar, connected graph of order p, if and only if, p is even, p exceeds ten, and p is not fourteen.

KEY WORDS

Regular, connected, planar, graphs

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Vertex Elimination Orderings and Completion Sequences in Classes of Chordal Graphs

Richard M. Odom, Craig W. Rasmussen* Naval Postgraduate School

Several classes of chordal graps are associated with special vertex elimination orderings. In some cases, these elimination orderings are fundamental to recognition algorithms. These elimination orderings share with conditional graph completion sequences the property that at each step some target attribute, e.g., chordality, is preserved. We show that cerain elimination orderings are closely related to efficient algorithms for constructing conditional completion sequences.

Keywords: Chordal graphs, elimination orders, completion problems.

Friday March 10,1995
11:50 a.m.

237 **Permutation Polynomials over Strange Rings**

Daniel Ashlock, Iowa State University, Math Department

A permutation polynomial is a polynomial that, when evaluated, yields a bijective function. Much of the work on permutation polynomials has been concentrated on polynomials over a Galois field for good and sensible reasons. In this talk I will give a very general method for finding permutation polynomials with coefficients in a commutative ring with 1 R that act on any ring S which has a scalar multiplication by R. I will use R=integers (mod n) and S=m-by-m matrices over the integers (mod n) as a motivating example. They key tool is a class of ideals, termed compositional attractors, in a polynomial ring which, in addition to the usual ideal properties have the property that for f(x) in the ideal and g(x) in the ring f(g(x)) is in the ideal.

Key Words: Permutation Polynomial, Integers (mod n), Ideal Theory

238 **Interval Graph Partitioning for Clique-width Maximization**

M. Sarrafzadeh* and Amir Farrahi
EE and CS, Northwestern U, Evanston, IL 60208 majid@eecs.nwu.edu

Clique-width of an interval graph in the the maximum width of one of its maximum cliques. It is straightforward to find the clique-width of a given interval graph. Here, we study the problem of bi-partitioning an interval graph. The goal is to maximize the sum of the clique-width of the two sub-graphs. We first study basic propertiesof the problem and establish theorems regarding the structure of feasible solutions. We prove a fundamental property of the problem which allows us to simply the problem by treating non-containing intervals separately, facilitates design of an efficient algorithm for the problem. We show the problem can be solved optimally. In particular, we propose an O(nlogn) time algorithm where n is the number of intervals. We solve the problem, with the same time complexity, when the size of the two partitions need to be equal. We will also prove that the problem is NP-complete for multi-interval graphs.

To the best of our knowledge, this problem has not been studied in the past. The problem finds applications in processor scheduling for low-power circuit design.

key words: Interval graphs, Geometry, Computational complexity, Algorithms

239 **Thwart Numbers of Some Bipartite Graphs**

D. G. Hoffman and P. D. Johnson Jr.*, Auburn University
E. B. Wantland, Western Connecticut State University

For a simple graph G, let $c(G)$ denote the choice number of G, and let $c_k(G)$ be defined as $c(G)$ is, except that there are only k colors available to form the lists on the vertices of G; $c_k(G)$ is defined only for $k \geq \chi(G)$. The *thwart number* of G, denoted $thw(G)$, is the smallest k such that $c_k(G) = c(G)$. To put it another way, $thw(G)$ is the smallest $k \geq \chi(G)$ such that there is an assignment of $(c(G)-1)$-subsets of a k-set to the vertices of G, so fiendishly contrived that all attempts to properly color G from these assigned lists will be *thwarted*.

We recycle some earlier work on choice numbers and restricted choice numbers to get results (some not entirely trivial) on thwart numbers of bipartite graphs. For instance, we show that $thw(K_{m,n}) = m^2$ for all $n \geq m^m$ provided $m \geq 2$, and that $(m - \frac{3}{2})(m-1) \leq thw(K_{m,n}) \leq (m-1)^2$ for $(m-1)^{m-1} - (m-2)^{m-1} \leq n < m^m$, if $m \geq 3$. We also make a start toward characterizing bipartite graphs with thwart number 3.

241 AMICABLE POLYNOMIALS OVER GF(q)

J.T.B. Beard, Jr.* and Martye Leanne Link Tennessee Technological University

In 1941, E.F. Canaday (the first doctoral student of L. Carlitz) investigated the sum of the divisors of polynomials over $GF(2)$. Since 1974, studies of polynomial analogs over $GF(q), q = p^d, d \geq 1$, of number-theoretic phenomena have yielded numerous results on perfect, unitary perfect, bi-unitary perfect, and non-unitary perfect polynomials. Here, monic polynomials $A, B \in GF[q,x]$ satisfying both $\sigma(A) = B, \sigma(B) = A$ are studied, where $\sigma(A)$ denotes the sum of the distinct monic divisors in $GF[q,x]$ of $A = A(x)$. New constructions of perfect polynomials $A = \sigma(A)$ are obtained, such polynomials being self-amicable. The primary results give constructive techniques yielding amicable pairs $(A, B)_{am}$; i.e., with $A \neq B$; infinite classes of which occur in a variety of ways. Key Words: Finite fields, arithmetic of polynomials.

242 **Perfect Domination and Symmetry in n-Cubes.**

Italo J. Dejter*, University of Puerto Rico
Jaume Pujol, Universitat Autónoma de Barcelona, Spain.

Recently, the existing perfect dominating sets in n-cubes were determined for $n \leq 8$. We determine the automorphism groups of the complements of those perfect dominating sets.

243 **On Pebbling Graphs**

Lior Pachter*, MIT Hunter Snevily, Bill Voxman, University of Idaho

The pebbling number of a graph G, f(G), is the least m such that, however m pebbles are placed on the vertices of G, one can move a pebble to any vertex by a sequence of moves, each move taking two pebbles off one vertex and placing one on an adjacent vertex. We present some results relating graph properties to pebbling. We also find f(G) explicitly for certain classes of graphs and characterize efficient graphs. Finally we present results pertaining to optimal pebbling which is a variant of pebbling.

Keywords: Pebbling, Optimal Pebbling.

Friday March 10, 1995
3:00 p.m.

Properties of the Rotation Graph of Binary Trees 245

Ronald D. Dutton, University of Central Florida, and Rodney O. Rogers*, Embry-Riddle Aeronautical University

The Rotation Graph of Binary Trees R_n has a node for each binary tree on n nodes. Two nodes are adjacent when their corresponding trees differ by a single rotation, a standard operation for maintaining balance in binary trees. It is known that R_n is Hamiltonian and that its diameter is bounded above by 2n-6 for $n \geq 11$. We establish additional bounds on the diameter, show the radius is n-1, and characterize nodes in the center. We also show that R_n is the union of n+2 induced copies of R_{n-1} and prove that R_n is (n-1)-connected.

A Quadratic Lower Bound for Reverse Card Shuffle

Linda Morales Gardner and Hal Sudborough*
Computer Science Program, University of Texas-Dallas

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Reverse Card Shuffle ("Topswaps"), proposed by J.H. Conway (Cambridge) and studied by D. E. Knuth (Stanford), and D. Berman, M. S. Klamkin (Waterloo), considers the maximum number of steps, r(n), over all permutations, before termination in the following:

A deck of cards is numbered 1 to n in random order. Whatever the number on the top card is, count down that many in the deck and turn the whole block over on top of the remaining cards. Then, whatever the number of the (new) top card, count down that many cards in deck and turn this whole block over. Repeat the process until 1 appears on top.

Knuth shows that r(n) is at most th(n+1)-th Fibonacci number and conjectures that r(n) is in fact linear. We show, to the contrary, a quadratic lower bound (and we give exact values for r(n), for n<16).

Keywords: reverse card shuffle, permutations, topswaps, pancake problem, upper and lower bounds.

Cycles in Q-Polynomial Distance Regular Graphs

Heather A. Lewis University of Wisconsin - Madison

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Let G=(X,R) be a Q-polynomial distance rular graph with diameter and valency at least three. We have proved that the girth of G is at most six. This improves upon a result of Brouwer, Cohen, and Neumaier which states that the girth is at most seven. We have also proved that if certain conditions on the parameters of G hold, then every cycle in G can be decomposed into cycles of length at most six. In all cases, every cycle of G can be decomposed into cycles of length at most six and cycles of length either 2D or 2D+1 where D is the diameter of G.

Key Words: Distance Regular, Q-Polynomial

Permutation Graphs and the Petersen Graph 249

John Goldwasser* and C.Q. Zhang West Virginia Univ.

A cubic graph G is called a permutation graph if G has a 2-factor F which is the union of two chordless cycles. It was proved by Ellingham (1984) that every permutation graph with no Petersen minor is edge 3-colorable. We prove that every permutation graph with no Petersen minor contains at least two 4-cycles, thereby strengthening Ellingham's theorem. We also characterize such graphs having precisely two 4-cycles and use this characterization to show that the cubic graph obtained by adding a chord to one cycle of F is edge 3-colorable.

Key words: Permutation graph, Petersen graph, edge 3-coloring

Elementary Constructions of Some Ramanujan Graphs 250

H.L. Janwa* MRI, India and UPR Rio Piedras, PR
O. Moreno, UPR, Rio Piedras, PR

Ramanujan graphs were introduced by Lubotzky, Phillips and Sarnak as examples of asymptotically optimal *expander graphs.* The notions of expanders, magnifiers and concentrators were introduced by Bassalygo, Pinsker, and Pippenger in early 1970s in the study of communication networks (in particular switching circuits). All the constructions of Ramanujan graphs known so far use deep results from Number Theory or from Arithmetic Algebraic Geometry. One such construction is due to Wen-Ching Winnie Li (J. of Num. Th., 1992). Let $q = p^m$ where p is a prime. She constructs $q+1$ regular Ramanujan graphs on q^2 vertices. The Ramanujan property of these graphs is derived from the bounds of Deligne on Kloosterman sums given in Deligne, *Publ. IHES, 1974.* Recently, we have given several constructions of Ramanujan graphs from combinatorics and algebraic coding. In this talk, we will present an elementary construction of Winnie Li's graphs for q even. By modifying our previous techniques, we derive better graphs than those given by Winnie Li for the particular case of q-even.

Key Words: Ramanujan graphs, expanders, combinatorics, codes, dual codes, exponential sums, Kloosterman sums

The Graph of Maximal Intersecting Families of Subsets 251

AARON D. MEYEROWITZ, Florida Atlantic University

A maximal intersecting family of susbsets of an n–set will always consist of exactly half the subsets. We define, for each n, a graph whose nodes are these families with two joined by an edge if one arises from the other by replacing a set by its complement. Thus, for $n = 3$, the graph is a star with 4 nodes; The central node is the family of all subsets with more than one member, while the other three nodes are the families of sets conaining a fixed point.

These graphs have proved useful in answering questions about maximal intersecting families. In this talk we discuss properties of the graphs themselves (radius, center, automorphisms etc.) We mention several conjectures, including one which depends on Chvatal's conjecture.

Key Words: Maximal Intersecting Family, Extremal Set Theory, Chvatal's conjecture

Friday March 10,1995
3:40 p.m.

On The Feedback Vertex Sets In Cubic Graphs 253

Cheng Zhao, The University of Reading, RG6 2AX UK

In this paper a new upper bound for the feedback set of cubic graphs is obtained. This result answers a question posed by E. Speckenmeyer in the field offeedback vertex set and improves several former results due to J.A. Bondy, G. Hopkins and W. Staton. Also this new bound is sharp in some cases.

The Automorphism Groups of 1-factorizations on K_{12} 254

E. Ihrig* & E. Petrie,Mathematics, Arizona State U

The enumeration, by computer, of all 1-factorizations (OF's) of K12 with more than two automorphisms was completed by Seah and Stinson in 1988. They gave the orders of all the automorphism groups, G, of these O-F's, but did not give the group structure of G. Just recently, Garnick and Dinitz enumerated all OF's of K12.

In this paper, we will compliment this work on the OF's of K12 by giving the group structure of all possible G. To this end, we make no use of the computer, but rather apply a number of structure results developed by ourselves and others to aid in the general classification of automorphism groups OF's of K2n.

Here is an example of one of the curious facts which is explained by our results. It is easy to see that the odd part of o(G) must divide 3(5)(11). The results of Seah and Stinson show there are OF's with the odd part of o(G) equal to each of 1, 3, 5, 11, 3(5), 5(11) and 3(5)(11). What happened to 3(11)? Using our theory, one finds that if the odd part of o(G) is 3(11), then G can not be solvable. The classification of simple groups can be used to show that any finite group, H, with the odd part of o(H) equal to 3(11) must be solvable. Hence, this slight numerical anomaly is a reflection of this deep classification theorem.

COVERING BY COMPLEMENTS OF SUBSPACES 255

W. Edwin Clark, Boris Shekhtman*, USF

Let V be an n-dimensional vector space over an algebraically closed field K. Define $\gamma(k, n, K)$ to be the least positive integer t for which there exists a family E_1, E_2, ..., E_t of k-dimensional subspaces of V such that every $(n-k)$-dimensional subspace F of V has at least one complement among the E_i's. Using algebraic geometry we prove that $\gamma(k, n, K) = k(n-k) + 1$.

Friday March 10,1995
4:00 p.m.

On graphs for which every maximal 2-packing is of the same size

G.Gunther, Sir Wilfred Grenfell College, B.Hartnell*, Saint Mary's University, C. Whitehead, Goldsmiths College

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A 2-packing of a graph G is a set P of vertices of the graph such that the distance between any pair of vertices in P is at least 3. We call a graph equipackable if every maximal 2-packing of G is of the same size. The cycles Cm are equipackable when m is less than 9 and also when m=11. In this paper we consider graphs of girth 9 or more and obtain a complete characterization of equipackable graphs of girth at least 11. Partial results are also obtained for girth 9 and 10.

Post-Optimization Methods for the DARPTW in Paratransit

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F. Fiala*, School of Computer Science, Carleton University, Ottawa, Ontario, K1S 5B6, Canada; fiala@scs.carleton.ca and Huashi Wang, Bell-Northern Research Ltd., P.O. Box 3511, Station C, Ottawa, Ontario, K1Y 4H7, Canada; dhwang@bnr.ca

The Dial-A-Ride Problem (DARP) is one of routing vehicles of given capacities from a depot to service customers, each wishing to go from an origin to a destination, while minimizing the total cost of the service. The DARP in paratransit is concened about service time windows (DARPTW). Customer precedence, vehicle capacity and other related paratransit constraints must be also satisfied. We report on our experience in developing and testing heuristic algorithms for DARPTW. Our post-optimization procedures consist of intra-route and inter-route methods. For intra-route post-optimization, we use 2-opt, 3-opt, Or-opt backward and forward interchange schemes to improve the initial routes, which are set up by using insertion heuristics, according to different objective functions. Finally, a comparison of the computational results obtained from six test cases is made among the different intra-route post-optimization procedures and some conclusions are drawn. For inter-route post-optimization, we develop and present a new algorithm based on the insertion heuristics.

Key words: vehicle routing and scheduling, time windows, optimization, heuristics.

Matroids, Generalized Orthogonal Arrays and Ideal Secret Sharing Schemes

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G.R.Blakley, Texas A&M University, USA, and
G.A.Kabatianski, IPPI, Moscow, Russia, and Texas A&M University

Informally speaking, perfect secret sharing schemes (PSSS) give shares (*i.e.* elements from some finite alphabets $S_i, i \in \{1,\ldots,n\}$) to n participants in such a way that allowed coalitions of participants can recover a secret, which is also an element from some finite alphabet S_0, and nonallowed coalitions have no additional *a posteriori* information about the value of the secret. We describe a PSSS as a pair (V,P), where V is an $M\times(n+1)$-matrix V such that the entries of its i-th column belong to S_i. We call its rows sharing rules. And we assume that P is some probability distribution on sharing rules. The relationships among ideal SSS (*i.e.* those PSSS for which all alphabets coincide), matroids and orthogonal arrays have been established recently under the restrictive, but simpler to investigate combinatorially, assumption of uniform distribution of sharing rules (see [1] for references and all details). We prove all these results for the general case, *i.e.* for an arbitrary probability distribution on sharing rules. This leads to a clearer explanation of previous results.

1. D.R.Stinson, An explication of secret sharing schemes, *Designs,Codes and Cryptography*, vol.2, 1992, pp. 357-390.

Friday March 10,1995
4:20 p.m.

On Gaussian Graphs 261

John W. Kennedy, Pace University, New York NY 10038
Brigitte Servatius and Herman Servatius, WPI, Worcester MA 01609

A graph is Gaussian if it is the graph of arcs and self intersections of a closed C-infinity curve in the plane. We will give a recursive construction for all 4-regular Gaussian graphs and conditions under which the Gaussian Property is a graph invariant.

On the Complexity of Stone's Task Assignment Problem 262

Larry Dunning* and Sub Ramakrishnan

Key words: complexity, scheduling, networks, minimum cuts.

The complexity of a task assignment problem for distributed computing systems is shown to be NP-Complete. For set consisting of *m* tasks, the assignment problem is to determine the processor from a set of *n* processors at which each task should reside for its lifetime. Given the problem parameters, one looks for an assignment that optimizes a system-wide objective function. We consider a well known objective function, originally proposed by H. S. Stone in 1977, known as the "throughput metric" to address the complexity issues of the assignment problem. We show that this problem is NP-Complete. Variants of the problem have previously been shown to be NP-Complete by D. Fernandez-Baca in 1989 and by the authors. Although it had been supposed that the general Stone task assignment problem is NP-Complete, Stone later suggested that for n=3 the problem may have a polynomial solution and that case alone is left unresolved.

Some Investigations on Certain Combinatorial Arrays 263

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An array T with s levels is merely a matrix of size $(m \times N)$ with elements from a certain finite set S of s elements (say, o, 1, 2,..., s-1), m is also called the number of constraints and N the number of runs or treatment combinations of T. T is called a binary array if S=2 i.e. T has elements 0 and 1. If we impose some combinatorial structure on these arrays, it will make them quite useful in combinatorics and statistical design of experiments. In this paper we restrict ourselves to binary arrays with the combinatorial structure that in every $(t \times N)$ sub matrix of $T(t \leq m)$, every $(t \times 1)$ column with i 1's in it appears a constant number (say) a μi times (i=0, 1, 2,...t). We call such an array a balanced array (B-array) of strength t. We will derive some inequalities which are necessary for the existence of such arrays, and briefly describe the usefulness of these arrays in statistical design of experiments and combinatorics.

Key words: Constraints, runs, arrays balanced array, strength of an array.

Friday March 10,1995
4:40 p.m.

On the Automorphism Group of a Factor Automaton of a Strongly Connected Automaton

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In algebraic automata theory, the research of automorphism groups of automaton A=(S, Σ ,M) plays an important role in the field of decomposition theory of automaton. In this paper a relation between automorphism group of strongly connected automaton A and that of factor automaton A/H is shown.

Let H be a nontrivial normal subgroup of the automorphism group G of a strongly connected automaton A. It is shown that the automorphism group of the factor automaton A/H is arbitrary with the sole condition that it contains a subgroup isomorphic to G/H. This can be also said that an automaton whose automorphism group contains a subgoup isomorphic to G/H can be extended by an automaton whose automorphism group is isomorphic to H to an automaton whose automorphism group contains a subgroup isomorphic to G. In group theory, a group which contains a subgroup isomorphic to G/H can not be always extended by H to a group which contains a subgroup isomorphic to G. So this result shows one structural difference between group extension and automaton extension.

Key words algebraic automaton, automaton automorphism, automaton decomposition

Mapping Parallel Divide-and-Conquer Algorithms to Multi-Processors With Hypercube or Two-Dimensional Mesh Network Using Binomial Tree as Computational Model

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A classical full binary tree is very often used as a computational model for parallel divide-and-conquer algorithms. This model of computation has two deficiencies: 1) for some problems of input size of n we need n-1 processors which is many and 2) when the leaves of binary tree do the computations, interior nodes are idle.

In order to remove these deficiencies and improve speed-up and efficiency of the parallel computations a new model for computation called Binomial tree is introduced. We present a binomial tree as a Graph Gc = (VC, EC), where VC are a set of nodes (tasks) and EC is a set of edges between two tasks which is called here as communication links.

We consider the multiprocessors with message-passing which each has a network of processors. Each processor has local memory with I/O facilities for sending and receiving a message to other processors. We assume here the interconnection networks are hypercube and two-dimensional mesh networks. We represent these systems each with a Graph GA = (VA, EA), where VA represents processors and EA represents processor-to-processor connections.

We will show the superiority of binomial tree structures over the classical full binary trees. Then we plan to build two algorithms for mapping the binomial trees to these two parallel systems. These two mappings have to minimize total interprocessor communication and response time and also achieve perfect load balancing among the processors. Implementing and analyzing speed-up of the algorithms leads to achieve these goals and choosing a "Good Mapping".