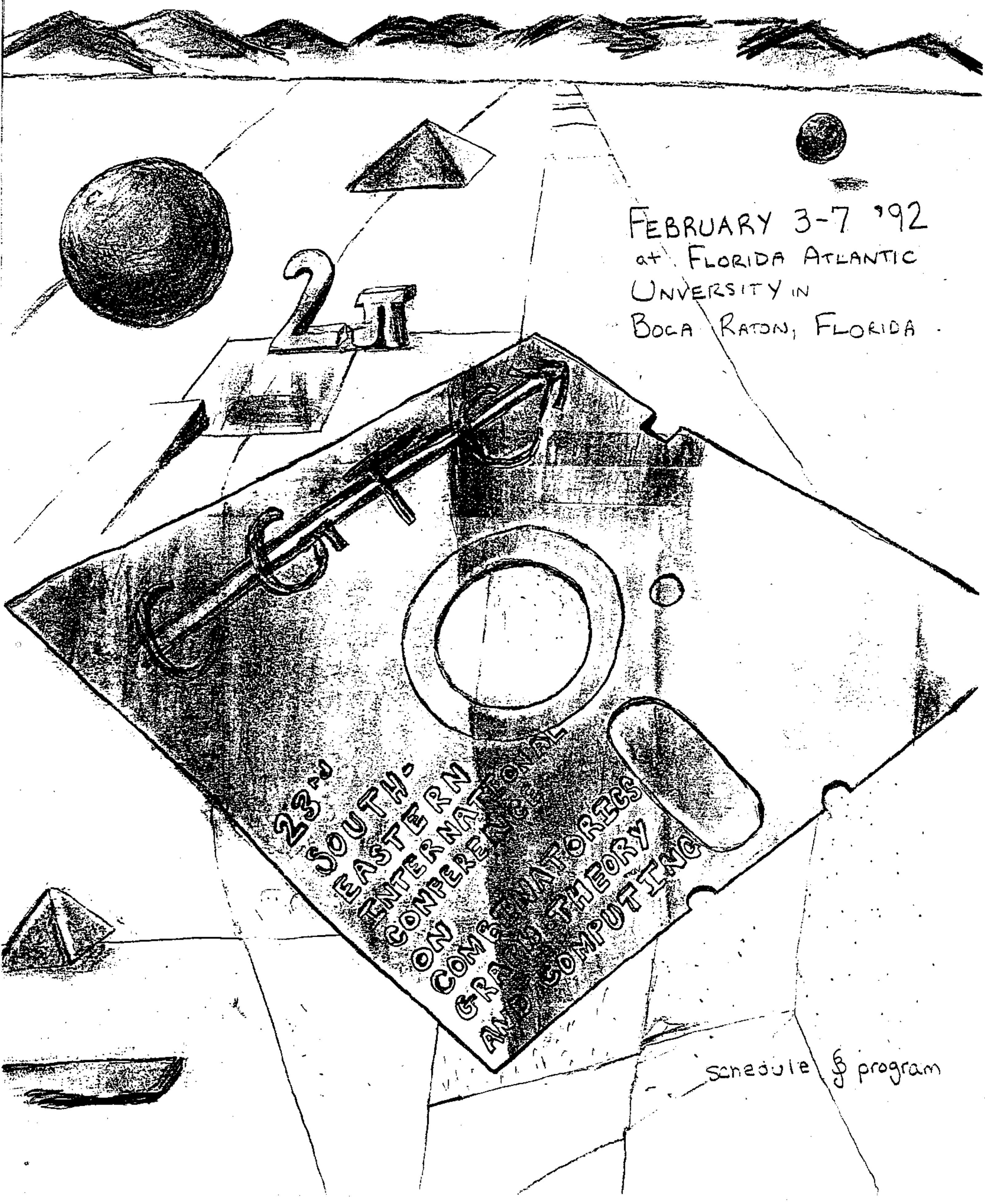

FEBRUARY 3-7 '92
at FLORIDA ATLANTIC
UNVERSITY IN
BOCA RATON, FLORIDA.
2π
23rd
SOUTH-
EASTERN
INTERNATIONAL
CONFERENCE
ON
COMBINATORICS
GRAPH THEORY
AND COMPUTING
schedule & program

INVITED INSTRUCTIONAL LECTURERS

Monday, February 3, 1992

Professor László Babai, Eötvös University and the University of Chicago, will speak on *Vertex-Transitive Graphs, Excluded Minors, and Hyperbolic Geometry,* at 9:30 a.m., and *Transparent Proofs,* at 2:00 p.m.

Tuesday, February 4, 1992

Professor Daniel Gorenstein, Rutgers University and DIMACS, will speak on *Characterizations of the Sporadic Groups,* at 9:30 a.m. and 2:00 p.m.

Professor Heinz-Otto Peitgen, Florida Atlantic University and University of Bremen, will speak on *Cellular Automata, Fractals, and Attractors,* at 3:15 p.m.

Wednesday, February 5, 1992

Professor Kathy O'Hara, University of Iowa, will speak on *Tableaux: A Survey of Combinatorial Problems,* at 9:30 a.m. and 2:00 p.m.

Thursday, February 6, 1992

Professor Paul Erdös,Hungarian Academy of Sciences, will speak on *My Favorite Problems in Combinatorics and Graph Theory*, at 9:30 a.m.

Professor Douglas Stinson, University of Nebraska, will speak on *Secret Sharing Schemes* - Part 1, at 2:00 p.m.

Friday, February 7, 1992

Professor Douglas Stinson, University of Nebraska, will speak on *Secret Sharing Schemes* - Part 2, at 9:30 a.m.

MONDAY, FEBRUARY 3, 1992

REGISTRATION begins at 8:00 A.M. in the downstairs lobby of the University Center, where COFFEE WILL BE SERVED. GCN (left or front) and GCS are the two Halves of the Gold Coast Room. FAU Rooms A and C are reached through the second floor Lounge.

	GCN	GCS	FAU A	FAU C
9:00 AM	OPENING and WELCOME DEANS CARRAHER AND HOLLAND			
9:30	BABAI			
10:30	COFFEE			
10:50	1 BARETY	2 SEN	3 FRICKE	4 BELL
11:10	5 BOLLMAN	6 TIAN	7 DC FISHER	8 SCEPANOVIC
11:30	9 DUFOUR	10 BOALS	11 MOAZZAMI	12 MARSHALL
11:50	13 KULKARNI	14 WOJCIECHOWSKI	15 DUNBAR	16 ROBINSON
12:10 PM	17 GRABLE	18 HARTSFIELD	19 EO HARE	20 GRIMALDI
12:30	LUNCH (On your own -- Cafeteria and Rathskellar open; there are many nearby restaurants)			
2:00	BABAI			
3:00	COFFEE			
3:20	21 ALSPACH	22 BERMAN	23 SCHELP	24 LEISS
3:40	25 GOODMAN	26 CHEN	27 RB GARDNER	28
4:00	29 C WALLIS	30 ROSENFELD	31 CALAHAN	32 STRAYER
4:20	33 ULLMAN	34 GUAN	35 REES	36 STIVAROS
4:40	37 JUDD	38 VARMA	39 SARVATE	40 HUANG
5:00	41 BRIGHAM	42 GYARFAS	43 FRANCEL	44 MCINTYRE
6:00	CONFERENCE RECEPTION in the LOBBY of the new SCIENCE AND ENGINEERING BUILDING.			

There will be Conference transportation back to the motels at 5:30 PM, returning to the reception about 6:15. There will be transportation from the reception back to the motels.

TUESDAY, FEBRUARY 4, 1992

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. FAU Rooms A and C are reached through the second floor Lounge. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	FAU-A	FAU-C
8:40 AM	45 HOLLIDAY	46 SANTORO	47 PETERS	48 LUNDGREN
9:00	49 HEMMETER	50 YODER	51 RAMANNA	52
9:30	GORENSTEIN			
10:30	COFFEE			
10:50	53 MEYEROWITZ	54 CIMIKOWSKI	55 W GU	56 J LIU
11:10	57 PROPP	58 MYRVOLD	59 GEORGES	60 HARTNELL
11:30	61 WE CLARK	62 ERICKSON	63 ELLINGHAM	64 GIMBEL
11:50	65 SNEVILY	66 LOO	67 HEMMINGER	68 CARRINGTON
12:10 PM	69 SAJAL DAS	70 SOIFER	71 ELMALLAH	72 AMIN
12:30	LUNCH BREAK (ON YOUR OWN)			
2:00	GORENSTEIN			
3:00	COFFEE			
3:15	PEITGEN			
4:20	73 BENNETT	74 GEWALI	75 PIAZZA	76 PINTER
4:40	77 MILLS	78 KOOSHEESH	79 GREEN	80 LASKAR
5:00	81 WIEDEMANN	82 ABELLO	83 HARBORTH	84 BAGGA

6:00 CONFERENCE PARTY at the home of JACK FREEMAN : 741 AZALEA ST, (but park on AURELIA ST) 395 - 7921.

CONFERENCE TRANSPORTATION will leave for the motels at 5:30. There will be transportation from the UNIVERSITY CENTER to the party at about 5:30, and from the motels at about 6:10. There will be transportation from the parties back to the motels. As always, we urge car-pooling. It is a pleasant walk to the Freeman home, should you be adventurous.

WEDNESDAY, FEBRUARY 5, 1992

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. FAU Rooms A and C are reached through the second floor Lounge. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	FAU-A	FAU-C
8:40 AM	85 CHOPRA	86 CASTRO	87 WILLIAMS	88
9:00	89 NAIR	90 HJ LAI	91 SLATER	92
9:30	O'HARA			
10:30	COFFEE			
10:50	93 SCHIAPPACASSE	94 OELLERMANN	95 CHINN	96 CHUNG
11:10	97 TAPIA	98 KNISELY	99 TOLMAN	100
11:30	101 ORAL	102 BOLAND	103 JEFFS	104
11:50	105 PAUTASSO	106 CURRAN	107 DR HARE	108

12:15 PM **CONFERENCE PHOTOGRAPH** at the OUTDOOR STAGE. We will lead you from the lobby, if you can't find it on your own, but PLEASE PARTICIPATE!

12:30 LUNCH BREAK (ON YOUR OWN)

	GCN	GCS	FAU-A	FAU-C
2:00	O'HARA			
3:00	COFFEE			
3:20	109 CATER	110 WACHS	111 G-H ZHANG	112 OLARIU
3:40	113 KUBICKA	114 COLLINS	115 SHREVE	116 NEL
4:00	117 ROELANTS	118 WILLIAMSON	119 OPATRNY	120 PENG
4:20	121 RUSKEY	122 ANDERSON	123 WOLDAR	124 LEE
4:40	125 KONG	126 SCHMITT	127 LAZEBNIK	128 MONROE
5:00	129 RIVERA-VEGA	130 TAFT	131 KELMANS	132 HATTINGH

The CONFERENCE BANQUET will be held in the new Cafeteria Building at 7:30 PM (seating at 7:15). There will be a cash bar (beer and wine only) from 6:30 to 7:30. Conference transportation will be available to the motels at 5:30. There will be transportation from the motels to the University at approximately 6:15. There will be transportation back to the motels after the banquet.

THURSDAY, FEBRUARY 6, 1992

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. FAU Rooms A and C are reached through the second floor Lounge. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	FAU-A	FAU-C
8:40 AM	133 PEDERSEN	134 SAKAI	135 NG	136 C ZHAO
9:00	137 GUIDULI	138 LANGLEY	139 JONES	140 KUBALE
9:30	ERDOS			
10:30	COFFEE			
10:50	141 BLAHA	142 SEAGER	143 W CAI	144 CULBERSON
11:10	145 NAYAK	146 WANG	147 WAXMAN	148 COSTA
11:30	149 PEMMARAJU	150 MCRAE	151 PORTER	152 KODA
11:50	153 SUNIL DAS	154 FINIZIO	155 WELLSCH	156 HIND
12:10 PM	157	158	159 KLASA	160 BREWSTER
12:30	LUNCH BREAK (ON YOUR OWN)			
2:00	STINSON			
3:00	COFFEE			
3:20	161 MCCOLM	162 GRINSTEAD	163 ZHAN	164 JOHNSON
3:40	165 ERGINCAN	166 DAWES	167 YU	168 RAJPAL
4:00	169 LIPMAN	170 ZAHID	171 WISEMAN	172 HU
4:20	173 L CAI	174 KNILL	175 TIMAR	176 SCHMALZ
4:40	177 X ZHU	178 PURDY	179 SANDERS	180 KWOK
5:00	181	182 PEI	183 GODDYN	184 STEINER

There will be an informal CONFERENCE PARTY 6:00-7:00 in the Cafeteria Patio area--to be moved indoors if weather dictates. There will be Conference transportation back to the motels at 5:30 PM and back to the party at 6:00. There will be transportation back to the motels after the party.

FRIDAY, FEBRUARY 7, 1992

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:30 A.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. FAU Rooms A and C are reached through the second floor Lounge. There will be book exhibits in Room 232 from 9:00 to 11:30.

	GCN	GCS	FAU-A	FAU-C
8:40 AM	185 EVANS	186 RIVERA-CAMPO	187 MCDOUGAL	188
9:00	189 MARK	190 HILTON	191 NEMHAUSER	192 ZHA
9:30	STINSON			
10:30	COFFEE			
10:50	193 ATKINSON	194 J LIU	195 HOELZEMAN	196 ASHLOCK
11:10	197	198 MUGAVERO	199 DRAPER	200 FAUDREE
11:30	201 KITTAPPA	202 MCKEON	203 RATHI	204 CHRISTOPHER
11:50	205 CHEROWITZO	206 RUBIN THOMAS	207 L BROWM	208 DEAN
12:10 PM	209 BEASLEY	210 SOARES	211	212 GARGANO
12:30	LUNCH (ON YOUR OWN)			
2:00	213 ALAVI	214 LANE	215 MCKEE	216 ADHAR
2:20	217 HEVIA	218 FISCHER	219 KUBICKI	220 BERMUDEZ
2:40	221 ABRHAM	222 FIGLISTER	223 VANDERJAGT	224 LI
3:00	225 SCHWENK	226 LANDMAN	227	228 EDWARDS

There will be transportation back to the motels following the last talks.

THANKS FOR COMING!!

There will be an informal after-dinner SURVIVORS PARTY, at the home of Aaron Meyerowitz and Andrea Schuver, 454 NE Third Street, beginning about 7PM. Tell us if you need transportation.

WE'LL SEE YOU HERE FOR THE TWENTY-FOURTH SOUTHEASTERN INTERNATIONAL CONFERENCE ON COMBINATORICS, GRAPH THEORY AND COMPUTING,

***** FEBRUARY 22-26, 1993 *****

INDEX OF ABSTRACTS

This index contains a list of all authors of abstracts, not just presenters. The list is complete as of 11:00 a.m. on Wednesday, January 29, 1992.

INDEX (continued)

Monday, February 3, 1992
10:50 a.m.

1 Communication complexity analysis on the hypercube topology

Jaime Seguel and Julio Barety *
University of Puerto Rico at Mayaguez, Mayaguez PR 00681

Communication complexity on the hypercube topology is normally measured in terms of the number of nearest neighbor communication exchanges (NNCE) of contiguous elements or *i-cycles*. In this work we propose a matrix representation of NNCEs and use matrix algebra techniques to analyse the communication complexity of a given algorithm. Our framework is especially suited for matrix computations and provides the basic mathematical techniques for an a-priori analysis able to determine which ones, among all possible algorithms, present a minimal communication burden.

2 Containment Digraphs And Overlap Digraphs

M.Sen[1*], B.K.Sanyal[2] and D.B.West[3]
1. Department of Mathematics, North Bengal University,
Darjeeling, West Bengal, India 734430 (present address same as 3)
2. Department of Mathematics,University College,
Raiganj, West Bengal, India 733134
3. Department of Mathematics,University of Illinois, Urbana, IL 61801

Interval containment digraphs are introduced and are shown to be equivalent to digraphs of Ferrers dimension 2. Relation between circular-arc digraphs and circular-arc containment digraphs is obtained. Overlap digraphs are introduced and are characterized in terms of their adjacency matrices. Finally equivalence of unit overlap digraphs and indifference digraphs is established.
Keywords: containment digraphs, overlap digraphs, circular-arc digraphs, indifference digraphs, Ferrers dimension

3 BISHOP DOMINATION AND IRREDUNDANCE ON A CHESS BOARD

Gerd H. Fricke*, Wright State University
and
S.T. Hedetniemi and Charles Wallis, Clemson University

What is the largest number of bishops that can be placed on an n x n chess board, so that any square is attacked by at least one bishop and every bishop is needed to attack all the squares? This is equivalent of finding the upper domination number $\Gamma(B_n)$ for the bishop graph on an n x n board. We will show that $\Gamma(B_n) = 2n - 2$.
For upper irredundance we have $IR(B_n) = \max\{4n-14, 2n-2\}$.

4 Decomposition of K_n into Cycles of Length Less Than or Equal to Fifty

Elaine T. Bell, Auburn University

It is shown that the necessary conditions for the existence of a k-cycle system of order n are sufficient for $k \in \{20, 24, 28, 30, 33, 35, 36, 39, 40, 42, 44, 45, 48\}$, thus settling the problem for all $k \leq 50$.

Monday, February 3, 1992
11:10 a.m.

5 DIGIT-INDEX PERMUTATIONS

Dorothy Bollman, University of Puerto Rico at Mayaguez

Digit-index permutations, which include perfect ripple shuffles, bit reversal, and matrix transposition, play an important role in various applications, especially in fast Fourier transform algorithms. In hypercube implementations the inter-processor communications imposed by these permutations can occupy a considerable portion of the total running time. The principal result of this work is an algorithm which digit-index permutes the contents of the processors of a hypercube with n processors using no more than logn parallel transmissions. Several implementations on the connection machine are discussed.

6 Directed Distance in Digraphs: Centers and Peripheries

Gary Chartrand, Western Michigan University
Garry L. Johns, Saginaw Valley State University
Songlin Tian*, Central Missouri State University

The (directed) distance d(u, v) from a vertex u to a vertex v in a digraph D is the length of a shortest (directed) u-v path in D if such paths exist; otherwise, $d(u, v) = \infty$. The eccentricity of a vertex v in D is the distance from v to a vertex furthest from v. The eccentricity set of D is the set of eccentricities of the vertices of D. Sets of positive integers that are eccentricity sets of digraphs are characterized. The center C(D) of D is the subdigraph induced by those vertices of minimun eccentricity, while the periphery P(D) of D is the subdigraph induced by those vertices of maximum eccentricity. It is shown that for every asymmetric digraph D, there exists a strong asymmetric digraph H_1 with $C(H_1) \cong D$ and a strong asymmetric digraph H_2 with $P(H_2) \cong D$.

Key words: digraph, strong digraph, distance, eccentricity, center, periphery.

7 2-Packing and Domination of Complete Grid Graphs

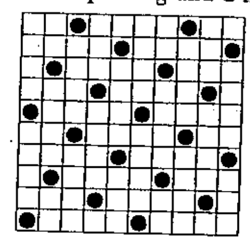
David C. Fisher – University of Colorado at Denver

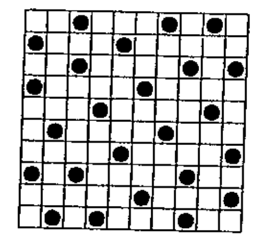
Here are two problems on an $m \times n$ checkerboard:

2-Packing. Place as many checkers as possible on the board so checkers are at least two squares apart (diagonals do not count).

Domination. Place as few checkers as possible on the board so each square has a checker on it or is next to a square with a checker on it (diagonals do not count).

Below are a maximal 2-packing and a minimal domination of a 10×10 board.





For all n, Hare and Hare found maximal 2-packings of $1 \times n$, $2 \times n$ and $3 \times n$ boards, and conjectured (based on their computer studies) maximal 2-packings for all sized boards. Here, these are verified. Jacobson and Kinch found minimal dominations of $1 \times n$, $2 \times n$, $3 \times n$ and $4 \times n$ boards for all n. Here, minimal dominations of $m \times n$ boards for all $m \leq 16$ and for all n are given. Minimality is proved by looking for periodicities in a dynamic programming algorithm of Hare, Hedetniemi and Hare.

Keywords: Domination, 2-packing, Complete Grid Graphs, Dynamic Programming.

8 Nonseparable Graphs With a Given Number of Cycles

G. L. Chia, Dragan Marušič and Ranko Šćepanović*

Department of Mathematics, University of California, Santa Cruz

At several combinatorics conferences (Oberwolfach 1990, Malta 1990, Bremen 1990) G. Ringel conjectured that for every positive integer n other than 2, 4, 5, 8, 9 and 16 there exists a nonseparable graph with n cycles. In fact the conjecture is true even with the restriction to planar and hamiltonian graphs. This recent result and some related problems will be discussed here.

Monday, February 3, 1992
11:30 a.m.

9 A Modular Theorem for Matrices.

Matthieu Dufour, University of Montreal

We shall prove the following theorem.

Let A=(a(i,j)) be a square matrix of order q with integral elements such that, for a certain integer n, G.C.D.(det A, n)=1. Let t be an integer congruent to det A modulo n. Then, there exists an integral matrix B=(b(i,j)) of order q such that

(i) a(i,j) is congruent to b(i,j) modulo n for i and j ranging from 1 to n;

(ii) det B=t.

10 **Arc Traffic Vectors**

Alfred J. Boals*, Gary Chartrand
Western Michigan University
Jamal Nouh
King Fahad University

The traffic vector of a set S of arcs of a digraph D of order n is the sequence

$$TV_D(S) = (\pi_1(S), \pi_2(S), \ldots, \pi_{n-1}(S))$$

where $\pi_i(S)$ is the number of directed paths of length i that contain every arc of S. In this paper we investigate some of the properties of traffic vectors in acyclic digraphs. We also show that the problem of finding a directed tree such that the set of traffic vectors of its edges is a given set of finite sequences is NP-complete.

11 **Tenacity and its Properties in Stability Calculation**

Margaret B. Cozzens — Northeastern University and N.S.F
Dara Moazzami — Barry University
Sam Stueckle — Northeastern University

In this paper, we introduce a new invariant for graphs. It measures how tightly various pieces of a graph hold together. If we think of the graph as modeling a network, the tenacity measures the resistance of the network to disruption of operation after the failure of certain stations or communication links. The tenacity of a graph G, T(G), is defined by T(G) = min{ (|A|+τ(G-A))/ω(G-A)}, where the minimum is taken over all vertex cutsets, A of G, G-A is the graph induced by the vertices of V-A, τ(G-A) is the number of vertices in the largest component of the graph induced by G-A and ω(G-A) is the number of components of G-A. A connected graph G is called T-tenacious if |A|+τ(G-A) ≥ T ω(G-A) holds for any subset A of vertices of G with ω(G-A) >1. If G is not complete, then there is a largest T such that G is T-tenacious; this T is the tenacity of G. On the other hand, a complete graph contains no vertex-cutset and so it is T-tenacious for every T. Accordingly, we define $T(K_n) = \infty$ for every n (n ≥ 1). A T-set (with respect to some prescribed graph G) means a set A⊆V(G) for which

T(G)=[|A|+τ(G-A)]/ω(G-A).

We will compare, integrity, connectivity, binding number, toughness and tenacity for several classes of graphs. The results suggest that T is a most suitable measure of stability in that it is best able to distinguish between graphs that intuitively should have different levels of stability.

12 Partitioning complete graphs into cycles of even length

Susan Marshall
Simon Fraser University

In this talk we consider the problem of partitioning the edges of the complete graph into cycles of a fixed length. This is a problem on which there has been much research, and for which there are still many cases left unsolved. We give here a decomposition of $K_{2n} - I$, the complete graph on $2n$ vertices with a one-factor removed, into cycles of fixed even length $2m$, for cases when n is even and $3m/2 \leq n < 2m$.

13 Hilbert Polynomial of a Certain Ladder-Determinantal Ideal

Devadatta M. Kulkarni
Department of Mathematics, Oakland University, Rochester, MI 48309

A ladder-shaped array is a subset of a rectangular array which looks like a Ferrers diagram corresponding to a partition of a positive integer. The determinantal ideals from a ladder-shaped array of indeterminates play an interesting role in the study of Schubert subvarieties of a flag manifold. In this talk, we give an explicit expression for the hilbert polynomial of the ideal generated by the two by two minors of a ladder-shaped array of indeterminates in the corresponding polynomial ring. Counting the number of paths in the ladder-shaped array having fixed number of "turning points" above the path corresponding to the ladder is a central idea of the combinatorial construction of the hilbert polynomial. We also make some relevant remarks about these counting expressions and related combinatorics. This gives a constructive proof of the hilbertianness of the ideal generated by the two by two minors of a ladder-shaped array of indeterminates. The hilbertianness of the determinantal ideals from the ladder-shaped array is proved by Abhyankar and Kulkarni in "On Hilbertian Ideals", Linear Algebra and Its Applications, 116, 53–76 (1989).

Keywords: hilbert polynomial of determinantal ideals, ladder-shaped array, counting paths above a given path, "turning points" of a path.

14 On a problem of Häggkvist

Jerzy Wojciechowski, West Virginia University

Let k be a positive integer and let G be a simple digraph with n vertices such that the outdegree of every vertex of G is at least n/k. Häggkvist conjectured that there must exist a directed cycle of length k or less in G. We give an easy proof that the conclusion is true provided that the outdegree of each vertex is at least $\alpha_k n$, where $\alpha_3 = \frac{3-\sqrt{5}}{2}$ and $\lim_{k\to\infty} \alpha_k k = \ln 4$.

15 Properties of Minus Domination in Graphs

Jean Dunbar*, Converse College, Spartanburg, SC
Stephen Hedetniemi, Alice McRae, Clemson University, Clemson, SC
Michael Henning, University of Natal, Pietermaritzburg, South Africa

Suppose a three-valued functiin on the vertices of a graph G=(V,E) is of the form f: V -> {-1,0,1}. Such a function is said to be a minus dominating function if for every vertex u in V, f(N[u]) >= 1, where N[u] denotes the closed neighborhood of vertex u. The weight of a minus dominating function equals the sum of the values f(u) for all u in V. The minus domination number of a graph G equals the minimum weight of a minus dominating function of G. Properties and complexity aspects of the minus domination number of a graph are discussed.

Keywords: domination, minus domination

16 Hamilton Cycles in Random r-Regular Graphs and Digraphs

R. W. Robinson*, Univ. of Georgia and N. C. Wormald, Univ. of Melbourne

For fixed $r \geq 3$ and rn even, choose a labeled r-regular graph on n nodes uniformly at random and let H denote the number of Hamilton cycles it contains. We show that as $n \to \infty$, $Pr(H \geq 1) \to 1$. For $r = 3$ this is done by analyzing the mean and variance of H. The conditional expectation of H for fixed numbers of short odd cycles is determined, and the variance of these expectations is found to account for almost all of the variance of H. For $r > 3$ we induct on r by removing 1-factors. This requires an analysis of the mean and variance of M, the number of 1-factors in a random r-regular graph. The variance of M is almost all accounted for by the effect of short cycles (even as well as odd length). As a corollary we prove that for fixed $r \geq 3$, almost all r-regular graphs on an even number of nodes are r-edge-colorable. Similar results are obtained for r-regular bipartite graphs and r-regular digraphs.

Key words: random regular graphs, Hamilton cycles, 1-factors, asymptotic analysis, edge-colorings

Monday, February 3, 1992
12:10 p.m.

17 Almost Every Finite Linear Space is Rigid

David A Grable, Auburn University

A linear space is called rigid if it admits only the identity permutation as an automorphism (i.e. it has a trivial automorphism group). We prove that the proportion of finite linear spaces which are rigid goes to 1 as the number of points goes to infinity. This result, though perhaps not surprising, has been conjectured for many years by Jean Doyen among others. The proof techniques generalize to give similar results about Steiner triple systems and partial t-designs as well.

18 FORBIDDEN MINORS FOR OUTER-PROJECTIVE-PLANAR GRAPHS

Dan Archdeacon, University of Vermont
Nora Hartsfield, Western Washington University
C. H. C. Little, Massey University
Bojan Mohar, University of Ljubljiana

A graph G is <u>outer-projective-planar</u> if G can be embedded in the projective plane so that all vertices appear on one face of the embedding. We present a set of 32 graphs with the property that any non-outer-projective-planar graph contains one of these graphs as a minor.

19 Fractional Domination of $P_m \times P_n$

L. S. Stewart and E. O. Hare*
Clemson University

A real-valued function $g : V(G) \to [0,1]$ is a dominating function if for every $v \in V(G)$, $g(N[v]) \geq 1$. A dominating function is minimal if for every $v \in V(G)$ with $g(v) > 0$, there exists a vertex $u \in N[v]$ such that $g(N[u]) = 1$. Then the fractional domination number of G, denoted $\gamma_f(G)$, is the minimum $\sum g(v)$, over all $v \in V(G)$, such that g is a minimal dominating function. This work is a continuation of results presented at this conference in 1990.

key words: domination, grid graph

20 INTEGER-DISTANCE GRAPHS

Ralph P. Grimaldi

Rose-Hulman Institute of Technology

This paper presents an extended discussion on a family of graphs and their complements. The discussion includes the derivations of several numeric parameters of the graphs, such as the sizes of their edge sets, their independence numbers, and their clique numbers. Furthermore, it is shown that these complements are unit interval graphs – hence chordal, and perfect, while the original family comprises graphs that are comparability graphs of semiorders and these are perfect though not generally chordal.

Monday, February 3, 1992
3:20 p.m.

21 1/2-transitive graphs

Brian Alspach*, Simon Fraser University
Dragan Marušič, University of Ljubljana
Lewis Nowitz, New York

A graph is said to be 1/2-transitive if its automorphism group acts transitively on vertices and edges, but not on arcs. We construct an infinite family of such graphs all of which have degree 4.

22 Computing Upper Bounds for the Independence Number of a Graph

Kenneth A. Berman* & Jerome L. Paul, University of Cincinnati

Let G be a graph with vertex set $V = \{v_1,...,v_n\}$, where each vertex v_i has been assigned a positive real weight w_i^2, and set $\mathbf{w} = (w_1,...w_n)$. For $U \subseteq V$, the *weight of* U is the sum of the weights over all the vertices in U. Let $\mathbf{w}_U = (w_1^{(U)},...,w_n^{(U)})$, where $w_i^{(U)} = w_i$ if $v_i \in U$ and $w_i^{(U)} = 0$, otherwise. Now consider any $k \times n$ real matrix $\mathbb{B}$. We say that a subset U of V is $\mathbb{B}$-*constrained* if $\mathbb{B}\mathbf{w}_U^t = 0$, and define the $(\mathbb{B},\mathbf{w})$-*independence number*, $\alpha_{\mathbb{B},\mathbf{w}}$, to be the maximum weight of a $\mathbb{B}$-constrained independent set of vertices (note that $\alpha_{\mathbb{B},\mathbf{w}}$ becomes the weighted independence number when $\mathbb{B} = 0$). For $\mathbb{A} = (a_{ij})$ any symmetric real $n \times n$ matrix such that $a_{ij} = 0$ whenever v_i and v_j are not joined with an edge of G, let λ be any real number no larger than the smallest eigenvalue of $\mathbb{A}$. Suppose $\mathbf{c} = (c_1,...,c_n)$, $\mathbf{d} = (d_1,...,d_k)$ satisfy the linear equations: $(\mathbb{A} - \lambda\mathbb{I})\mathbf{c}^t + \mathbb{B}^t\mathbf{d}^t = \mathbf{w}^t$, and $\mathbb{B}\mathbf{c}^t = 0$. In this paper, we show that if U is any independent set of vertices such that $\mathbf{d}\mathbb{B}\mathbf{w}_U^t \leq 0$, then U has weight at most $-\lambda\mathbf{c}\mathbf{w}^t$. In particular, $\alpha_{\mathbb{B},\mathbf{w}} \leq -\lambda\mathbf{c}\mathbf{w}^t$. The latter result has applications to electrical networks and random walks.

key words: graph, independence number, eigenvalue

23 Directed Steiner Triples and a Communication Problem

A. Gyárfás, R.H. Schelp*, Hungarian Academy of Sciences, Memphis State University.

Motivated by a communication problem, a certain factorization of complete bipartite graphs is considered. The solution is obtained by proving that oriented complete graphs have balanced decompositions into edge disjoint transitive triples.

Keywords: Factorization, Steiner triples, communication networks.

24 OPTIMIZING SHORTEST PATH CALCULATIONS

C. L. Elford, E. L. Leiss*, and H.-W. Zhou
Allied Geophysical Laboratories
University of Houston
Houston Texas 77204-4231

Ray tracing occurs in many applications, with tomography in general an important representative. Specifically, in geotomography the portion of the image to be ray-traced is partitioned into rectangular cells of equal size. This discretization reduces the problem of ray-tracing to one of finding shortest paths between discrete points. This talk starts by sketching a history of network shortest path optimization, followed by an example of the underlying technique. Memory requirements are discussed and a memory-saving modification to the method based on regular node distribution is presented. Examples are given to demonstrate the use of this technique, together with the requisite formulae. The memory requirements of the new method are derived and compared with those of the original. Comments on implementation and run time performance of the modified approach conclude the talk.

Keywords: Ray-tracing, finding shortest paths.

25 Edge-Minimal Graphs with Given Automorphism Group

Albert J. Goodman, University of Chicago

A graph X is said to *represent* the group G if the full automorphism group of X is isomorphic to G. It is known that (with three exceptions) every finite group G can be represented by a graph with at most $2|G|$ vertices (Babai, 1974). This talk reports joint work with Lászlό Babai which answers the corresponding question for edges: Every finite group G can be represented by a graph with at most $c|G|$ edges (for some constant $c < 300$). This is despite the fact that an unbounded number of edge-orbits is required (Goodman, 1990), as conjectured by Babai (1981). In contrast to most prior results on graphs representing arbitrary groups, the edge-minimal result delves deeply into the structure of the groups, using two inductive constructions and the following result: Every finite group is generated by at most three subdirectly reducible subgroups and six other elements. (A group is *subdirectly reducible* if it has non-trivial normal subgroups with trivial intersection.) The proof of this involves separate arguments for p-groups and for solvable groups, together with a theorem of Aschbacher and Guralnick (1982) which is a consequence of the classification of finite simple groups.

Key words: automorphism group, graph, edge-minimal, finite, subdirectly reducible group

26 Common Moment Sets of Graphs and Their Complements

Hang Chen, Western Michigan University

The k^{th} moment of a graph G is the sum of the k^{th} powers of its degrees, that is, $M_k(G) = \sum_{i=1}^{n} d_i^k$. A graph G and its complement $\overline{G}$ are said to share the k^{th} moment if $M_k(G) = M_k(\overline{G})$. We define the common moment set of G and $\overline{G}$ as $P = \{k \in \mathrm{N} \mid M_k(G) = M_k(\overline{G})\}$. When G and $\overline{G}$ have distinct degree sequences, P is a finite subset of N. In fact, $P = \{1, 2, \ldots, 2p\} \cup A$ where $p \geq 0$ and $2p+1, 2p+2 \notin A$. For every even integer $2p$, we explicitly construct a graph which shares the first $2p$ common moments with its complement, and furthermore, we seek the smallest such graph. We characterize the sets A that can occur and thus we also characterize sets P which can be the common moment set of G and $\overline{G}$. These results closely parallel the analogous problems for tournaments analyzed in the paper " Tournaments That Share Several Common Moments with Their Complements " by Chen, Erdős, and Schwenk.

Key words: common moment, complement, degree sequence

27 TRANSROTATIONAL DIRECTED TRIPLE SYSTEMS

Robert B. Gardner
Department of Mathematics
Louisiana State University in Shreveport
Shreveport, Louisiana 71115

A directed triple system of order v, denoted $DTS(v)$, is said to be k-transrotational if it admits an automorphism consisting of a fixed point, a transposition and k cycles of length $\frac{v-3}{k}$. In this talk, we give necessary and sufficient conditions for the existence of k-transrotational $DTS(v)$s for several values of k.

Keywords: directed triple system, automorphisms of designs, transrotational automorphisms

28 Calculations of some shapes using Fourier Fractals

S. Klasa, Concordia University, MONTR$AL

Last year at the Baton Rouge Conference, we introduced the concept of Fourier Fractals, together with an indication of their use in Computer Vision and Pattern Recognition.
This time we present an algebraic study of shapes in the plane and give a Fourier Fractal characterization (necessary and sufficient conditions) of some simple shapes (such as regular polygons, etc.) and their positions in the plane.

Key words: fractal, Fourier Fractals, shape, polygon, Pattern Recognition, Computer Vision

Monday, February 3, 1992
4:00 p.m.

29 *THE PARTITIONED GRAPH ISOMORPHISM PROBLEM*

Charles Wallis, Dept. of Mathematical Sciences, Clemson University

Can the edge set of a given tree T=(V,E) be partitioned into two sets ($E=E_1 \cup E_2$) such that the graphs $<E_1>$ and $<E_2>$ induced by the two edge sets are isomorphic? The above problem has long been known to be NP-complete. We therefore ask for which sub-families of trees the problem can easily be solved. Two sub-families of trees, Fibonacci trees and spiders, are discussed, and the above problem is solved for these types of trees.

keywords: *graph isomorphism, Fibonacci tree, spider*

30 **Defending the honor of the icosahedron**

Moshe Rosenfeld, Pacific Lutheran University.

The regular icosahedron, one of the five regular 3-polytopes, is definitely an attractive object. The M.A.A. chose the icosahedron as its logo, proudly rendered on the cover of all its magazines. The East Germans marked the bicentennial of Euler's death with a rendition of the regular icosahedron. Unfortunately, these renditions did a mathematical injustice to the icosahedron (while the M.A.A. did correct its rendition of the icosahedron, the East Germans will not be able to do so in the foreseeable future).
In this note we'll attempt to restore some of the "lost honor" of the icosahedron. The graph of the icosahedron has diameter 3. If we construct a graph by connecting by an edge vertices of distance 2, we obtain another copy of the icosahedron. There are 6 disjoint pairs of vertices at distance. Thus K12 is decomposable into two edge disjoint copies of an icosahedron plus a 1-factor. A similar "geometric" decomposition can be obtained from the 12 vertices of the regular icosahedron. In this note we'll show that this decomposition pattern applies to infinitely many graphs.

31 *BICYCLIC STEINER TRIPLE SYSTEMS*

Rebecca S. Calahan*
Middle Tennessee State University

and

Robert B. Gardner
Louisiana State University in Shreveport

A Steiner triple system admitting an automorphism whose disjoint cyclic decomposition consists of two cycles is said to be bicyclic. Necessary and sufficient conditions are given for the existence of bicyclic Steiner triple systems.

32 Transforming Upper Bounds on Reliability into Lower Bounds

Heidi J. Strayer(*) and Charles J. Colbourn
University of Waterloo — University of Waterloo
hjs@caddac.waterloo.edu — cjcolbourn@watmath.waterloo.edu

Exact determination of the two-terminal reliability of a network is a #P-complete problem. Consequently, much research has concentrated on finding efficient bounding strategies. Computational results tell us that in general, existing upper bound techniques tend to produce tighter bounds than their lower bound counterparts. Thus, we develop a scheme employing topological dualization that allows the use of upper bound techniques to compute lower bounds for the two-terminal reliability of 2-connected networks. The results we obtain via this approach are competitive with other existing lower bound techniques.

Monday, February 3, 1992
4:20 p.m.

33 Fractional Isomorphism of Graphs

Motakuri V. Ramana and Edward R. Scheinerman, Johns Hopkins University; Daniel Ullman*, George Washington University.

Graphs G and H, with adjacency matrices A and B respectively, are called fractionally isomorphic provided there exists a doubly stochastic matrix S so that AS=SB. A readily computable condition, based on partitioning the vertex set according to vertex degrees, is provided characterizing when two graphs are fractionally isomorphic. The connection with other equivalence relations on graphs is explored.

34 Maximum Squareless Subgraphs of Hypercube of Dimension 5 and 6

R. Emamy-K, P. Guan*, P. Rivera Dept. of Math. UPR Rio Piedras

We prove that there are exactly 2 type of maximum squareless subgraphs of hypercube of dimension 5, each type contains 4 non isomorphic graphs with 56 edges. The maximum squareless subgraph of hypercube of dimension 6 is 132.

35 COMPLETING THE SPECTRUM FOR RESOLVABLE GDDS WITH BLOCK SIZE THREE

ROLF S. REES, MEMORIAL UNIVERSITY OF NEWFOUNDLAND

ABSTRACT. We construct resolvable group-divisible designs with block size three and type g^6 for all $g \equiv 2$ or 10 (mod 12), $g \geq 22$, thus completing the spectrum for these designs.

Key words: design, resolvable, group-divisible.

36 The Residual Reliability of the complete multipartite networks.

Constantine Stivaros, FDU, Madison, NJ 07940

The regular and almost regular complete multipartite networks are known to optimize many of the reliability criteria. We investigate whether these structures optimize this reliability measure. We show their performance in various aspects of this optimization problem along with their optimality in the all-equal-failures case. In the general case we present a counterexample to their uniform optimality on the Kn,n graph.

Keywords: network reliability, multipartite graphs, combinatorial optimization, regular graphs

Monday, February 3, 1992
4:40 p.m.

37 Domination and Perpetual Evasiveness

Stephen Judd, Siemens Corporate Research

A generic NP-complete graph problem is described. The calculation of a certain predicate on the graph is shown to be both necessary and sufficient to solve the problem and hence the calculation must be embedded in every algorithm solving NP problems. This observation gives rise to a metric on the difficulty of solving *single instances* of the problem. The combinatorics appear to yield an interesting phase transition in this metric when the graphs are generated at random in a ''2-dimensional'' extension. The metric is sensitive to 2 parameters governing the way graphs are generated: p, the density of edges in the graph, and K, related to the number of points in the graph. The metric seems to be finite in part of the (p,K)-space and infinite in the rest. If true, this phenomenon would demonstrate that NP-complete problems are truly monolithic and can easily exhibit strong intrinsic coupling of their variables throughout the entire instance.

38 **Decomposition of Complete Symmetric Digraphs into Orientations of a Cycle of Length k: Odd and Even case**

Badri N. Varma
Department of Mathematics
University of Wisconsin Center-Fox Valley

Sufficient conditions are found for a self-converse orientation of a cycle of length k, k odd or even integer, to divide DK_n, a symmetric complete digraph, for all $n \geq k$ and $n \equiv 0$ or $1 \pmod k$. Furthermore, these results are used to show that the necessary conditions for such a decomposition are also sufficient if $k = 6$ or $k = 7$.

39 **Some Necessary Existence Conditions, Constructions, and a List of Partially Balanced Ternary Designs**

Jaideep Mirchandani, University of North Carolina, Chapel Hill and Dinesh G. Sarvate*, College of Charleston.

We will restrict our attention to partially balanced ternary designs with two association classes; any two points in a design can either be first or second associates of each other. A partially balanced ternary design PBTD(m, V; B; ρ_1, ρ_2, R; K, Λ_1, Λ_2) is a collection of V points, say {1,2, ..., V} in B blocks each of size K such that each point occurs $R=\rho_1+2\rho_2$ times in the design; singly in ρ_1 blocks and doubly in ρ_2 blocks. If any two distinct points are first associates, they occur together in Λ_1 blocks and if they are second associates, they occur together in Λ_2 blocks. The first parameter m gives the number of first associates of a point in the design. Some new constructions and necessary conditions for the existence of partially balanced ternary designs are presented. A complete list of partially balanced ternary designs with $R \leq 5$, $V \leq 5$ and any Λ_1 and Λ_2 and a list with $\Lambda_1 + \Lambda_2 \leq R \leq 7$ and $V \leq 7$ are given. We have reported the existence or non-existence of each entry in these lists.

Key words: Balanced ternary designs, Partially balanced ternary designs.

40 Optimal Fault-Tolerant Ring Networks

Ke Huang* and Jie Wu
Department of Computer Science and Engineering
Florida Atlantic University

Adding fault tolerance features to a distributed computer system has been an active research topic. Normally a distributed computer system is represented as a simple undirected graph G, where nodes represent processing elements and edges present the links between processing elements. A fault-tolerant extension of a graph G is to design a new graph G' by adding redundant nodes and edges to G, such that when some of the nodes in G' fail there is a subgraph isomorphic to G. In this paper, we propose an optimal fault-tolerant extension of ring network which can tolerate one node failure, where optimal means the least node and link redundancy. This extension has the proporties of small diameter and perfect symmetry. Other features of this extension are also explored.

Index Terms:
Distributed computer systems, fault-tolerant interconnection networks

Monday, February 3, 1992
5:00 p.m.

On Pascal Graphs

41 Robert C. Brigham*, Ronald D. Dutton, Narsingh Deo, University of Central Florida

The Pascal graph PG(n) of order n is formed from the symmetric adjacency matrix whose lower triangular part is composed of the first n-1 rows of Pascal's triangle with entries taken modulo 2. A parity condition of binomial coefficients is used to compute several properties of Pascal graphs including the values of the node independence number, node clique cover number, chromatic number, clique number, minimum degree, and edge connectivity number.

42 Minimal Path-pairable Graphs

A. Gyarfas Memphis State University

A graph of order 2n is called path-pairable if for any ordering v_1,v_2,...,v_2n of its vertices there exist n edge disjoint paths from v_2i-1 to v_2i, for i=1,2,...,n. A graph with at least 2k vertices is called k-path pairable if for any k pairs of distinct vertices there are k edge disjoint paths between the pairs. The talk addresses the following extremal problem: what is the minimum number of edges in a path pairable (or k-path pairable) graph of given maximum degree? The results are from joint researh with Faudree, Lehel and Schelp.

43 A Comparison of n-ary design definitions

Margaret Ann Francel*, The Citadel
and Dinesh G. Sarvate, College of Charleston

A generalization of (binary) balanced incomplete block designs is to allow a treatment to occur in a block more than once, that is, instead of having blocks of the design as sets, allow multisets as blocks. Such a generalization is referred to as n-ary designs. There are at least three such generalizations studied in the literature. The present note studies the relationship between these three definitions. We also give some results for the special case when n is 3.

Key words: Balanced ternary designs, Balanced n-ary designs.

44 An Efficient Implementation Of Huffman Decode Tables

D. R. McIntyre* and F. Wolff
Cleveland State University & Case Western Reserve University

Suppose a finite file, F, consists of a set of elements of equal length and that the occurrence frequencies of each element in F are known in advance. Then probably the best known technique for data compression is the static Huffman compression algorithm which produces a minimal prefix code for the elements of F in the sense that no other prefix code compresses F more. However, because Huffman codes are dependent on the element frequencies, decode information must be stored along with the compressed file. We examine the use of canonical trees to more efficiently store the decode information in the compressed file.

Key words: data, compression, prefix codes, Huffman.

Tuesday, February 4, 1992
8:40 a.m.

45 Some Nonexistence Results for Affine α-Resolvable Designs

Robert L. Holliday, Lake Forest College (Illinois)

A block design is affine α-resolvable if: i) the blocks can be partitioned into n sets of m blocks each such that each set contains every point exactly α times; and ii) any two blocks belonging to the same set have x points in common and any two blocks in different sets have y points in common. Relationships on the parameters of such designs are derived using elementary counting arguments, a determinant argument on the incidence matrix, and the parameters of an auxiliary design constructed by taking the dual of a set of blocks in the original design. These relationships lead to straightforward proofs of the nonexistence of affine α-resolvable designs with smaller intersection number x=1, 2, 3, 4, 5, 6. Moreover, if attention is restricted to designs with $k \leq v/2$, then the nonexistence of affine 3-resolvable designs is established.

key words: quasisymmetric block design; strongly regular graph; affine α-resolvable design

46 YO-YO: A DISTRIBUTED ELECTION ALGORITHM FOR ARBITRARY GRAPHS

NICOLA SANTORO
School of Computer Science, Carleton University, Ottawa, Canada

The aim of this talk is to formally describe the "Yo-Yo" algorithm for leader election in networks of arbitrary topology. This algorithm has a very simple structure: after an initialization phase which transforms the graph into a (logical) dag, the algorithm proceeds through a sequence of downward (the "Yo-") and upward (the "-Yo") sweeps of the dag; during each sweep the dag is transformed into a different dag with fewer source nodes; the process terminates when there is only one source node which becomes the leader. The communication complexity (i.e., the total number of messages) of the Yo-Yo algorithm is still not known despite the fact that the algorithm is almost a decade old. It has been conjectured that Yo-Yo has a O(e+n log n) complexity, where e and n denote the number of arcs and nodes of the network, respectively. An O(e log n) upper-bound will be presented.

47 A Category of Transition Diagrams Over Time for Typed Timed Automata*

James F. Peters Computer Science, Univ. of Arkansas, Fayetteville

This paper presents a new model of interpretation for concurrent typed timed automata as a category of transition graphs over time. The intention is to axiomatize concurrent automaton graphs in terms of functions and their compositions. A set of finite, directed, labeled transition graphs representing concurrent automata is given. A Concurrent automaton is an extension of the DeJong-Muller predicate automaton, which is typed in the tradition of Martin-Lof and provides a provably correct specification for the behavior of communicating processes. An application of this model in terms of a controller for a real-time system is given.

Keywords: Category Theory, Transition Diagrams, Automata, Type Theory.

48 The 2-Step Number of a Graph

Kathryn F. Jones and J. Richard Lundgren*,
University of Colorado at Denver
Suzanne Seager, Mount Saint Vincent University

The 2-step graph $S_2(G)$ of a graph $G = (V, E)$ is the graph with vertices V in which distinct vertices are joined by an edge if and only if in G there is a path of length two between them. These graphs are also known as neighborhood graphs and have been characterized by Acharya and Vartak. Not all graphs are 2-step graphs, but every graph is the induced subgraph of a 2-step graph. The 2-step number $s(G)$ of a graph G is the minimum number k of vertices such that G is an induced subgraph of the 2-step graph of a graph on $|V| + k$ vertices. This number, which was introduced in a slightly different form by Boland, Brigham, and Dutton, is similar to competition numbers and niche numbers, concepts which have received considerable attention. In fact, every 2-step graph is a competition graph. In this talk we will discuss the 2-step numbers for several classes of graphs including paths, cycles, caterpillars, triangle-free graphs, and interval graphs. We also will give some bounds for arbitrary graphs and discuss open problems.

49 GRAPHS RELATED TO THE DUAL POLAR GRAPHS

Joe Hemmeter, University of Delaware

Suppose that V is a finite–dimensional vector space over a finite field, equipped with a quadratic, Hermitian or symplectic form. The maximal subspaces on which the form vanishes form the vertex set of a very nice graph, called the dual polar graph. Such a graph has been known for some time to be distance–regular. Recently, several distance–regular graphs related to the dual polar graphs have been discovered. We present a survey of recent work.

Key words: Distance-regular graphs, dual polar graphs

50 A new solution to the word problem for the braid groups

Margaret Yoder* and John Pedersen
University of South Florida, Tampa, FL 33620-5700
mty@gauss.math.usf.edu

We exhibit a new and simpler algorithm for solving the word problem in the braid groups B_n. The normal forms obtained are the same as those obtained through Garside's method, but the process is much more transparent. Our algorithm uses a complete set of reductions, which were generated using rewrite completion under the Knuth-Bendix ordering.
Key Words: braid groups, Knuth-Bendix, rewrite completion

51 A Transition Graph Model for a Typed Knowledgebase*

S. Ramanna CIS Department, Idaho State University, Pocatello

This paper presents a transition graph model for a typed knowledgebase (tKB). In this model, a knowledgebase is realized as a logic program, which is specified with a finite, directed, labeled transition graph. Such a transition graph represents a Peters-DeJong-Muller typed timed input/output automaton (tTAi/o) useful in designing a tKB. In addition to the typing of states in a tTAi/o, the input and output to a tTAi/o are formulas prescribing knowledge types. The typing of i/o formulas for a tTAi/o partitions a tKB. The practical benefit of this approach to the design of knowledgebases is that they are more efficient to use by a knowledgebased controller for a real-time system.

Keywords: Transition Graph, Artificial Intelligence, Knowledgebase, Automata

Tuesday, February 4, 1992
10:50 a.m.

53 Sequence Reconstruction

Aaron Meyerowitz, Florida Atlantic University

The sequences **abbac** and **baabc** are 2-*equivalent* in that they have the same $\binom{5}{2}$ subsequences of length 2: **aa ab ab ac ac ba ba bb bc bc**. For which k and n are there distinct n-sequences which are k-equivalent? This talk will sketch results of a forthcoming paper of the author and others as well as new results and many open questions. An extended version of this talk is available.

54 Graph Planarization and Skewness

Robert J. Cimikowski
Computer Science Department
Montana State University, U.S.A.

Keywords: skewness, maximum planar subgraph, planarity, NP-complete.

The problem of finding a *maximum planar subgraph* of a nonplanar graph is computationally intractable. The graph-theoretic version of this problem is that of determining the *skewness*, i.e., minimum number of edges to remove to make the graph planar. Several heuristics for the problem have been developed but none have guaranteed performance better than the size of a spanning tree, or 1/3 the optimum number of edges. We examine known results about skewness for special families of graphs, including complete graphs and the n-dimensional hypercube Q_n. We also discuss the two major heuristics for the problem, one based on path embedding and the other based on the P,Q-tree data structure, and present new ideas for obtaining approximation algorithms with improved performance bounds. Finally, we explore the relationship between the skewness and *crossing number* of a graph.

55 On Minimal Embedding of Two Graphs as Center and Periphery

Weizhen Gu
Dept. of Math., Southwest Texas State University

For any graphs G_1 and G_2, and an integer d $(2 \le d \le r(G_2))$, define $\beta(G_1, G_2)$ (or $\beta(G_1, G_2; d)$) to be the minimum number of vertices of the graph H which contains G_1 as its center and G_2 as its periphery (and $dia(H) = d$, respcetively). In this paper, the values of $\beta(G_1, G_2)$ and the upper bounds for $\beta(G_1, G_2; d)$ are obtained when G_2 is not 3-self-centered.

56 Dominating Property of i - centers in P_t - free Graphs

Jiping Liu* and Qinglin Yu
Simon Fraser University, B.C., Canada

Let $C_i(G)$ be the i - center of a graph G. We proved that for a P_t - free graph G, if $i \ge \lfloor t/2 \rfloor$, then $C_{i+p}(G)$ is (p+1) - dominated by $C_i(G)$. This solved a problem posed by O. Favaron and J. L. Fouquet.

Tuesday, February 4, 1992
11:10 a.m.

57 **Partial Ordering for Matchings of Bipartite Planar Graphs**

James Propp, Massachusetts Institute of Technology

A partial ordering will be defined on the set of matchings of a bipartite planar graph G. This ordering will be used to show that (under a very mild additional hypothesis on G) any matching of G can be obtained from any other by means of a sequence of ''elementary moves'' in which a set of edges forming an alternating cycle around a face of G is replaced by the edges of the complementary alternating cycle. This generalizes Thurston's recent results on domino and lozenge tilings.

Key words: bipartite graphs, planar graphs, matching, partially ordered set.

58 **Ranking and Unranking Spanning Trees**

Wendy Myrvold, University of Victoria

Colbourn, Day and Nel proposed the first $O(n^3)$ algorithm for ranking and unranking spanning trees of a graph. Their algorithm is a 6-way divide and conquer. We suggest a simpler $O(n^3)$ algorithm based on rank-one updates. Our approach can equally well be used to rank and unrank spanning arborescences of a directed graph which are rooted at a specific vertex.

All results of this type are based on the famous Kirchoff Matrix Tree theorem. As a side note, we explain why row and column permutations are not required when applying Gaussian elimination to a Kirchoff matrix of a connected graph.

Keywords: ranking, unranking, random spanning tree, spanning arborescence, Kirchoff matrix, rank-one update

59 **On the λ-Number of Q_n and Related Graphs**

M.A. Whittlesey, J.P. Georges*, D.W. Mauro; Trinity College, Hartford, CT

A λ-labelling of graph G is an integer labelling of $V(G)$ such that adjacent vertices have labels that differ by at least two and vertices distance two apart have labels that differ by at least one. The λ-number of G, $\lambda(G)$, is the minimum range of labels over all such labellings. We examine the properties of $\lambda-$labellings of the $n-$cube. Griggs and Yeh have determined $\lambda(Q_n)$ for $n \leq 5$ and have established $n+3 \leq \lambda(Q_n) \leq n+1$ for $n \geq 6$. We modify a technique used in coding theory to improve the upper bound of $2n+1$. We also examine the $\lambda-$labellings of related graphs such as the subdivision of the $n-$cube and the Cartesian products of paths and complete graphs.

Keywords: $\lambda-$labelling, $n-$cube.

60 **A CHARACTERIZATION OF GRAPHS IN WHICH SOME MINIMUM DOMINATING SET COVERS ALL THE EDGES**

B. Hartnell*, Saint Mary's University & D. Rall, Furman University

In general, the minimum number of vertices required to dominate (cover) all the vertices of a graph is less than or equal to the minimum number of vertices required to cover (dominate) all the edges of the graph. We shall characterize those graphs for which equality holds (e.g., C_4, P_4 and P_5).

It is of interest to note that this collection of graphs is one for which it is easy to compute both the domination and the independence number.

As a corollary of our work we also obtain a characterization of bipartite graphs having the property that one part of the bipartition is a minimum dominating set.

61 SUM-FREE SETS IN VECTOR SPACES OVER GF(2).

W. Edwin Clark* and John Pedersen, Department of Mathematics, University of South Florida, Tampa, FL 33620-5700, eclark@math.usf.edu

A subset S of an abelian group G is *sum-free* if whenever a, b $\in$ S, then a + b $\notin$ S. A *maximal sum-free (msf)* set S in G is a sum-free set which is not properly contained in another sum-free subset of G. We consider only the case where G is the vector space V(n) of dimension n over GF(2). A sum-free set in V(n) may be interpreted as a cap (no three points collinear) in the projective space PG(n - 1, 2). There is also a coding theory interpretation of msf sets: Let H be a parity check matrix for a binary [n, n-r, d] code C and let S be the set of columns of H. Then S is a msf set in V(r) if and only if $d \geq 4$ and the covering radius of C is 2.

We are concerned with the problem of determining all msf sets in V(n). It is well-known that if S is a msf set then $|S| \leq 2^{n-1}$. We prove that there are no msf sets S in V(n) with $5 \cdot 2^{n-4} < |S| < 2^{n-1}$ (This bound is sharp at both ends.) Further, we construct msf sets S in V(n), $n \geq 4$, with $|S| = 2^{n-s} + 2^{s+t} - 3 \cdot 2^t$ for $0 \leq t \leq n-4$ and $2 \leq s \leq [(n-t)/2]$. These methods suffice to construct msf sets of all possible cardinalities for $n \leq 6$. We also present some of the results of our computer searches for msf sets in V(n). Up to equivalence we found all msf sets for $n \leq 6$. For $n > 6$ our searches used random sampling and in this case we find many more msf sets than our present methods of construction can account for.

Keywords and phrases: sum-free set, cap in PG(n,2), linear code, covering radius 2.

62 Conflict Free Access to Rectangular Subarrays

Doreen L. Erickson(*), University of Waterloo, dlericks@watmath.waterloo.edu
and
Charles J. Colbourn, University of Waterloo, cjcolbourn@watmath.waterloo.edu

In a synchronized parallel environment, we consider conflict free access to rectangular subarrays, using a natural formulation in terms of latin squares. To provide conflict free access to all rectangular subarrays of size (k-i) by (k+i), we have found that the necessary number of memories for k odd is 2k(k-1)+1 and for k even is 2k(k-1). For all k, 2k(k-1)+1 memories are sufficient. Furthermore, there is a linear skewing scheme which can realize this upper bound. These lower bounds also imply a lower bound for rectangular subarrays under other size constraints.

63 Contractible Edges in Longest Cycles in Nonhamiltonian Graphs

M.N. Ellingham*, R.L. Hemminger, and Kathryn E. Johnson
Vanderbilt University

An edge of a 3-connected graph is *contractible* if its contraction results in a graph which is still 3-connected. Dean, Hemminger and Ota showed that all 3-connected graphs with seven or more vertices have at least three contractible edges on any longest cycle. Recently it was conjectured that any nonhamiltonian 3-connected graph has at least six contractible edges on any longest cycle. We prove this conjecture and provide a construction to show that it is best possible.

64 Independent Dominating Sets of Objects

John Gimbel*, University of Alaska, USA and Preben Dahl Vestergaard, Aalborg University, Denmark

Given a graph, we refer to both edges and vertices as objects. A set of objects is independent if no pair is adjacent nor incident. Also, such a set dominates a graph if each object not in the set is adjacent or incident to an object in the set. We prove several results about independent dominating sets of objects. Specifically, we show that with a finite number of exceptions, if G is a connected graph of order n then G contains an independent dominating set of cardinality no more than $n-2\sqrt{n}+2$.

Tuesday, February 4, 1992992
11:50 a.m.

65 **The Snake-in the-Box Problem: A New Upper Bound**

Hunter S. Snevily, Department of Mathematics, Caltech.

We give a new upper bound for the length of the largest induced cycle in the hypercube.

66 ON RADO NUMBERS

Stefan A. Burr, City College, New York, NY 10031
Saoping Loo*, West Virginia University, Morgantown, WV 26506

A linear equation (L) with integer coefficients is said to be c--regular if there is a positive integer m such that any c--coloring of [m]={1,2,...,m} contains a monochromatic solution to (L). It is regular if it is c--regular for every positive integer c. When (L) is c--regular, use r(L,c) denote the least integer m with the above property and call it the c--color Rado number of (L). In a series of papers beginning in 1933, R. Rado published a beautiful collection of results which completely solved the problem of determing whether a system of homogeneous linear equations is regular. However, almost no actual values of Rado numbers have been determined. We initiate the systematic study of such numbers and have given the 2--color Rado numbers of certain linear equations.

67 On contractible edges in longest cycles

R.E.L. Aldred, Otago University, N.Z. and
R.L. Hemminger*, Vanderbilt University, U.S.A.

We will describe the 3-connected graphs having a longest cycle that contains at most five contractible edges.

68 Global Domination of Simple Factors

*Julie R. Carrington, Dept. of Computer Science
Robert C. Brigham, Depts. of Mathematics and Computer Science
University of Central Florida, Orlando, FL. 32816

A factoring of a graph G = (V, E) is a collection of spanning subgraphs F_1, F_2, ..., F_k, known as factors, into which the edge set E has been partitioned. A global dominating set is a smallest set of nodes which dominates all of the factors. It is clear that the problem of finding a minimum global dominating set for an arbitrary factoring of some graph is NP-hard. We investigate the tractability of this problem where all of the factors are simple structures such as trees or even paths and report some surprising results.

Tuesday, February 4, 1992
12:10 p.m.

69 **Recursive Networks: Degree-Sequence, Fault-Tolerance, and Embedding**

*Sajal K. Das** and *Aisheng Mao*
Department of Computer Science, University of North Texas, Denton, TX 76203

This paper investigates degree-sequences of vertices in bipartite, recursive networks (RNs) which are derived from combinatorial principles, with unit incrementability. The degree-sequence of a vertex in such irregular networks of various sizes is obtained from the *starting element* and *increment distance*, both of which depend on the binary representation of the label of that vertex. We provide an algorithm for computing vertex degrees. By identifying vertices of minimum degrees, called *referred vertices*, the fault-tolerance properties of RNs are discussed. Furthermore, two isomorphic and disjoint full binary trees are embedded in RNs, a congestion-free routing scheme is designed, and data broadcasting is shown to require constant time.

Key Words: Recursive networks, degree sequence, connectivity, embedding, routing.

70 **CHROMATIC NUMBER OF THE PLANE**

by Alexander Soifer, University of Colorado at Colorado Springs

Find the smallest number χ (called *chromatic number of the plane*) of colors that are required to color the plane in such a way that no two points of the same color are distance 1 apart.

Five mathematicians have been credited with the creation of this great open problem: Paul Erdös, Martin Gardner, Hugo Hadwiger, Leo Moser, and Edward Nelson. Who really created it and when? This question will be answered.

Apparently in 1958, Paul Erdös asked the following related question: find the smallest number χ_p (polychromatic number of the plane) of colors that are required to color the plane in such a way that no color realizes all distances. In 1970, D. E. Raiskii proved that $4 \leq \chi_p$, and S. B. Stechkin showed that $\chi_p \leq 6$. A new six-coloring of the plane will be shown in which distance 1 is not realized by any color except one, the sixth color does not realize the distance $\frac{1}{\sqrt{5}}$. This seems to be the first relevant example of 6- (or 7-) coloring not based on a regular triangular lattice.

Key words: chromatic number, coloring, plane

71 On the Recognition of k-Polygon Graphs

Ehab Elmallah* and L. Stewart
Department of Computing Science, University of Alberta, Edmonton

This talk presents some preliminary work on the recognition of k-polygon graph (the subset of circle graphs that can be embedded in a convex k-gon). Our main result is an approximate algorithm for embedding a k-polygon graph in a k'-gon where k' is no more than k plus half the number of the leaves in a given (join) decomposition tree of G.

72 **Neighborhood Parity Conditions for Graphs**

A. T. Amin* and P. J. Slater, University of Alabama in Huntsville

An odd neighborhood dominating set is a vertex set S such that each close neighborhood N[v] contains an odd number of vertices in S. Sutner has shown that such a set exists for every graph G. Clearly the empty set is an even neighborhood dominating set. We generalize this problem to the case where parity requirements for each vertex can be arbitrarily specified. Not all parity requirements can be met for every graph. Theoretical and algorithmic results for this problem will be presented.

Tuesday, February 4, 1992
4:20 p.m.

73 PERFECT MENDELSOHN COVERING DESIGNS WITH BLOCK SIZE FOUR

F.E. Bennett*, Mount Saint Vincent University
H. Shen, Shanghai Jiao Tong University
J. Yin, Suzhou University

Let v and k be positive integers. A (v,k,1)–<u>**perfect Mendelsohn covering design**</u>, briefly (v,k,1)–PMCD, is a pair $(X,\mathcal{B})$ where X is a v–set (of <u>**points**</u>) and $\mathcal{B}$ is a collection of cyclically ordered k–subsets of X (called <u>**blocks**</u>) such that every ordered pair of points of X appears t–apart in at least one block of $\mathcal{B}$ for all t = 1, 2, ..., k–1. A (v,k,1)–PMCD is called a minimum covering if no other (v,k,1)–PMCD has fewer blocks; and the number of blocks in a minimum covering is called the covering number, denoted by c(v). The general problem is to determine c(v) for each integer $v \geq k$. This paper is concerned mainly with the case k = 4. If we define $C(v) = \lceil v(v-1)/4 \rceil$, then it is shown that c(v) = C(v) holds for all integers $v \geq 20$.

74 The External Grazing Problem

Laxmi P Gewali* and R Venkatasubramanian,
HRH College of Engineering, University of Nevada, Las Vegas
Las Vegas, NEVADA 89154-4019

Given a rope of length l and a convex polygon of n sides, the External Grazing Problem (EGP) asks for locating a point p on the boundary of the polygon such that when one end of the rope is tied at p, the total area swept (grazed) by the rope (in the exterior of the polygon) is maximized. We prove that the solution for EGP lies on one of the vertices of the polygon and present an O(n**2) time algorithm to obtain it. We also extend this algorithm (without increasing the time complexity) to compute the point from which the swept area is minimized. However, the point yielding the minimum grazing area may not lie on the vertices of the polygon. We conclude by discussing variations of EGP and approaches for developing faster algorithms.

Key Words: Computational Geometry, Visibility

75 A Circular-arc Characterization of Certain Rectilinear Drawings

B. L. Piazza*, U. of Southern Miss.; R. D. Ringeisen, Clemson U.;
S. K. Stueckle, Northeastern U.

The maximum crossing number, $\nu_m(G)$, is the maximum number of crossings among all good drawings of G and the maximum rectilinear crossing number, $\nu_m'(G)$, is the maximum number of crossings among all good drawings of G in which each edge is drawn as a straight line segment. In a previous paper, an upper bound was established for $\nu_m(G)$, i.e. $\nu_m(G) \leq U^* = M - N + \Sigma[|E| - \deg(u) - \deg(v) + 1]/2$, where M and N are the number of subgraphs of G isomorphic to K_4 and C_4, respectively, and the sum is taken over all edges uv in E.

Last year at the CGTC conference in Baton Rouge, we gave a characterization of those graphs for which $\nu_m'(G) = \nu_m(G) = U^*$ which was constructive in the sense that they were obtained by deleting appropriate edges from a certain drawing of a complete graph. However, if you are given a graph there is no easy way to determine if it could be obtained by such a construction. In this paper, we present a second characterization of these graphs in terms of the complements of proper circular-arc graphs.

KEYWORDS: maximum crossing number, proper circular-arc graph

76 Planar Regular One-Well-Covered Graphs

Michael R. Pinter, Belmont University Nashville, TN 37212

An independent set in a graph is a subset of vertices with the property that no two of the vertices are joined by an edge, and a maximum independent set in a graph is an independent set of the largest possible size. A graph is called well-covered if every independent set that is maximal with respect to set inclusion is also a maximum independent set. If G is a well-covered graph and G - v is also well-covered for all vertices v in G, then we say G is 1-well-covered. By making use of a characterization of cubic well-covered graphs, it is straightforward to determination all cubic 1-well-covered graphs. Since there is no known characterization of k-regular well-covered graphs for $k \geq 4$, it is more difficult to determine the k-regular 1-well-covered graphs for $k \geq 4$. The main result in this regard is the determination of all 3-connected 4-regular planar 1-well-covered graphs.

KEY WORDS: Well-covered graph, Regular, Planar, Independent set

77 PACKING PAIRS BY QUINTUPLES

W. H. Mills

Institute for Defense Analyses
Princeton, N. J. 08540

A packing of pairs by quintuples of a v-set V with index λ is a family of quintuples of V such that every pair is contained in at most λ of them. We discuss the problem of determining the maximum number of quintuples in such a packing, and we solve this problem for small values of v and arbitrary index.

Key words: Packing, Quintuples

78 Euclidean Shortest paths in the Presence of Isosethic Obstacles

Ali A. Kooshesh* and Bernard M. E. Moret, Univ of New Mexico

The Euclidean shortest-path problem between two distinguished points asks to find a path, interior to a specified subset of two-dimensional space (such as a simple polygon), of least total length. An extension of this problem allows the specified subset of 2D space to contain holes; for intance, the two distinguished points are within an enclosing simple polygon that contains a set of polygonal obstacles. The shortest-path must then avoid the obstacles (holes) while remaining within the enclosing polygon. In this talk, we will discuss the evolution of the shortest-path problems and present an O(n long n) algorithm for solving a special case in which the obstacles are isosethic line segments.

79 Subgraphs, Drawings, and the Thrackle Conjecture

J. E. Green* and R. D. Ringeisen, Clemson University

The crossing number of a graph G, ν (G), and the maximum crossing number, ν_M (G), are the minimum and maximum number of crossings among all good drawings of G, respectively. The Subgraph Problem asks the question, "Given a graph G and a subgraph H of G, must it be true that $\nu_M (H) \leq \nu_M (G)$?" . In this paper we give several subgraphs which satisfy this inequality. Study of Woodall's famous Thrackle Conjecture also led us to look at thrackles on orientable surfaces other than the plane.

80 *Global Parameters in Graphs*

Jean Dunbar, Converse College and Renu Laskar*, Clemson University.

Let G be a graph. A subset S of V(G) is a global P-set if S has property P in both G and its complement $\overline{G}$. We discuss global P-sets of graphs for some properties P such as domination, and irredundance and show some new results for total domination.

KEY WORDS: DOMINATION, GLOBAL, TOTAL DOMINATION

Tuesday, February 4, 1992
5:00 p.m.

81 Cyclic Difference Covers Through 133

Doug Wiedemann, Thinking Machines Corporation

A cyclic difference cover is a set of residues modulo m such that every residue modulo m can be written as the difference between two elements of the set. Here we are interested only in covers with a minimum number of elements. W.H. Mills and the author had previously computed lexicographically first covers through modulus 110 on the Cray-1 computer. The present paper extends this through modulus 133, using the CM-5 computer. Modulus 133 has a cover with 12 elements due to the Singer difference set construction. The computer shows that modulus 132 requires 14 elements, and this is the smallest modulus where the elementary lower bound of $(1+\sqrt{4m-3})/2$ is exceeded by two.

82 On Visibility Graphs Of Simple Polygons

James Abello* & Hua Lin
Computer Science Department
Texas A&M University
College Station, Tx77843

A simple polygon in the plane is a closed chain of line segments with no two non-adjacent line segments intersecting. Two vertices of the polygon are called visible if the closed line between them is either a segment on the polygon boundary, or is completely contained in the interior of the polygon. The visibility graph of a polygon is the graph whose vertices correspond to the vertices of the polygon and edges correspond to visible pairs of vertices in the polygon.
Several issues about visibility graphs are addressed in this paper. First, a novel efficient algorithm to find the maximum clique in a visibility graph is presented. Second, Ghosh's conjecture, about sufficient conditions for a 3-connected graph to become a visibility graph, is disproved. Third, it is proved that for a 3-connected planar graph to be a visibility graph it must be maximally planar.

Key words: visibility graph, maximum clique, maximally planar, 3-connectivity, Hamiltonian cycle

83 Drawings of the complete graph with maximum number of crossings

Heiko Harborth, Techn. Univ. Braunschweig, Germany

A drawing $D(K_n)$ of the complete graph K_n uses different points of the plane for vertices, and for edges simple connecting arcs which have at most one point in common either a vertex or a crossing. The maximum number of crossings in $D(K_n)$ is $\binom{n}{4}$. Here different classes of $D(K_n)$ with $\binom{n}{4}$ crossings are described, and the distribution of edges without crossings is discussed.

84 The Pure Edge-Integrity of Graphs

K. S. Bagga*, Ball State University
J. S. Deogun*, University of Nebraska

Recently, several papers have appeared on the topics of integrity and edge-integrity. In this paper, we present a related but new measure of vulnerability in graphs. For a graph G, the pure edge-integrity of G is defined as $\min\{ |S| + m(G-S) \}$, where, for a subset S of the edge-set E(G) of G, m(G-S) denotes the size(the number of edges) of a component of G-S of largest size, and the minimum is taken over all subsets S of E(G). We present some properties of this measure, study extremal cases, and give values for some well-known classes of graphs, including trees, complete graphs, and cubes.

85 On Some Combinatorial Arrays

D.V. Chopra, Wichita State University

Arrays are merely matrices of size $(m \times N)$ with elements from a finite set, the rows often referred as the constraints and columns as the runs (treatment combinations) of the arrays. Under various conditions and/or combinatorial structures these arrays provide us with different kinds of fractional factorial designs, and are also related to other combinatorial areas of design of experiments. In this paper we consider and discuss arrays T with two elements (say) 0 and 1 with the combinatorial structure that in every $(t \times N)$ submatrix of T, every $(t \times 1)$ column with i 1's appears $\mu_i (i=0,1,2,\ldots,t)$ times. Such an array T is called a binary balanced array (B-array) of strength t.

Key Words: Arrays, constraint, balanced arrays, strength of an array, fractional factorial designs.

86 Small Critical Squareless Subgraph of Hypercube

F. Castro*, P.Guan Dept. of Math. U.P.R. Rio Piedras, PR 00931

We present a recurssive construction of small critical squareless subgraphs of hypercubes. For n tends to infinity, the edge density of our construction tends to be 5/16. For the dimension of the hypercube is less than 5, we got the minimal critical subgraph.

87 **Determining Bandwidth Sum for Certain Graph Sums**

Kenneth Williams, Western Michigan University

For graph $G = (V, E)$, 1-1 mapping $f : V \longrightarrow \{1, 2, \ldots, |V|\}$ is called a *proper numbering* of G. The *bandwidth sum of* G, denoted $s(G)$, is the number

$$min\{\sum_{uv \in E} |f(u) - f(v)| : \text{ f is a proper numbering of G}\}.$$

It is well known that determination of the bandwidth sum for arbitrary graphs is NP-complete. The sum of two graphs G_1 and G_2, denoted $G_1 + G_2$, is defined to be the graph with vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2) \cup \{uw : u \in V(G_1), w \in V(G_2)\}$. We define a class of graphs called the *sum − deterministic* graphs and show that K_n, $\overline{K_n}$, C_n, P_n and $K_{1,n}$ are each sum-deterministic. For sum-deterministic graphs G_1 and G_2, $s(G_1 + G_2)$ is established. The bandwidth sum of the complete bipartite graph $K_{n,m}$ follows as an immediate consequence.

KEY WORDS: band width sum, proper numbering, graph sum

Wednesday, February 5, 1992
9:00 a.m.

89 **The unique structure of Perpendicular Array PA(3, 8, 8)**

P. S. Nair* and C. S. Peng
Department of Computer Science
Creighton University, Omaha, NE 68178-0109

The algebraic structure of perpendicular array PA(3, 8, 8) is presented. It has been shown that if interchange of rows is allowed then there exists exactly 240 perpendicular arrays PA(3, 8, 8) with identity permutation as first row. The structure of PA(3, 8, 8) provides a simple algorithm to generate these 240 perpendicular arrays directly. Further, if renaming of symbols together with corresponding column interchanges are allowed, then there is exactly one equivalence class. Further, the structure of PA(3, 8,8) is unique in the set perpendicular arrays PA(t,v,n).

Index terms: Perpendicular Arrays, Combinatorial Designs.

90 Large survivable nets and the generalized prisms

Arthur M. Hobbs
Texas A & M University, College Station, TX 77843
Hong-Jian Lai*
West Virginia University, Morgantown, WV 26506
Hongyuan Lai
Wayae State University, Detroit, MI 48202

November 4, 1991

For a graph G with vertices labeled 1, 2, ..., n and a permutation α in S_n, the *generalized prism* over G, $\alpha(G)$, consists of two copies of G, say G_x and G_y, along with the edges $(x_i, y_{\alpha(i)})$, for $1 \le i \le n$. In [Discrete Appl. Math. 30 (1991) 229 - 233], the importance of building larger graph by using generalized prisms is indicated, and the connectivity of the generalized prisms is discussed. Let $f(G)$ be a vulerability measure of G and let $\overline{f}(G)$ be the maximum value of $f(H)$ taken over all subgraphs H of G. A network G is *f-survivable* if $f(G) = \overline{f}(G)$, for a knowledgeable enemy would find no especially attractive targets. We shall investigate sufficent conditions to guarantee $f(\alpha(G)) = \overline{f}(\alpha(G))$, for all permutation $\alpha \in S_{|V(G)|}$, where f is the connectivity, edge-connectivity, and the strength.

91 The Additive Bandwidth of a Graph

M.E.Bascunan and S.Ruiz
Universidad Catolica de Valparaiso
P.J.Slater * - Univ. of Alabama in Huntsville

The principal application of bandwidth numbering problems is the compact representation of sparse n-by-n matrices. Here we introduce the additive bandwidth of a graph which offers up to twice as compact a representation for some important classes of graphs.

Wednesday, February 5, 1992
10:50 a.m.

93 OPTICAL ORTHOGONAL CODES FOR A LARGE NUMBER OF USERS

Oscar Moreno and Oscar Schiappacasse*
University of Puerto Rico
Department of Mathematics
Rio Piedras, PR 00931

University of Puerto Rico
Department of Mathematics
Cayey, PR 00733

Optical Orthogonal Codes (OOC) are families of 0-1 sequences with good correlation. They find applications in Local Area Network where a large family size and low correlation is required. Actually there are several constructions with low corrrelation but low family size. A new simple construction of OOC based on a special class of polynomials over GF(p) is given in this paper. Our construction given here has large family size and therefore has potential value for the applications.

94 Local Edge-domination Critical Graphs

Michael A. Henning, University of Natal - Pietermaritzburg
Ortrud R. Oellermann* and Henda C. Swart, University of Natal - Durban

Sumner and Blitch defined a graph to be k-γ-critical if $\gamma(G) = k$ and $\gamma(G + uv) = k-1$ for each pair u, v of nonadjacent vertices of G. We define a graph to be k-(γ, d)-critical if $\gamma(G) = k$ and $\gamma(G + uv) = k-1$ for each pair u, v of nonadjacent vertices of G that are at distance at most d apart. The 2-(γ, 2)-critical graphs are characterized. Sharp upper bounds on the diameter of 3-(γ, 2)- and 4-(γ, 2)-critical graphs are established and partial characterizations of 3-(γ, 2)-critical graphs are obtained.

95 THE BANDWIDTH OF THE CORONA OF TWO GRAPHS

Phyllis Zweig Chinn*
Humboldt State University
Arcata, CA 95521

Lin Yixun
Yuan Jinjiang
Zhengzhou University
Zhengzhou, Henan, P.R.C.

The bandwidth problem for a graph G is to label its p vertices v_i with distinct integers $f(v_i)$ so that the quantity

$$\max \{|f(v_i) - f(v_j)| : (v_i,v_j) \in E(G)\}$$

is minimized. Determining the bandwidth B(G) of an arbitrary graph is NP-complete. Many upper and lower bounds are known which relate bandwidth to various graph invariants. The exact value of the bandwidth is known for very few families of graphs.

Harary and Frucht defined the corona of G with H, G • H, as follows: Take one copy of G and |G| = p copies of H. Join the i^{th} vertex of G to each vertex in the i^{th} copy of H.

This paper determines an upper bound for the bandwidth of the corona G • H of two graphs, where the bound is based on the bandwidths of G and H. The exact values of the bandwidth of the corona of several families of graphs are determined, which achieve the given upper bound.

96 Large $2P_3$-free Graphs With Bounded Vertex Degree

Myung S. Chung* and Douglas B. West
Department of Mathmatics, University of Illinois, Urbana, IL 61801

An *H-free graph* is a graph that does not have H as an induced subgraph. Let $ex^*(D;H)$ be the maximum number of edges in a connected H-free graph G with maximum vertex degree D; this is bounded if and only if H is the disjoint union of paths.
We have proved that if H is the disjoint union of two 3-vertex paths ($H = 2P_3$), then $ex^*(D;2P_3) = \lfloor\frac{(D^3+D+1)^2}{8(D^2-D+1)}\rfloor$. We also show that if H is a 4-vertex path ($H = P_4$), then $ex^*(D;P_4) = D^2$.
Keywords: forbidden subgraph, $2P_3$-free, extremal problem

97 A lifting of the Reed-Solomon code to a Hermite curve

H. Tapia* (U. Autónoma Metropolitana-I, México)*
C. Rentería (I. Politécnico Nacional, México)

(Keywords: Hermite curve, Goppa codes, Reed-Solomon codes)

Let X be the hermitian curve $x^{q+1} + y^{q+1} + 1 = 0$ defined over the finite field $K = \mathbf{F}_{q^2}$ (q a power of a prime). With a suitable change of coordinates, this relation can be transformed to $y^q + y = x^{q+1}$. If $Q = (\alpha, 1) \in \mathbf{P}_1(\mathbf{K})$, $(\alpha \in K)$, the relation $y^q + y = \alpha^{q+1}$ has q solutions β_i in K, and we call the K-rational points $P_i = (\alpha, \beta_i)$ on the curve X, the "lifting" points of the point Q. The points on X thus obtained together with the point to infinity, P_∞, give all the $q^3 + 1$ K-rational points of the curve X. Let $C^*(D = \sum_{i=1}^{q^2} Q_i, kQ_\infty)$, be the dual of the (rational) extended Reed-Solomon code, where $0 \le k \le q^2 - 1$, $Q_i = (\alpha_i, 1)$, $\alpha_i \in K$, and Q_∞ is the point to infinity on the projective line. In this talk we describe the geometric code $C^*(\tilde{D}, \tilde{G})$ on the hermitian curve X, where the support of the divisor $\tilde{D}$ is the "lifting" of the support of the divisor D, and $\tilde{G} = [q(k+1) - 1]P_\infty$. Furthermore it is shown that this code can be interpreted as a (left) ideal in the group algebra $\mathbf{F}_{q^2}[\Gamma]$, where Γ is the group of automorphisms of the code under consideration.

98 CYCLIC DIAMETER OF GRAPHS

RENU LASKAR, JAMES A. KNISELY* CLEMSON UNIVERSITY

Let G be a finite, connected, undirected graph and χ be a proper edge coloring of G. The cyclic length of a path, the cyclic distance from one vertex to another, and the cyclic diameter of the graph can all be calculated with respect to χ. The cyclic diameter of G is defined to be the minimum cyclic diameter generated by any proper coloring. An algorithm for calculating the cyclic diameter with respect to a given coloring, the characteristics of colorings that give the cyclic diameter of G, and classes of graphs which have cyclic diameter equal to $\log_2 n$ are discussed.

KEY WORDS: BROADCASTING, GOSSIPING, CYCLIC DISTANCE, REGULAR GRAPHS

99 BAND-PROBLEMS

L.K.Tolman BYU

Alteration of the definition of bandwidth yields a large class of problems similar to those posed by "bandwidth." We consider three alternatives with some results, interpretations and unanswered questions.

Wednesday, February 5, 1992
11:30 a.m.

101 Self-dual codes and graphs

Haluk Oral, Bosphorus University, Istanbul, Turkey

We present recent constructions of self-dual codes from graphs and show how these constructions are useful in settleing a number of interesting questions about self-dual codes.

102 Observations Regarding D_n-graphs

J. Boland*, T. Haynes and L. Lawson
East Tennessee State University
Johnson City, Tennessee 37614

Given a graph G we define the distance-n graph of G, $D_n(G)$, to be the graph with $V(D_n(G)) = V(G)$ and two vertices adjacent in $D_n(G)$ if and only if they are a distance n apart in G. Several properties of D_n-graphs are examined including D_n-graphs with unique generators and conditions on G which assure that $D_n(G)$ is connected. Results specific to D_2 and D_3-graphs are examined.

103 Effects of a Local Change on the Bandwidth of a Graph

Janice Jeffs, St. Francis Xavier University

Schmidt and Haynes studied a graph transformation where a vertex of degree k is replaced by a clique of size k. They found several relationships between invariants of the original graph and invariants of the transformed graph. When k=2 this transformation is an elementary refinement, but not all elementary refinements can be modelled using this transformation. As with refinements, this transformation can increase or decrease the bandwidth of the graph. Conditions are presented that ensure:

(i) the bandwidth does not increase;
(ii) the bandwidth does not decrease.

The bandwidth of the transformed graph is bounded in terms of the bandwidth of the original graph. We compare the lower bound to Chvatalova's bounds obtained with respect to refinement.

Keywords: bandwidth, operations on graphs, refinement

Wednesday, February 5, 1992
11:50 a.m.

105 On Ternary Codes Generated by Hadamard Matrices of order 24

C. W. H. Lam and L. Thiel, Concordia University, and A. Pautasso*, C. M. R. Saint-Jean.

In 1979, N. Ito, J. S. Leon, and J. Q. Longyear showed that there are 59 Hadamard matrices of order 24. In 1981, J. S. Leon, V. Pless, and N. J. A. Sloane proved that these Hadamard matrices generated at least nine inequivalent ternary codes. In 1989, H. Kimura discovered another Hadamard matrix of order 24. In this paper, we show that these 60 Hadamard matrices generated exactly nine inequivalent ternary codes.
If two Hadamard matrices generate equivalent ternary codes, then, after applying signed column permutations, the rows of one matrix can be represented as a linear combination of the rows of the other. As an aid to demonstrate their equivalence, we also list the signed column permutation for each Hadamard matrix. After these signed column permutations are known, finding the linear combinations of rows is a simple exercise in linear algebra.

106 Disjoint Circuits in the Cartesian Product of Two Directed Cycles

Stephen J. Curran, University of Pittsburgh at Johnstown

We show that the Cartesian product of two directed cycles $\mathbf{Z}_a \times \mathbf{Z}_b$ has r disjointly embedded circuits $C_1, C_2, \ldots, C_r$ with specified knot classes $knot(C_i) = (m_i, n_i)$, for $i = 1, 2, \ldots, r$, if and only if there exist relatively prime nonnegative integers m and n such that $knot(C_i) = (m, n)$, for $i = 1, 2, \ldots, r$, and $r(am + bn) \leq ab$. A generalization of this result to the Cayley digraph of a finite abelian group with a two-element generating set will also be discussed.

107 An improved lower bound for the bandwidth of a graph

David R. Hare*, Lander College (Greenwood, SC)
William R. Hare, Clemson University (Clemson, SC)

An improvement on Harper's lower bound (*JCT*, **1** (1966) 385-393) for the bandwidth of a graph is given. Harper's bound has been used (Chvatalova, *Disc. Math.*, **11**(1975)249-253; Hare, Hare, Hedetniemi, *Congr. Num.* **50**(1985)67-76) to show that the bandwidth of the n x n grid-graph is n. The new lower bound is used analogously to show that the bandwidth of the n x n grid-graph with one corner vertex deleted remains unchanged. Hare, et al earlier showed that removal of two opposite corner vertices reduces the bandwidth to n-1.

109 Commutative Order Eight Semigroup Generation — Progress Report

Steven C. Cater, Computer Science Department, University of Georgia

A program to generate all commutative semigroups of order eight has been running since September, 1989. As of mid-January, 1992, it has produced approximately 208,000 semigroups. The algorithm used and the progress to date will be discussed. Experiments with different implementations and data structures designed to speed the calculation will also be described.
Keywords: finite semigroups, enumeration, algorithms.

A Basis for the Homology of d-Divisible Partition Lattices

110 **Michelle L. Wachs,** Department of Mathematics and Computer Science, University of Miami, Coral Gables, FL 33124

A partition of a set in which each block size is divisible by some fixed positive integer d shall be called a ***d-divisible*** partition. For n a multiple of d, let Π_n^d be the lattice of d-divisible partitions of the set $\{1,2,\dots,n\}$. We construct an explicit natural basis for the top homology group of Π_n^d. Each basis element turns out to be the fundamental cycle of the barycentric subdivision of the boundary of an $\frac{n}{d}-1$-dimensional cube. Moreover, these cycles correspond in a natural way to permutations in the symmetric group $\mathcal{S}_{md-1}$ with descent set $\{d, 2d, \dots, n-d\}$. The basis constructed here yields a direct combinatorial derivation of a result conjectured by Stanley and proved by Calderbank, Hanlon, and Robinson, relating the character of $\mathcal{S}_n$ acting on the top homology of Π_n^d to a certain skew character.

111 REGULAR GRAPHS OF CLASS TWO WITH GIVEN ODD GIRTH

Guo-Hui Zhang
Sonoma State University, Rohnert Park, CA 94928

The *odd girth* of a regular graph G is the length of a shortest odd cycle in G. A k-regular graph of order n and odd girth g is called an *(n, k, g)-graph*. Let *d(n, g)* denote the largest k for which there exists an (n, k, g)-graph, and *f(n, g)* denote the largest k for which there exists an (n, k, g)-graph without one-factorization. The number d(n, g) has been determined by the author. This talk will focus on the question of estimating f(n, g).

112 Optimizing P4-sparse Graphs in Linear Time

Beverly Jamison
Dept. of Math. and Computer Science
University of Hartford
Hartford, CT 06117

Stephan Olariu*
Dept. of Computer Science
Old Dominion University
Norfolk, VA 23529-0162

Quite often, real-life applications suggest the study of graphs that feature some "local density" properties. In particular, graphs that are unlikely to have more than a few chordless paths of length three (also referred to as P_4s) appear in a number of contexts. A graph G is P4–sparse if no set of five vertices in G induces more than one chordless path of length three. P4–sparse graphs generalize both the class of cographs and the class of P4-reducible graphs. One remarkable feature of P4–sparse graphs is that they admit a tree representation unique up to isomorphism. It has been shown that this tree representation can be obtained in linear time in the size of the graph. The purpose of this work is to shows how the data structures returned by the recognition algorithm can be used to produce optimal solutions to the four classic optimization problems for P4–sparse graphs.

Wednesday, February 5, 1992
3:40 p.m.

113 CONSTANT TIME ALGORITHM FOR GENERATING BINARY ROOTED TREES

Ewa Kubicka* and Grzegorz Kubicki, University of Louisville

A binary rooted tree is a rooted tree where each node has at most two children. In this paper an efficient algorithm for generating all binary trees of a given order is presented. Its complexity is evaluated and it is shown that the average number of steps per tree is bounded by a constant independent of the order of the trees.

Key words: algorithm, binary tree.

114 Planar lattices are lexicographically shellable

Karen L. Collins, Wesleyan Univ, Middletown, CT

The special properties of planar posets have been studied, particularly in the 1970's, by I. Rival and others. More recently, the connection between posets, their corresponding polynomial rings and corresponding simplicial complexes has been studied by R. Stanley and others. This paper, using work of A. Bjorner, provides a connection between the two bodies of work, by characterizing when planar posets are Cohen-Macaulay. In fact, we show that rank-connected planar lattices are lexicographically shellable.

Key words: poset, lattice, planar, shellable, EL-labelling, Cohen-Macaulay

115 On k-girth Graphs

G.T. Chen, North Dakota State University
R.H. Schelp, Memphis State University
W.E. Shreve*, North Dakota State University

Let G be a graph and $\mathcal{C}$ be the set of all cycles in G. Fix k, a positive integer. Define the subset $\mathcal{C}' \subseteq \mathcal{C}$ as follows: $C \in \mathcal{C}'$ if for some vertex $x \in C$, there exist $k-1$ edge disjoint cycles $C_1, C_2, \cdots, C_{k-1}$ each containing x but edge disjoint from C so that $\ell(C_1) \leq \ell(C_2) \leq \cdots \leq \ell(C_{k-1}) \leq \ell(C)$, where $\ell(C)$ is the length of the cycle C.
The k-girth of G, $g_k(G) = min\{\ell(C) : C \in \mathcal{C}'\}$. Clearly, $g_1(G) = g(G)$, the girth of G and $g_1(G) \leq g_2(G) \leq \cdots$, where $min\{a : a \in \phi\} = \infty$.
Let r and k be two positive integers with $2k \leq r$. Let g be a positive integer with $g \geq 3$. Define $f_k(r,g)$ to be the smallest positive integer n such that there is an r-regular graph on n vertices for which the k-girth equals g.
Erdös and Sachs have proven the existence of $f_1(r,g)$. The purpose of this paper is to consider the existence of $f_k(r,g)$ and give upper bounds of $f_k(r,g)$ for several values of k, r and g, as well as the values of $f_2(4,g)$ for $3 \leq g \leq 6$.

Key Words: Graph, Cycle, Girth.

116 A Graphical User Interface for Graph Algorithm Programs

Louis D. Nel*, and Janet B. Noye
Carleton University Ottawa, Ontario, Canada K1S 5B6

This paper describes a prototype graphical user-interface facility, GraphUI, for the development of graph algorithm programs. GraphUI, provides a graphical graph editor and display service which runs as a separate application and communicates with the client graph algorithm program through message passing, and a graph-based command language. Thus the client program or programmer requires limited knowledge of, or interaction with, the user-interface aspects of the program. GraphUI is implemented as a Macintosh computer application which can be used either by itself or on behalf of other graph algorithm programs (which may be run on other types of machines).

Wednesday, February 5, 1992
4:00 p.m.

117 Constant Time Generation of Involutions

Dominique Roelants van Baronaigien

An involution is a permutation that is its own inverse. Involutions are known be in one-to-one correspondence with Young Tableau. There is no published algorithm for listing involutions, however, Nijenhuis and Wilf give a linear average time algorithm for listing all Young Tableau of a given shape.
A variable length integer sequence representation of involutions is presented and used to develop a constant amortized time algorithm for generating involutions. The representation is also useful for ranking and unranking. Efficient algorithms are presented for listing, ranking and unranking involutions.

118 Fixed Point Properties in Ordered Sets

Dwight Duffus
**Sylvia Williamson*
Emory University
Atlanta, GA 30322

Key words: *dismantlable, irreducible, truncated lattice*

Motivated by the question, is the fixed point property preserved by direct products, Duffus and Sauer introduced the more restrictive *strong fixed point property* which is known to be preserved by direct products. We explore the extent of the strong fixed point property.

Using algebraic topology, Baclawski and Bjorner showed that truncated noncomplemented lattices have the fixed point property. We seek a combinatorial proof of this result and we wish to show that truncated noncomplemented lattices have the strong fixed point property. We make general observations and provide an infinite family of truncated lattices with the strong fixed point property.

119 Large Regular Graph of Small Diameter

J. Opatrny*, Concordia University, Montreal
N. Srinivasan, A. J. Jain College, Madras
R. Thularisaman, Concordia University, Montreal

We present several generalizations of de Bruijn graphs that can be used to obtain very large regular graphs using a simple set of generators. In particular, we have obtained in this way for many values of k and d a regular graph of degree k and diameter d, which has more vertices than any known graph of degree k, and diameter d-1. Some properties of these graphs and open problems are discussed.

120 Fuzzy Net and Fuzzy Maxflow Problem

C. S. Peng* and P. S. Nair
Department of Math/Computer Science
Creighton University, Omaha, NE 68178-0109

We present Fuzzy Net, Vertex-Labeled Fuzzy Net, General Fuzzy Net, Fuzzy α-Maxflow problem, and Accumulative Fuzzy α-Maxflow problem. It is proved that (1) the value of any α-(s,t)-flow does not exceed the α-capacity of any (s,t)-cutset, (2) given a total α-flow v and a cutset (S,T), if $v = C_\alpha(S,T)$, the α-capacity of (S,T), then v is a α-maxflow and (S,T) is a α-mincut, (3) an α-flow has a maximum value if and only if there are no α-augmenting paths from source to sink, and (4) the maximum value of an α-flow equals the minimum α-capacity of any (s,t)-cutset, where s is source vertex, t is sink vertex, and α is a given fuzzy relationship constraint.

Key Words: Fuzzy Net, Fuzzy Maxflow, α-Path, α-Augmenting Path, α-Capacity.

Wednesday, February 5, 1992
4:20 p.m.

121 Listing All k-Paths

Andrzej Proskurowski – University of Oregon
Frank Ruskey (*) and *Malcolm Smith* – University of Victoria

We give efficient algorithms for listing simple representations of k-paths as k-ary strings inequivalent with respect to string reversal and permutation of symbols. These algorithms use space $O(n)$ and time $O(\sqrt{k}N)$, where N is the total number of strings generated and n is the length of each string (the corresponding k-paths have $n + k + 2$ vertices). For $k = 2$, we obtain a recursive decomposition of the set of binary strings that allows the strings to be generated without rejecting any strings. For $k \geq 3$, some strings must be rejected. The algorithm is simple but its exact analysis is rather complicated. The analysis involves the cycle structure of the symmetric group S_k, finite Markov chains, and Bessel functions.

Keywords: Strings, listing, algorithm analysis.

122 MORE CONSTRUCTIONS OF 2-SEQUENCINGS OF GROUPS

B. A. Anderson Arizona State University

Recently 2-sequencings of finite groups have been used to characterize all finite solvable groups that have symmetric sequencings and to find infinite classes of self-orthogonal Hamiltonian path decompositions of $2K_n$. Bailey conjectured that all finite groups except $Z_2{}^n$, $n \geq 2$, have 2-sequencings. Some new constructions of 2-sequencings are given. One is useful in finding symmetric sequencings for finite non-solvable groups with a unique element of order 2.

123 A Family of Regular Graphs of High Edge Density and Girth at Least 16

Vasiliy A. Ustimenko and Andrew J. Woldar*

Using properties of the natural geometry of Lie group "twisted F4" (generalized octagon) we obtain a family of regular graphs of (constant) girth at least 16 whose edge density is the largest known in the sense of the Erdos-Simonovits bound. Attempts to meet the bound are presently being studied by the authors.

124 On edge-graceful cubic graphs conjecture.

Sin-Min Lee* and Eric Seah, Department of Mathematics and Computer Science San Jose State University, San Jose. Department of Acturial and Management Science, University of Manitoba, Winnipeg, Canada.

A graph G=(V,E) is said to be edge-graceful if there exists a bijection f: E-> {1,2,..., |E|} such that the induced mapping f^+: V -> {0,1,..., |V|-1} , defined by $f^+(v) = \Sigma\{f(u,v) : (u,v)$ in E } (mod |V|) , is a bijection.

The first author conjectured that every cubic graph of order 4k is edge-graceful.

We show that the conjecture is true for the class of generalized Petersen graphs.

Key words: graph labeling, edge-graceful, cubic, generalized Petersen graphs.

Wednesday, February 5, 1992
4:40 p.m.

125 Generating All Maximal Cliques and Related Problems for Certain Perfect Graphs

Yang Cai, Dept. of Computer Science, University of Texas at Austin
M.C. Kong*, Dept. of Computer Science, University of Kansas

We present in this paper fast algorithms for generating all maximal cliques and/or independent sets for certain perfect graphs, including comparability graphs, cocomparability graphs, and permutation graphs. These algorithms can generate a maximal cliques (independent set) of the given graph in time proportional to its cardinality after a polynomial time preprocessing step.
Our algorithms are based on a unified approach which can also be used to develop efficient polynomial algorithms for other related problems. These include the problem of counting the number of maximum (maximal) cliques and/or independent sets for these graphs, which is a #P-complete problem for general graphs, and the problem of computing a minimum weight independent dominating set for cocomparability graphs and permutation graphs, which is a NP-hard problem for general graphs as well as for comparability graphs. Applications of our results to problems in string matching are also discussed.

Keywords: Perfect graphs, Algorithms, Complexity.

126 Hopf Algebras in Combinatorics

William R. Schmitt, Memphis State University

It is shown how families of partially ordered sets give rise to Hopf Algebras, and the structure of these Hopf algebras is investigated. In particular, their algebra structure is completely determined, combinatorial formulas for antipodes are given, and in the cocommutative case, a technique for finding generating sets of primitive elements is given. These results are applied to examples arising from families of graphs and matroids.

Key words: Antipodes, Hopf algebras, Incidence Algebras, Partially ordered sets.

127 **New Examples of Graphs of Large Girth and of Large Size.**

Felix Lazebnik(*), Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, USA;
Vasiliy A. Ustimenko, Department of Mathematics and Mechanics, Kiev State University, Kiev 252127, Ukraine.

We investigate the problem of finding graphs with the given girth (the length of the shortest cycle) having the greatest possible number of edges. For any prime power $q \geq 3$, we exibit two new infinite series of bipartite q–regular edge–transitive graphs which are induced subgraphs of generalized 4–gon and 6–gon, respectively. The first is of order $2q^3$ and girth 8; for $q = 2^n \geq 4$, graphs of this series are also vertex–transitive. The second is of order $2q^5$ and girth 12; for $q = 3^n \geq 9$, graphs of this series are also vertex transitive.
Then, for infinitely many values of q, we construct a new infinite series of bipartite q–regular edge–transitive graphs of order $2q^5$ and girth 10.
To our knowledge, all graphs described above have greater edge density than known extremal graphs. These constructions were motivated by some results on embedding of Chevalley group geometries in the corresponding Lie algebras, and a construction of a covering for an incident system and a graph.

128 Irredundant Full Graphs

Ted Monroe Wofford College

A vertex u in a set of vertices S from the graph G is said to have a **private neighbor with respect to S** if at least one of two conditions is satisfied. One of these is that u is an independent vertex in the subgraph induced by S; the other is that there exists a vertex v outside S so that v is adjacent to u but not adjacent to any other vertex in S. If every vertex in S has a private neighbor with respect to S, then S is said to be **irredundant**. The **upper irredundance number of G,** devoted IR(G) , is the largest cardinality of any such irredundant set. G is said to be **k-IR-full** if IR(G) = k and IR(G+e) = k - 1 for every missing edge e. The purpose of this paper is to characterize k-IR-full graphs.

129 An efficient Approximation Algorithm for the File Redistribution Scheduling Problem in Fully Connected Networks

Ravi Varadarajan* Pedro I. Rivera-Vega† *

File Redistribution Scheduling problem is the problem of finding routes and a schedule for transferring a given set of files in a network from their respective sources to their respective destinations in minimum time (makespan). It is assumed that it takes unit time to transmit a file along a communication link and no more than one file can use a communication link at any time. We have previously shown this problem to be NP-hard even when the network has a fully connected topology and when each file can have at most one hop in its route. It is conjectured that there always exists an optimal schedule wherein every file has at most one hop in its route. In this work, we first prove the conjecture in the case of a fully connected network with four nodes. Then we present an efficient polynomial time approximation algorithm that is guaranteed to give a solution with a makespan no more than twice the optimal value; in this solution, every file has at most one hop in its route to its destination.

*Department of Computer and Information Sciences, University of Florida, Gainesville FL 32611 ravi@cis.ufl.edu

†Department of Mathematics, University of Puerto Rico, Rio Piedras, P.R. 0093 p_rivera@upr1.upr.clu.edu

130 Linearly recursive sequences in several variables

Earl J. Taft, Rutgers University

We study the continuous dual of the algebra of polynomials over a field k in n variables $x_1,\cdots,x_n$. These are multisequences $f = (f_{i_1 i_2\cdots i_n})$, where f is a linear function on $k[x_1,\cdots,x_n]$ with $f(x_1^{i_1}x_2^{i_2}\cdots x_n^{i_n}) = f_{i_1 i_2\cdots i_n}$. For each j, $1\le j\le n$, the rows of f parallel to the j-th axis must satisfy a linearly recursive relation $p_j(x_j)$ with constant coefficients. Such multisequences with all $i_j\ge 0$ form a bialgebra, whose product is the Hadamard, or pointwise, product. We determine the invertible elements under this product. They are all multisequences with all coordinates non-zero such that every row is eventually an interlacing of geometric sequences. The proof uses Laurent polynomials in $x_1,\cdots,x_n$, and its continuous Hopf algebra dual of all $f = (f_{i_1 i_2\cdots i_n})$ for all i_j in the integers, where the recursive relations $p_j(x_j)$ for the rows satisfy $p_j(0) \neq 0$. Moreover, the procedure is effective in the sense that given certain knowledge about the roots of the $p_j(x_j)$, there is a finite algorithm to determine whether or not a given f is invertible. This generalizes earlier work of Larson and the author for $n=1$ (Israel J. Math. 72 (1990), 118–132).

131 OPTIMAL PACKING OF INDUCED STARS IN A GRAPH

A. Kelmans, RUTCOR, Rutgers University

We consider undirected graphs without loops or parallel edges. A graph G is called *a star* if it is connected and has a vertex which is an end-point of every edge of G. An edge subset Y of G is called an *induced n–star packing* of G if every component of the subgraph $G[Y]$ induced by Y is a star with at most n edges and is an induced subgraph of G. An induced n–star packing of G is *optimal* if it maximizes the number of covered vertices of G. A graph F is called an *odd clique tree* if F is connected and every block of F is a complete graph with an odd number of vertices. Let $\mathcal{P}_n(G)$ denote the set of all induced n–star packings of G, and $\mathcal{T}(G)$ denote the set of all components of G which are odd clique trees. A vertex set X of G is called *n–star independent* if there exists an induced n–star packing Y of G which covers X, i.e. $X \subseteq V(G[Y])$. Let $\mathcal{I}_n(G)$ denote the set of all n–star independent sets of G and put $\mathcal{S}_n(G) = (V(G),\mathcal{I}_n(G))$. We also introduce a concept of an *Y–augmenting path* similar to that of for the matchings in a graph. We prove the following:

(a1) *An induced n–star packing Y of G is optimal if and only if there is no Y–augmenting path in G.*

(a2) *Let $n \ge 2$. Then*
$min\{|V(G) \setminus V(G[Y])| : Y \in \mathcal{P}_n(G)\} = max\{0, |\mathcal{T}(G \setminus X)| - n|X| : X \subseteq V(G)\}$.

(a3) *$\mathcal{S}_n(G)$ is a matroid defined on the set of vertices of G, $n = 1, 2, \ldots$.*

The proof of **(a2)** provides a polynomial time algorithm which finds an optimal induced n–star packing and a corresponding obstuction X described in **(a2)**.

132 Distance irredundance in graphs: complexity issues

J. H. Hattingh (*) Rand Afrikaans University and
M. A. Henning, University of Natal

Let $n \ge 1$ be an integer. The closed n- neighbourhood $N_n u|$ of a vertex u in a graph $G = (V, E)$ is the set of vertices $\{vd(u,v) \le n\}$. The closed n-neighbourhood of a set X of vertices, denoted by $N_n X|$, is the union of the closed n-neighbourhoods $N_n u|$ of vertices u in X. For $x \in X \subseteq V(G)$, if $N_n x| - N_n X - \{x\}| = \emptyset$, then x is said to be n- redundant in X. A set X containing no n-redundant vertex is called n-irredundant. The n- irredundance number of G, denoted by $ir_n(G)$, is the minimum cardinality taken over all maximal n- irredundant sets of vertices of G. In this paper we show that computing $ir_n(G)$ for bipartite graphs G is NP-complete. We then show that computing $ir_n(G)$ for $(n-1)$-path augmented split graphs is NP-complete, and also give an alternative proof for this result. These results extend those of Hedetniemi, Laskar and Pfaff and Laskar and Pfaff for the case $n = 1$.

Keywords: Distance irredundance, graphs

133 Global synchronization of local oscillators coupled via metric random graphs

John Pedersen, University of South Florida, Tampa, FL 3362-5700

Discrete oscillators are placed at the vertices of a metric random graph. The oscillators may be influenced by the multiset of states of their immediate neighbors. Time progresses discretely. We investigate programs for achieving global synchronization of the oscillators. This work is partially motivated by biological phenomena such as synchronization of flashes in colonies of some species of fireflies.

134 Unit Interval Graphs, ñ-Graphs and Uniform n-Extensions

Denise Sakai
RUTCOR - Rutgers University
New Brunswick - NJ 08903
and
Universidade de São Paulo
São Paulo - Brazil

A graph G is an *$\tilde{n}$-graph* (respectively, a *unit interval graph*) if we can assign a set of n consecutive integers (respectively, an interval of unit length) to each vertex so that edges correspond to pairs of sets (respectively, intervals) which overlap. Roberts [1979], using a result by Fine and Harop [1957] about uniform n-extensions, showed that these two concepts are equivalent in the sense that every unit interval graph is an $\tilde{n}$-graph for some positive integer n, and conversely, every $\tilde{n}$-graph is unit interval. This paper studies the problem of finding the minimum n such that a given unit interval graph is an $\tilde{n}$-graph. A linear time algorithm to compute this number in a particular case is given, improving the earlier algorithms by Fine and Harop [1957]. An (integer) linear programming formulation is also presented.

Key words: Unit interval graphs, $\tilde{n}$-graphs, uniform n-extensions.

135 Minimizing Maximal Weighted Earliness and Tardiness around a Common Due Date

Chung W. Ng* and Gilbert H. Young
Tulane University

We consider the problem of scheduling independent jobs on a single processor, with the objective of minimizing the maximal weighted earliness and tardiness around a known common due date, d. Each job has arbitrary execution time and a weight which specifies the penalty cost per unit time that the job completes earlier or later than d. The value of d can be arbitrarily large, or small enough to constrain the scheduling decision. In both cases, we prove that finding a nonpreemptive schedule with minimum maximal weighted earliness and tardiness is NP-hard in the ordinary sense even on one processor. When all the jobs have equal weight, we show that the corresponding problem can be solved in $O(n)$ time on one processor, but remains NP-hard in the ordinary sense on two or more processors.

136 The total chromatic number of graphs with bipartite complement

A.J.W.Hilton and Cheng Zhao*, Department of Mathematics,West Virginia University, Morgantown, WV 26506

A simple graph G is called Type 1 if the total chromatic number equals to it's maximum degree plus 1. In this paper we obtain a sufficent condition for G to be Type 1 in the case when the complement of G is bipartite. This generalizes an earlier result of Dugdale and Hilton.

137

Eigenvalues of switching classes of graphs

Barry Guiduli

Let G be a graph on the vertex set $[n] = \{1, 2, ..., n\}$. Define the switching matrix of G to be the symmetric $n \times n$ matrix $A = (a_{ij})$ given by

$$a_{ij} = \begin{cases} 0 & \text{if } i = j, \\ 1 & \text{if } i \text{ is adjacent to } j, \\ -1 & \text{otherwise.} \end{cases}$$

By eigenvalues of G, we mean eigenvalues of this matrix. Define $\lambda(G)$ to be the largest eigenvalue of G. Let $X \subset [n]$ and define the switch of a graph G on X to be the graph G^X obtained from G by switching all edges/nonedges between vertices of X and vertices of $[n] - X$. This operation preserves the eigenvalues of G. We define an equivalence relation on graphs as follows: given two graphs G and H, we say that G is switching equivalent to H if there is a set X so that $G^X \cong H$. For the trivial graph (no edges), $\lambda(G) = 1$ with multiplicity $n - 1$ and for the single edge graph, $\lambda(G) < 3$ is simple (multiplicity one). We show that for all graphs G with $\lambda(G)$ simple, if $\lambda(G)$ has minimum possible value, then G is switching equivalent to the single edge graph. We prove the additional result that if G is not switching equivalent to the trivial graph or the single edge graph, and $n \geq 8$, then $\lambda(G) \geq 3$.

38

Proper and Unit Tolerance Graphs

Kenneth P. Bogart Dartmouth College, Hanover NH 03755
Peter Fishburn AT&T Bell Laboratories, Murry Hill NJ 07974
Garth Isaak Dartmouth College, Hanover NH 03755
Larry Langley * Dartmouth College, Hanover NH 03755

The class of unit interval graphs is equivalent to the class of proper interval graphs. We answer the question posed by Golumbic, Monma and Trotter, whether the same holds for the more general interval tolerance graphs, by presenting a proper interval tolerance graph that is not a unit tolerance graph. This example generalizes to an infinite class of such graphs.

Keywords: interval graph, intersection graph, interval order

139

Path Assignment for Call Routing: An Application of Tabu Search

Charles Anderson, Kathryn Jones*, Mark Parker and Jennifer Ryan
University of Colorado at Denver

We describe an implementation of tabu search that solves the path assignment problem, which is the problem of routing video data through an undercapacitated telecommunications network. Two versions of the tabu search are studied. We compare our results with those from a genetic algorithm approach and with an integer programming solution using a restricted solution space. Our results compare very favorably with those from these other methods.

140

The complexity of restricted colorings of graphs

M. Kubale, Technical University of Gdańsk, Poland

We consider the complexity of two related coloring problems: restricted coloring of vertices and restricted coloring of edges of a graph in which each vertex (edge) receives one color from a list of permissible colors associated with that vertex (edge). Since both problems are strongly NP-complete, we assume various restrictions imposed on the number and form of permissible colors and the structure of a graph. In this way we obtain some evidence for comparing the complexity of restricted vertex coloring problem versus that of edge coloring problem and arrive at a number of results about special cases that are either positive (polynomial algorithms) or negative (NP-completeness proofs).

Keywords: chromatic index, chromatic number, NP-completeness, polynomial-time algorithm, restricted coloring

141 An Algebraic View of the Moebius Graph

Kenneth Blaha, Pacific Lutheran Univ., Tacoma, WA

Leland and Solomon introduced a family of trivalent graphs, called Moebius graphs, and because of their small diameter suggested that they be used as an effective interconnection scheme for multiprocessor networks.

We state necessary and sufficient conditions needed to determine if a graph is isomorphic to a quotient Cayley graph (QCG). We use this characterization to show that the Moebius graph is a QCG. This observation affords an algebraic analysis of the Moebius graph. In particular, two problems proposed by Leland and Solomon in their original paper are reduced to tractable group theory problems and solved.

Key words: Moebius graph, multiprocessor network, quotient Cayley graph

142 Niche Graph Trees

Suzanne Seager, Mt. St. Vincent Univ.
Halifax NS, B3M 2J6

It is known that all but two trees of maximal degree at most 3 are niche graphs, and that no trees of maximal degree at least 5 are niche graphs. Classification of trees of maximal degree 4 appears to be difficult. We examine the effect of relaxing the definition of niche graphs to allow cycles and/or loops in the food web, for trees and for more general classes of graphs.

Keywords: niche graphs, niche trees

143 A Fast Algorithm for Edge Connectivity of Undirected Graph

Weiqing Cai* and David Matula, Computer Science Department
Southern Methodist University

We describe a fast algorithm to find the edge connectivity by iteratively using maximum adjacency search, graph contraction and bounds on each successive iteration, which takes linear time. Althoug the number of iterations is worst case bouded by O(n), our test graphs virtually all take at most 2 or 3 iterations.

Key word: connectivity, maximum adjacency sear

144 Iterated Greedy Graph Coloring and the Difficulty Landscape

Joseph Culberson, University of Alberta

Of the many heuristic algorithms which have been proposed for graph coloring, the simplest is the greedy algorithm. Many variations have been proposed for this algorithm, but it is generally assumed that the coloring will occur in a single pass. We note that if a new permutation of the vertices is chosen which respects the independent sets of some previous coloring, then applying the greedy algorithm will result in a new coloring no worse than the previous one. We introduce several heuristics for generating new permutations that are fast when implemented and effective in reducing the coloring number, and investigate the trade-offs over various classes of graphs. We couple this algorithm with several other coloring algorithms, including a modified Tabu search, and report on the trade-offs in different classes of graphs. Using these techniques we can optimally color graphs in G(1000,1/2,K) (i.e. K-colorable random graphs)for K up to 60, and can color graphs in G(1000,1/2) with as few as 86 colors in some cases.

Keywords:heuristic graph coloring algorithms, computational experience

145 COMBINATORIAL AND GRAPH PROBLEMS ARISING IN THE ANALYSIS OF CATASTROPHIC FAULT PATTERNS

A. NAYAK* (Carleton University), L. PAGLI (University of Pisa, Italy), N. SANTORO(Carleton University)

Faults occurring at strategic locations in a regular VLSI system may have catastrophic effects and cannot be overcome by any amount of redundancy or clever reconfiguration algorithms. The characterization of such patterns of faults is obviously crucial for the identification, testing and detection of catastrophic system events. In the analysis of catastrophic fault patterns several combinatorial and graph problems arise. We present a variety of such problems and, for specific cases, their solution. In their general formulation, many of these problems are still open.

146 **Competition and resource graphs of food webs**

Chi Wang
Department of Mathematics
University of Louisville
Louisville, KY 40292
c0wang01@ulkyvax.bitnet

The competition graph of a food web has species as its vertices and there is an edge between species a and b in the competition graph if and only if a, b have a common prey. The resource graph of a food web (also called common enemy graph) has species as its vertices and there is an edge between species a and b in the resource graph if and only if there are arcs from a vertex v to both a and b. The digraphs obtained from most real food webs are acyclic and have very small indegrees and outdegrees. We characterize competition graphs and resource graphs of acyclic digraphs of indegree at most two. We study the fundamental problem of characterizing those digraphs whose competition graph and resource graphs are interval graphs. We introduce the notion of restricted competition numbers and study the relationship between competition number and restricted competition number of those digraphs.

147 **Near-Point Optimality in Rank Constrained Level Graphs**

Jacob Shapiro, Baruch College, CUNY; Jerry Waxman*, Queens College, CUNY

A **level graph**, denoted by G = (V,E,k,f), is a structure consisting of a set of vertices V, a set of edges E, a positive integer k, and an onto function f: E → { 1,2,3, ..., k }. A level graph is called **rank constrained**, if for all $v \in V$, $|els(v)| \leq 2$ and for all $i,j \in els(v)$, $|i-j| \leq 1$. An algorithm, LGS previously introduced by the authors, exhibited asymptotic optimality with respect to shortest path generation. This paper studies a heuristic extension of LGS, within the context of rank constrained level graphs, which converges rapidly even for points $u,v \in V$ close in V.

Keywords: level graph, rank constrained level graph, approximate shortest path algorithms.

148 Some heuristics for T-colorings of Graphs

Daniel Costa, Swiss Federal Institute of Technology, Lausanne and Simon Fraser University:

A generalization of the classical graph coloring model is discussed in this talk. We want to color the vertices of a graph in such a way that the two colors assigned to two adjacent vertices i and j differ by at least a parameter tij. The goal is to minimize the spectrum of colors used. We present the results produced by well-known heuristics such as Tabu Search and Simulated Annealing. The results are compared with optimal colorings obtained by a branch and bound algorithm.

Thursday, February 6, 1992
11:30 a.m.

149 **Stack and Queue Layouts of Directed Acyclic Graphs**

Lenwood S. Heath and Sriram V. Pemmaraju*
Virginia Polytechnic Institute and State University

Stack layouts and queuelayouts of *undirected* graphs have been used to model problems in fault tolerant computing, VLSI, and in managing the flow of data in a parallel processing system. In certain applications, such as managing the flow of data in a parallel processing system, it is more realistic to use layouts of *directed acyclic graphs* (DAGs) as a model. A *stack layout* of a DAG consists of a topological ordering σ of the nodes of the graph along with an assignment of arcs to stacks such that if the nodes are laid out in a line according to σ and the arcs are all drawn above the line, then no two arcs assigned to the same stack cross. A *queue layout* is defined similarly, except that arcs are assigned to queues with the condition that no two arcs assigned to a queue nest. The *stacknumber* of a DAG is the smallest number of stacks required for its stack layout, while the *queuenumber* of a DAG is the smallest number of queues required for its queue layout. We study families of DAGs classified according to the structure of the underlying undirected graphs. We determine the stacknumber and queuenumber of families of DAGs whose underlying graphs are trees, unicyclic graphs, and outerplanar graphs. We give examples of families of DAGs whose stacknumber and queuenumber are unbounded, an example of a family of DAGs whose stacknumber is large compared to the queuenumber, and an example of a family of DAGs whose queuenumber is large compared to the stacknumber. Syslo defines the stacknumber of a poset (transitive DAG) as the stacknumber of the corresponding Hasse diagram. He then, suggests the stacknumber of a poset as a measure of its structure and establishes connections with other measures. We define the queuenumber of posets analogously and establish connections between the queuenumber of posets and other measures such as jump number and bump number.

150 **Edge Irredundance in Graphs**

Alice McRae, Department of Computer Science, Clemson University

For a graph G=(V,E) a set of vertices S is irredundant if for all vertices v in S, the closed neighborhood of v is not contained in the union of the closed neighborhoods of the vertices in S-v. We investigate irredundance parameters of line graphs and study their relationship to edge domination and matching parameters. We also show that the decision problems associated with the upper and lower maximal edge irredundance numbers are NP-complete.

Keyword: irredundance

151 **A Generalization of a Graph Partition Conjecture of Paul Erdos**

T.D. Porter, Southern Illinois University, GA3982@SIUCVMB.bitnet

A generalization of a graph partition conjecture of Paul Erdos is presented. The result yields a measure as to how k-chromatic a given graph is. The discrepancy between the multi-partite max cut and chromatic number is discussed and a lower bound on the multi-partite max cut of a graph is given.

152 **Orderly Algorithms for Generating K-Colored Graphs**

Yasunori Koda * and Ronald Read
University of Victoria *University of Waterloo*

We show orderly algorithms for generating k-colored graphs. Further, for $k = 2$, we give an algorithm for generating connected bicolorable graphs. We also describe some properties of a canonic matrix representing combinatorial objects which might be useful for improving the performance of orderly algorithms generally.

Key words: orderly algorithm, k-colored graph, bicolorable graph

Thursday, February 6, 1992
11:50 a.m.

153 RESULTS ON PROBABILISTIC EXHAUSTIVE TESTING OF VLSI CIRCUITS

Wen-Ben Jone and Sunil R. Das*

The performance of probabilistic exhaustive testing (PET) of VLSI circuits is first evaluated in the paper based on irredundant fanout-free AND-gate and EXOR-gate networks. The general case is examined next and the results from pseudorandom testing are employed *to estimate* the expected fault coverage of probabilistic exhaustive test patterns (PETPs). The paper shows that the effectiveness of PET is heavily dependent on the *function* and also on the *structure* of the circuit under test (CUT). If the number of test patterns that detect the circuit faults is *not* sufficiently large, then almost 100% of PETPs might be needed before a high fault coverage can be guaranteed. A relation between the expected fault coverage and percentage of desired PETPs is derived in the paper in terms of the detectability profile of the CUT. The paper provides illustrative examples and includes experimental results in support of the major conclusions as well.

Key Words : Built-in self-testing (BIST), random testing, pseudorandom testing, exhaustive testing, probabilistic exhaustive testing (PET)

---

*Wen-Ben Jone is with the Department of Computer Science, New Mexico Tech, Socorro, NM 87801, and Sunil R. Das is with the Department of Electrical Engineering, University of Ottawa, Ottawa, Ontario K1N 6N5, Canada.

154 An Infinite Class of Cyclic Triplewhist Tournaments

Ian Anderson
Department of Mathematics
University of Glasgow
Glasgow, Scotland
G12 8QW

Norman J. Finizio*
Department of Mathematics
University of Rhode Island
Kingston, Rhode Island
02881

A triplewhist tournament on v players, TWh(v), is a specialization of the whist tournament problem and was introduced by E. H. Moore in 1896. At the same time Moore provided a construction of cyclic TWh(3p+1) for all prime $p \equiv 1 \pmod 4$. In the present study, Moore's construction is adapted and extended to produce cyclic TWh($3p_1p_2+1$) for all prime p_1, p_2 such that $p_i = 4mk_i + 1$, k_i odd, $i = 1,2$, and $(k_1,k_2) = 1$.

KEY WORDS : block designs, cyclic block designs, whist tournaments, triplewhist tournaments, cyclic triplewhist tournaments

155 The Genetic Algorithm applied to the Traveling Salesman Problem

Ken Wellsch, University of Waterloo, kcwellsc@watdragon.waterloo.edu

The Genetic Algorithm (GA) is used to solve search and optimization problems and consists of a "population" of solutions, an evaluation function and a set of operators that transform the population into a new one. The transformation process selects members of the current population based on fitness (fitness is derived from the evaluation function) and the new population is generated via crossover and mutation to produce children that differ from their parents. Existing applications of the GA to the Traveling Salesman Problem (TSP) have used tour solutions to represent population members. This work takes a different approach by instead representing each member as a string of tour permutations. This has two benefits: there is no need to contrive a complex crossover operation that preserves the TSP solution validity of children, and the potential of being able to exploit the implicit learning capability of the GA technique. Representing tour permutations rather than explicit solutions provides the necessary information to extract transitions that may be more successful to other instances. Several implementations based on this new approach as well as various experimental results will be presented.

Adapting Vizing's Theorem for Total Colouring

Hugh R. Hind, University of Reading (England)

Let G be a finite graph. A function $\psi : V(G) \cup E(G) \longrightarrow \mathbb{N}$ is called a (proper) total colouring of G if no two adjacent or incident elements are assigned the same colour from $\mathbb{N}$. The total chromatic number of G, denoted $\chi''(G)$, is the smallest positive integer k for which there exists a total colouring $\psi : V(G) \cup E(G) \longrightarrow \{1, 2, \ldots, k\}$. A recolouring technique for total colourings will be discussed. The technique is derived from that used in the proof of Vizing's edge-colouring theorem. A theorem will be presented which has, as corollaries, the following two results:

Corollary 1: If G is a finite simple graph, then $\chi''(G) \leq \Delta(G) + 2\left\lceil \frac{|V(G)|}{\Delta(G)} \right\rceil + 1$.

Corollary 2: Almost every graph G_p (in $\mathcal{G}_{n,p}$) has $\chi''(G_p) = \Delta(G_p) + 1$.

(Note that Corollary 2 was first proved, independently, by McDiarmid and Reed.)

KEYWORDS: Total Colouring, Random Graphs

159 Calculations of some shapes using Fourier Fractals

S. Klasa, Concordia University, MONTR$AL

Last year at the Baton Rouge Conference, we introduced the concept of Fourier Fractals, together with an indication of their use in Computer Vision and Pattern Recognition.
This time we present an algebraic study of shapes in the plane and give a Fourier Fractal characterization (necessary and sufficient conditions) of some simple shapes (such as regular polygons, etc.) and their positions in the plane.

Key words: fractal, Fourier Fractals, shape, polygon, Pattern Recognition, Computer Vision

160 Edge-coloured H-colourings.

Richard Brewster

Graph homomorphisms (or H-colourings) have proven to be a powerful tool for working on graph colouring problems. In this talk we study the homomorphism problem for edge-coloured graphs. A homomorphism from an edge-coloured graph G to an edge-coloured graph H is a mapping on the vertices that preserves both edges and colours.
A result of Hell and Nešetřil completely classifies the complexity of H-colouring for graphs. As yet, no such classification exists for the edge-coloured version. We study H-colourings for different families of edge-coloured graphs and demonstrate a very rich behaviour even in small examples.

Thursday, February 6, 1992
3:20 p.m.

161
GAMES LOGICIANS PLAY

presented by Greg McColm; Dept. of, Mathematics, U.S.F.-Tampa

A formula can be regarded as a set of rules for a two-player game, to be played on any of a large class of boards. For example, we can construct a (infinitary) formula representing path reachability on graphs from a vertex a to a vertex b, and it corresponds to a game where one player---call her Eloise---starts at a and tries to reach b, while the other---call him Abelard---makes sure she doesn't cheat; Eloise can win iff a path from a to b exists. We investigate several logics, and their games, and find that games "naturally" partition large program-like formulas into subroutines. We also find that these "definition" games can serve as a foundation for the "comparison" games of Ehrenfeucht, Barwise & Immerman, etc.

162 The Queen Domination Problem

M. Eisenstein, C. Grinstead*, B. Hahne, D. Van Stone

On a chessboard, a queen attacks, or dominates, all squares which are in the same row, column, or diagonal as that queen. A natural question arises: How few queens are necessary to place on a chessboard so that each square is attacked? Define $f(n)$ to be the answer for an n x n chessboard. One may also add the requirement that no two queens attack one another. The minimum number in this case we will denote by $g(n)$. Clearly, we have $f(n) \leq g(n)$. We will show that

$$g(n) \leq \frac{7}{12}n + O(1) .$$

Key Words: Queen domination problem

163 **Siming Zhan,** Simon Fraser University, Burnaby, BC Canada, V5A 1S6.
Edge-Disjoint Hamilton Circuits in Line Graphs.

We discuss the number of edge-disjoint Hamilton circuits in line graphs. If a graph is almost r-regular and Hamiltonian, then its line graph has at least $2\lfloor \frac{r}{2} \rfloor - 2$ edge-disjoint Hamilton circuits. Related results but without the Hamilton restriction or without the regulartity restriction are presented.

164 MAXIMAL SETS OF TRANSLATIONS FORBIDDABLE BY A TWO-COLORING

PETER D. JOHNSON JR., AUBURN UNIVERSITY

A subset S of an abelian group A is a forbidden set of translations (or forbidden, for short) by a coloring of A if and only if a and $a+s$ have different colors, for each $a \in A, s \in S$. Forbiddable sets (of translations) play a role in certain approaches to various Euclidean Ramsey problems. For each positive integer n, each abelian group contains maximal sets forbiddable by an n-coloring of the group. In this study we deal mainly with the case $n = 2$. Results: let S be a maximal subset of A forbiddable by a two-coloring of A. We conclude the following.

1. S is closed under the taking of odd-summing integer conditions (i.e., $s_1, \ldots, s_k \in S$, $m_1, \ldots, m_k \in \mathbb{Z}$ and $\sum_{i=1}^{k} m_i$ is odd imply that $\sum_{i=1}^{k} m_i s_i \in S$).
2. If A is a vector space over $\mathbb{Q}$, then S is closed under multiplication by scalars $\frac{p}{q}$, with p, q odd.
3. In case 2, $A = (S+S) \bigcup S \bigcup (\bigcup_{m=1}^{\infty} 2^{-m} S)$, and the union is disjoint; further, $S+S$ is a translation of S.
4. If A is a field containing $\mathbb{Q}$, and S is maximal with respect to the properties of being two-forbiddable and being closed under multiplication, then S is closed under the operations mentioned in 1 and 2, above, under the taking of reciprocals, and under the taking of square roots, when they exist in A.

Thursday, February 6, 1992
3:40 p.m.

165 DE BRUIJN HYPERGRAPHS

Fahir Ergincan* and Robin Dawes, Queen's University, Canada

Our aim is to find bus networks which connect as many processors as possible, with given bounds on the number of connections per processor, on the number of processors per bus and on the network diameter. Other design parameters are the network connectivity and the ease of routing. We represent the underlying structure of bus networks with hypergraphs.
In this perspective, we proposed de Bruijn hypergraphs as models for bus networks. De Bruijn hypergraphs are obtained from de Bruijn graphs which are already shown to possess attractive features such as small network diameter, fixed degree, high connectivity, and simple routing algorithms. In this paper, we show that the aforementioned features are inherited by de Bruijn hypergraphs.

Keywords: hypergraphs, maximum degree, rank, diameter, order, connectivity.

166 Pursuit on Grid-Graphs

Robin W. Dawes, Queen's University

On a grid-graph, a pursuit-evasion "game" is played according to simple rules governing the movement of the players along the edges of the graph. One player wins if both players ever simultaneously occupy either the same row or the same column of the grid. The other player wins if the first player cannot force a win. Movement is simultaneous, and the speeds at which the players can move are parameters of the problem. We present a new upper bound for the minimum relative speed required to guarantee a win for the first player, and consider some variations on the problem. The problem has applications in collision-avoidance for robotics systems.

Key words: pursuit, evasion, grid-graphs, motion-planning, collision-avoidance

167 Cycles in Weighted Graphs

Xingxing Yu, Georgia Tech

We study several problems involving cycles in weighted graphs. First, we solve a conjecture of Bondy about optimum cycles in a weighted cubic graph. Then, we improve a result of Zhang on cycle covers of graphs with some flow conditions. Finally, we prove a vertex cycle cover conjecture (posed by Bermond et al) for graphs with no Petersen minors. This work has been supported by an NSF grant.

168 A Generalization of M.D.S. Codes

Sanjay Rajpal, Dartmouth College

Using the theory of matroids, we study a class of codes which are a generalization of M.D.S. codes and call them "paving" codes because they correspond to a class of matroids called paving matroids. We prove the following about paving codes:
Let C be a (n,k) paving code over GF(q) and suppose k >= q+1. Then
(i) the dual of C is a paving code.
(ii) the minimum weight of C is (n-k) or (n-k+1).
(iii) n <= 4q and k <= 2q.
From (iii), a natural question which crops up is whether there exists a (4q,2q) paving code over GF(q) for all prime powers q. For q=2 and q=3, this is known to be true. We show that this is false for q=4; i.e., there does not exist a (16,8) paving code over GF(4).

Thursday, February 6, 1992
4:00 p.m.

Integer Realizations of Sphere-of-Influence Graphs

169 Marc J. Lipman, Office of Naval Research, Arlington, VA

The sphere-of-influence graph of a set of points in the plane is the intersection graph of the interiors of a set of circles, one for each point in the set. The circle is centered at the point and has radius equal to the distance from that point to its nearest neighbor in the set. A graph is a sphere-of-influence graph, or SIG, if it is such for some set of points. SIGs are used in pattern recognition and computer vision to help separate objects or otherwise capture perceptual relevance. For the purposes of these original applications it is useful to know which graphs are the SIGs of sets of pixels. In this talk it is shown that every SIG has a realization as a set of points with integer coordinates.

Key words: intersection graph, sphere-of-influence graph, proximity graph, pattern recognition, computer vision

170 WARNSDORFF'S TOURS OF A KNIGHT ON A 3XN BOARD

Marco Valtorta
Dept. of Computer Science,
University of South Carolina,
Columbia, SC 29208.
mgv@usceast.cs.scarolina.edu

*M. Ishaq Zahid
Dept. of Math and Comp. Sc.,
The Citadel,
Charleston, SC 29409.
ZAHIDI@CITADEL.BITNET

Warnsdorff's algorithm for the knight's tour problem, proposed in 1823, has not yet been proved correct for all boards for which a knight's tour exists. In this paper, after some preliminary observations on the nondeterminism of Warnsdorff, we show that a fixed tie-breaking rule and a fixed start position yield a Warnsdorff's tour for all 3 x n, n > 14 boards.

TERMS: Knight's tour, Warnsdorff's rule for a knight's move,

171 GRAPHS WITH WEAKLY SELF-DUAL CYCLE SPACES

James A. Wiseman, Rochester Institute of Technology

This paper investigates the structure of graphs where circuits intersect circuits in an even number of edges. A coding theorist would call the cycle space of such a graph weakly self-dual. The principal results obtained are: 1) an identification of the maximal simple "weakly self-dual" graph; 2) a clarification of the relationship between multigraphs and simple graphs with this property; 3) a constructive characterization of such graphs in terms of their blocks; and 4) a demonstration that the average maximal valence of these graphs is asymptotically independent of girth.

172 The Embedding-Code Problem

ZHU-XIN HU, University of Illinois at Urbana-Champaign

Let $X = ABCDE$, $Y = ADCBE$ be two words on $\{0,1,2,...,t\}$, where A, B, C, D, E are blocks (or subwords) of X, Y. If (i) $|B| = |D|$, i.e., B and D have the same length, (ii) $D = 00...0$, i.e. D contains only $0's$, (iii) A, B, C, E contain no $0's$, then we define embedding-permutations T_k, T_{-k} by $T_k(X) = Y$, and $T_{-k}(Y) = X$, where $k = |C|$. For any two words X, Y, we say that X and Y are EP-connected if we can transform X into Y by performing a series of (finitely many) of embedding-permutations on X.

Let $X = (1,2)^n0^3$, $Y = 0^31^n2^n$, $n \geq 4$. We have found an algorithm showing that we only need at most $n+1$ embedding-permutations to transform X to Y. Many others results about the embedding-code problem have also been obtained. For any X and Y, we have found a sufficient condition for X and Y to be EP-connected, and (under this condition)an algorithm with $f(n)$ embedding-permutations to transform X into Y, where $f(n)$ is a polynomial and n is the length of X and Y.

Example

$$\begin{aligned}&1212121212000\\ \longrightarrow&1210001212212\\ \longrightarrow&1212211210002\\ \longrightarrow&1200011211222\\ \longrightarrow&1221111000222\\ \longrightarrow&0001111122222\end{aligned}$$

Therefore $\quad T_4T_{-2}T_4T_{-3}T_4((12)^50^3) = 0^31^52^5$.

Thursday, February 6, 1992
4:20 p.m.

173 **Tree Spanners**

Leizhen Cai, Dept. of Computer Science, Univ. of Toronto, Canada

A tree t-spanner T of a graph G is a spanning tree such that for any edge xy of G, the distance in T between x and y is at most t. This notion is motivated by applications in distributed systems and communication networks. We study the problem of deciding whether a graph contains a tree t-spanner, and the problem of determining whether a graph contains a tree t-spanner isomorphic to a given tree. In particular, we present a linear time algorithm for finding a tree 2-spanner in a graph, and an $O(n^{3.5})$ algorithm for deciding whether a 2-connected graph contains a tree 2-spanner isomorphic to a given tree. Generalizations of tree spanners to edge weighted graphs are also considered.

Key words: Tree spanner, isomorphism.

174 The P-density Problem.

Emanuel Knill, Technical University of Nova Scotia.

The P-density problem is a generalization of the union-closed sets conjecture. Let P be a poset. For a poset Q, let Q^P denote the family of order-preserving maps from P to Q. Let L be a semilattice. For $x \in L$, the ***P*-density** of x is given by $|[x)^P|/|L^P|$, where $[x)$ denotes the order filter of L generated by x. Let p be the number of order filters of P. L has the ***P*-density property** iff there is a join-irreducible $x \in L$ such that the P-density of x is at most $1/p$. The P-density problem asks: Which semilattices L with $|L| \geq 2$ have the P-density property? For $P = \{1\}$, $p = 2$ and $|Q^P| = |Q|$ so that the P-density of x in L is $|[x)|/|L|$. The union-closed sets conjecture asserts that all non-trivial semilattices have the $\{1\}$-density property. No non-trivial semilattices without the P-density property are known. Most results related to the union-closed sets conjecture can be generalized to posets P other than $\{1\}$. A survey is provided. New results include the fact that a fixed lattice has the $(1, \ldots, n)$-density property for n large enough and that if L is a union-closed family of sets generated by two-element sets and $\emptyset$, then L has the $\{1\}$-density property.

Keywords: Union-closed sets conjecture, posets, lattices, order-preserving maps.

176 **t-Designs With Prescribed Automorphism Group**

Bernd Schmalz, Universität Bayreuth, 8580 Bayreuth, Germany

Key words: designs, t-designs, double cosets

Kramer and Mesner published in 1976 that the $t-(v,k,\lambda)$ designs admitting automorphism group A are the 0-1-solutions $\vec{x}$ of a system of linear equations

$$M^A_{t,k} \cdot \vec{x} = (\lambda, \ldots, \lambda)^t.$$

$M^A_{t,k}$ are incidence matrices which we compute by means of double cosets. Representing the set of all solutions of the above system of equations implicitly by a graph gives us the possibility either to extract the solutions explicitly or to compute their numbers. We use the structure of overgroups of A in the full symmetric group S_v for the construction of the isomorphism types of the t-designs with full automorphism group A.

To the best of our knowledge our approach for the first time allows to compute the precise number of isomorphism types or even these designs themselves for substantial numbers. We determined the isomorphism types for many known parameter sets and found new 6-designs with parameters

$$6-(28,8,\lambda),\ \lambda = 42, 63, 84, 105,$$

and full automorphism group $P\Gamma L2\{27\}$. We constructed all isomorphism types; their precise numbers are 3 , 367 , 21,743 , and 38,277 repectively.

175 CYCLES AND PATHS ADMITTED BY INDEPENDENT EDGES

C. Cary Timar, Vanderbilt University

Dirac proved that, given k vertices in a k-connected graph, there is a cycle passing through all of them. Lovasz conjectured that the same is true for k independent edges, unless they form an odd edge-cut. Haggkvist and Thomassen proved this for up to k-1 independent edges. I present an improvement on their proof.

Thursday, February 6, 1992
4:40 p.m.

177 THE EXISTENCE OF HOMOMORPHISMS TO ORIENTED PATHS AND CYCLES.

Pavol Hell and Xuding ZHU*, Simon Fraser University

Given an oriented path P, we characterize those digraphs G which are homomorphic to P. The characterization equates the non-existence of homomorphisms G -> P with the existence of homomorphisms W -> G, for some oriented path W which is not homomorphic to P. This result complements the recent polynomial time algorithm (of Welzl et al.) to find a homomorphism to a fixed P. Similar results will be given for oriented cycles C, culminating in a new polynomial time algorithm to find a homomorphism to a fixed unbalanced C.

KEY WORDS: homomorphisms, H-colourings, oriented graphs, characterization results, polynomial algorithms.

178 **Symmetric Chain Decompositions and Protecting Privacy in Data Banks**

George Purdy, University of Cincinnati

In the May 1991 issue of the **Bulletin of the Institute of Combinatorics and its Applications**, M.Miller, I.Roberts and J.Simpson discuss the maximum number of queries that can be made to a Statistical Database without compromising the data of any one individual. After giving an incomplete proof of a correctly guessed solution, using the Sperner theory of finite sets, they pose the "Relative Compromise Problem": How many queries can be made before the difference between the data of two individuals is revealed? We solve this problem using a result by Richard Stanley on Symmetric Chain Decompositons of the lattice M_n, we give a complete proof of the result of Miller et al, and we discuss some generalizations.

Key Words: Chain Decompositions, Posets, Statistical Databases, Sperner's Theorem, Finite Sets.

179 **On Paths and Cycles in Planar Graphs**

Daniel P. Sanders - Georgia Institute of Technology

We prove the following lemma: Let G be a 3-connected plane graph with exterior cycle X_G. Given any three edges of X_G, G has a cycle C through those edges such that every non-trivial bridge of C contains no edge of C and has three vertices of attachment.

We use this lemma to prove some results, two of which are presented. An interior component 3-cut of a 2-connected plane graph G is a minimal vertex cut X of two or three vertices such that there is a component of $G \setminus X$ containing no vertices of the exterior cycle of G. A sufficient condition is given for a 2-connected graph without interior component 3-cuts to be Hamiltonian. We then generalize Thomassen's 1983 Theorem on Paths in Planar Graphs. As a corollary we get that 5-connected toroidal graphs embedded with representativity at most one are Hamiltonian-connected.

180 Character Tables of Association Schemes of Affine Type

Wing Man Kwok, Department of Mathematics, Christopher Newport College, Newport News, Virginia 23606

We define association schemes of affine type. These are the association schemes defined by the action of the semidirect products of finite classical groups and the underlying vector spaces on the same vector spaces. We show that the entries in the character tables of association schemes of affine type are expressed in terms of Gaussian periods. Furthermore, we show that the character tables of association schemes of affine type defined by higher dimensional classical groups are controlled by the character tables of association schemes of affine type defined by a lower dimensional classical group.

Key Words: Association schemes. Gaussian periods.

182 An Isomorphism Invariance For Some Cubical Complexes In Small Dimensions

M.R. Emamy-K., P. Pei*, Dept Of Math. U. Puerto Rico Rio Piedras

Recently, we have characterized all nonisomorphic cut-complexes of the 5- and 6-cubes by an algorithmic method. Here we present some special structure complexes for which the algorithm is much faster. Our computational results show that an isomorphism invariance can replace the isomorphism subroutine of the algorithm with a high degree of efficiecy. Finally, by a counterexample for the 7-cube we show that this invariance does not work for n-cube with n greater than 6.

Key Words. Polytopes, Cubical Complexes, Threshold Boolean Functions.

183 Some Open Problems Regarding Circuit Covers of Graphs

Luis Goddyn,* Math. and Stats., Simon Fraser University, Burnaby, BC; goddyn@cs.sfu.ca

Let (G,w) be an edge-weighted graph. We say that (G,w) has a circuit cover if there exists a list X of circuits in G such that each edge e of G belongs to exactly w(e) circuits in X. For the constant function w(e)=k we have the following.

(1) If k>0 is odd then (G,w) has a circuit cover if and only if G is Eulerian.

(2) If k>2 is even then (G,w) has a circuit cover if and only if G is bridgeless.

(3) The Circuit Double Cover Conjecture asserts that (2) holds for k=2.

We consider the situation when the range of w contains 2 or more integers. For example, if the range of w consists of consecutive integers {k,k+1}, then the following holds:

(4) If k>2 then (G,w) has a circuit cover if and only if

(a) G is bridgeless,

(b) w(e) = w(f) for any 2-edge-cut {e,f},

(c) the edges of odd weight induce an Eulerian subgraph of G.

Statement (4) is equivalent to (3) when k=2, and is false when k=1. If the range of w has the form {k,k+2}, then very little is known. In this case, (4) is conjectured to hold for all k>2, though it is not yet known to hold for any fixed value of k.

184 A POSET-BASED METHOD FOR COUNTING PARTITIONS

George Steiner, McMaster University, Hamilton, Ont., Canada

Traditionally, the counting of various types of partitions of a natural number is done from their generating functions. In this paper we discuss a new, efficient method for counting partitions, which is based on counting the ideals of certain partially ordered sets. This enables us to use the same program for different kinds of partitions.

Friday, February 7, 1992
8:40 a.m.

185 The existence of complete mappings of finite groups.

Anthony B. Evans, Wright State University

In 1955 Hall and Paige proved that a finite group with a non-trivial, cyclic Sylow 2-subgroup cannot admit complete mappings. They conjectured the converse which they proved for solvable groups, symmetric groups, and alternating groups.

Very little progress was made toward solving this conjecture until quite recently. I will survey work on this conjecture and attempt to answer the question, "How close are we to solving the Hall-Paige conjecture?".

186 An Ore Type Condition for the Existence of Spanning Trees with Bounded Degrees

Eduardo Rivera-Campo, Departamento de Matematicas, Universidad Autonoma Metropolitana-Iztapalapa

Let G be a k-connected graph with p vertices. If $r > 1$ is an integer such that the degree sum of every set of $1 + k(r-1)$ independent vertices of G is at least $1 + k(p-k-1)$, then G contains a spanning tree with maximum degree at most r. Moreover if $r > 3$ and the degree sum of any set of $1 + k(r-1)$ independent vertices of G is at least $1 + k(1+((r-1)/(r-2))(p-2k-2))$, then G contains a spanning tree with maximum degree not greater than r and with at most k vertices with degree r.

187 JOINT REALIZABILITY OF CLASSES OF (0,1)–MATRICES WITH GIVEN ROW AND COLUMN SUMS

Kevin McDougal, University of Wisconsin–Oshkosh

Let $R = (r_1, r_2, \ldots, r_m)$, $S = (s_1, s_2, \ldots, s_n)$, $R' = (r_1', r_2', \ldots, r_m')$, and $S' = (s_1', s_2', \ldots, s_n')$ be nonnegative integral vectors. Let $A(R,S)$ denote the class of (0,1)–matrices with row sum vector R and column sum vector S. The three classes $A(R,S)$, $A(R',S')$, and $A(R+R',S+S')$ are called jointly realizable if there exists a matrix X in $A(R,S)$ and a matrix Y in $A(R',S')$ such that $X+Y$ is in $A(R+R',S+S')$. We give examples of classes where joint realizability fails and very special classes were joint realizability holds.

Friday, February 7, 1992
9:00 a.m.

189 Sylow's Theorem and Parallel Computation
Peter D. Mark, University of Oregon

We present parallel algorithms for constructive versions of Sylow's theorem. This includes:
Given: generators for a permutation group G and a prime p,
Find: a Sylow p-subgroup of G.
These results parallelize W. Kantor's polynomial-time sequential algorithms for Sylow's theorem. They add to a growing repertoire of algorithms developed by L. Babai, W. Kantor, E. Luks, A. Seress, and other researchers to solve a variety of problems for permutation groups. Membership testing, finding orders, normal closures, centers, and composition factors are all examples of problems for which not only have polynomial time sequential algorithms been found, but, more surprisingly, efficient parallel algorithms have been given as well by Babai, Luks, and Seress. The talk will concentrate mainly on the solvable case, where the problems are effectively reduced to linear algebra. For general groups, we indicate how a constructive version of the Frattini argument reduces the problem for general groups to the simple case. To solve the problem for simple groups, one proceeds by case analysis as dictated by the classification of finite simple groups. The classification actually arises in a variety of essential ways in the machinery for parallel computation in permutation groups.

190 Continuous k-to-1 maps between graphs.
A.J.W.Hilton, West Virginia University and Reading University

A graph is viewed as a topological space in which each edge is homeomorphic to the closed unit interval [0, 1].

Given G and H let $\ell(G, H)$ be the least integer k such that there is an exactly k-to-1 continuous map from G to H; if there is no such least integer let $\ell(G, H) = \infty$. Let t(G, H) be the threshold number, i.e. the least integer such that for all $k \geqslant t(G, H)$ there is an exactly k-to-1 map from G to H; if there is no such least integer, let $t(G, H) = \infty$. Various bounds for $\ell(G, H)$ and t(G, H) are presented.

191 Min-Cut Clustering
George Nemhauser*, Ellis Johnson, Anuj Mehrota, Georgia Tech

We introduce three classes of clustering problems and study the Min-Cut clustering problems in this paper. We describe a decomposition framework using a column generation scheme to solve these problems. The subproblem to generate additional columns here is itself an NP-hard mixed integer programming problem. We discuss some strong valid inequalities for the subproblem and describe some efficient solution strategies. Some computational results on compiler construction problems are also reported.

Keywords: clustering, column generation, subproblem optimization, valid inequality, compiler design.

192 **On the Non-null Separating Circuits in Embedded Graphs**
Xiaoya Zha and Yue Zhao*, The Ohio State University

In this paper, we try to approach the following question: "Under what conditions do there exist circuits in embedded graphs which separate the surface into two non-trivial parts?"
Suppose the surface has orientable genus $g \geq 2$ or non-orientable genus $k \geq 2$. We prove that representativity at least 6 for an orientable surface and at least 5 for a non-orientable surface are sufficient conditions for the existence of such circuits in embedded graphs. The proof yields a polynomially bounded algorithm to find such separating circuits. Counterexamples are given when the representativity is 2.

Friday, February 7, 1992
10:50 a.m.

193 A NEW ALGORITHM FOR TESTING THE REGULARITY OF A PERMUTATION GROUP

V. Acciaro*, Istituto di Scienze dell' Informazione, Bari, Italy
M.D. Atkinson, School of Computer Science, Carleton University, Ottawa, Canada K1S 5B6

An algorithm is presented for testing whether the group G generated by a given set of m permutations of degree n is regular. The algorithm has a worst case time complexity of O(nm^2). Then a probabilistic modification is proposed which is designed to reduce the execution time in cases where the generating set is redundant. The group parameters which control the execution time of the modified algorithm are discussed.

194 The Maximum Number of Maximal Independent Sets in k-Connected Graphs

Jiuqiang Liu, Dept of Math and Stat
Western Michigan University

A maximal independent set of a graph G is an independent set which is not contained properly in any other independent set of G. Denote $i(G)$ to be the number of maximal independent sets of G. Here, we estabilish two properties about $i(G)$ suggested by Erdős and show that if G is a graph of order n such that either $\Delta(G) = o(n)$ or G has a path of length $l(n) \longrightarrow \infty$ as $n \longrightarrow \infty$, then $i(G) = o(3^{n/3})$. Consequently, we give the maximum number of maximal independent sets possible in a k-connected graph of order n and the extremal graphs, for large n.

195 On the Genus of Star Graphs

David Hoelzeman* and Said Bettayeb Louisiana State University

The star interconnection network has recently been suggested as an alternative to the hypercube. The star graph has rich structure and symmetry properties as well as desirable fault tolerant characteristics. The star graph's maximum vertex degree and diameter, viewed as functions of network size, grow less rapidly than the corresponding measure in a hypercube. We investigate the genus of the star graph and compare it with the genus of the hypercube.

Key words: Genus, Star Graph, Permutation

196 The Costas Invarient on Graphs

Dan Ashlock, Iowa State University Department of Mathematics

The Costas problem has had an infinite number of unsolved cases for a substantial length of time. In this paper I present a generalization of the Costas problem as a first step in building new tools to attack the original problem. The Costas Invarient C(G) of a graph is the dimention of the smallest square array into which the vertecies of the graph can be mapped so that no two adjacent vertecies are in the same row or column and so that no two pairs of adjacent vertecies are connected by congruent vectors. In this paper the elementary properties of the Costas Invarient are developed including bounds in terms of the chromatic number, vertex count, and edge count of the graph, a basis theorem, and the Costas Invarient is computed for some interesting graphs.

Friday, February 7, 1992
11:10 a.m.

198 Random Walks Through Musical Graphs

Ann C. Mugavero, St. John's Univ., S.I. Campus

Graphs, digraphs, semigraphs and networks can be used to clarify the analysis of the harmonizations of musical compositions, as described in "Graphs for Musical Analysis", Advances in Graph Theory, V. R. Kulli, Ed..

These graphs provide a mathematical model for an equivalence class of musical compositions with the same harmonic patterns. Using the graph (digraph or network) of any composition, other compositions can be generated by randomly picking paths through the graphs (digraphs or networks).

This methodology is demonstrated using a program written in the C programming language. The seed networks are models of two harmonizations of the "Passion Chorale". One composed by J. S. Bach and the other by Marcel Dupre, a modern French composer.

199 EMBEDDING HYPERCUBES IN SUPERTOROIDAL NETWORKS

Richard Draper, Supercomputing Research Center
17100 Science Drive, Bowie, MD 20715
e-mail:draper@super.org phone:(301) 805-7366

The supertoroidal network with c^3 nodes is based upon a Cayley graph of a semidirect product of a cyclic group of order c with a cyclic group of order c^2. This construction is analogous to that of the well-known cube-connected cycle which is based upon a wreath product. In this talk, we explore embeddings of hypercubes into supertoroidal networks. These embeddings provide natural means of implementing ascend/descend algorithms (e.g. FFTs) on supertoroidal networks. Our approach is to introduce a mapping which we call an exponential mapping. This approach allows us to capitalize on the underlying group structure in heretofore undiscovered (or at least rarely exploited) ways. In particular, we show how the group structure can reveal that an exponential mapping lends itself to "masked" pipelining of the type commonly used in implementing ascend/descend algorithms on the cube-connected cycle.

200 **Grid Number of a Graph**

Ralph Faudree
Memphis State University

A finite graph G is a grid graph if it can be embedded in a rectangular grid (vertices are lattice points and edges are piecewise vertical and horizontal lines of the lattice). It is know that a graph G is a grid graph if and only if it is a planar graph of maximum degree at most 4. The grid number $gr(G)$ of a graph G is the minimum number of grid subgraphs whose union is G. Thus, for any graph G, $gr(G) \leq th(G)$, the thickness of a graph. The grid numbers for some classical graphs will be determined and bounds on the grid number for some special classes of graphs will be given.

201 Unlabeled Incidence Structures with 2 Points and n Lines

R. Kit Kittappa, Millersville University, PA 17551.

Sets P, L and $I \subseteq P \times L$ (called sets of points, lines and incidences, respectively) determine an incidence structure. It is shown that the number of nonisomorphic unlabeled incidence structures with 2 points and n lines is $(1/24)(n+2)(n+4)(2n+3)$, if n is even and $(1/24)(n+1)(n+3)(2n+7)$, if n is odd. Dually, the same formula holds for incidence structures with n points and 2 lines. Determining the corresponding formula for m points and n lines is an open problem.

Key words: Combinatorics, finite geometry, incidence structures.

202 On Signed Graphs with Prescribed Positive and Negative Graphs

Gary Chartrand, Western Michigan University
Frank Harary, New Mexico State University
Héctor Hevia, Grand Valley State University
*Kathleen A. McKeon, Connecticut College

In a signed graph S, each edge is either positive or negative. The subgraph S^+ of S is induced by the positive edges of S, while S^- is induced by its negative edges. For graphs F and H without isolated vertices, the sign number s(F, H) is the minimum order of a signed graph S such that S^+ is isomorphic to F and S^- is isomorphic to H. Some bounds for s(F, H) are presented. Also, the number s(F, H) is determined when one of the graphs is a star or complete. Moreover, s(F, H) is computed when both graphs are forests.

Key Words: signed graphs, trees, forests

203 Generalized Hypercube Embeddings

M. Rathi* and R. Bagai
Dept. of Computer Science
Wichita State University
Wichita, Kansas

A. Boals and A. Gupta
Dept. of Computer Science
Western Michigan University
Kalamazoo, Michigan

The problem of embedding an N-processor architecture G into an M-processor architecture H for $N > M$ arises when algorithms designed for architectures of an ideal size are simulated on existing architectures which are of a fixed size. In [1] we presented optimal embeddings for the case when both architectures are boolean (2-ary) hypercubes and the embeddings are to achieve a balanced load. An embedding achieves a balanced load if every processor of H simulates at most $\lceil \frac{N}{M} \rceil$ processors of G.

In this paper we generalize the results and show that an N-processor b-ary hypercube G can be embedded into an M-processor b-ary hypercube H for $b > 2$ with a balanced load, unit dilation and an optimal congestion of $\lceil \frac{N}{M} \rceil$. The main contribution of the paper is the lower bound on the congestion. The lower bound proof uses the key idea of counting the number of edges in a *compact b-ary hypercube*. We thus also introduce compact b-ary hypercubes that are defined to have any arbitrary number of processors (in contrast traditional complete b-ary hypercubes have only b^p-processors for some $p \geq 0$).

[1] A. Boals, A. Gupta, S. Hambrusch and N. Sherwani, "A Lower Bound on Embedding Large Hypercubes into Small Hypercubes" *Congressus Numerentium*, 78:141-151, 1990.

204 Highly Irregular Asymmetric Graphs

Peter R. Christopher, Worcester Polytechnic Institute
Worcester,MA 01609

A connected graph is said to be highly irregular if each of vertices has the property that the degrees of its neighbors are all distinct. A graph is asymmetric if the only automorphism of its vertex set is the identity. The existence of graphs which are both highly irregular and asymmetric is estabished for all possible orders. Corresponding results are determined for highly irregular asymmetric trees.

Keywords: highly irregular graphs, asymmetric graphs

205 DESARGUESIAN HYPEROVALS AND THE CREMONA TRANSFORMATION
William Cherowitzo, University of Colorado at Denver

The classical Cremona transformation, (x,y,z) --> (xz,yz,xy), is used to study hyperovals in finite Desarguesian planes. The latest results on the existence of these hyperovals will also be presented.

Keywords: Hyperovals, conics, Cremona transformation

206 **On the Edge Number of Graphs with No Dense k-Subgraphs**
Jerrold R. Griggs, University of South Carolina, Columbia, SC
George Rubin Thomas*, Benedict College, Columbia, SC

Let $\varepsilon(G)$ denote the number of edges of the simple graph G. Consider n-vertex graphs G such that for every k-vertex subgraph H of G, $\varepsilon(H) \leq l$. We determine the maximum value of $\varepsilon(G)$ for such graphs, denoted by $E(n,k,l)$, when l is large or small. Let $t_j(n) := \varepsilon(T_j(n))$, where $T_j(n)$ is the balanced complete j-partite graph on n vertices. For $l = \binom{k}{2} - \lambda$, $k \geq 2\lambda > 0$, we obtain that $E = t_{k-\lambda}(n)$. Moreover, $T_{k-\lambda}(n)$ is the unique extremal graph provided that

(1) $k > 2\lambda, \quad n \geq k+1,$

(2) $k = 2\lambda, \quad n \geq 2k-1,$

(3) $\lambda = 1, \quad n \geq k.$

Case (3) is the familiar theorem of Turán. For $0 \leq l < k-1 < n$, we determine that $E = l + \lceil (1-\frac{1}{\gamma})(n-k) \rceil$, where $\gamma = \lfloor (k-1)/(k-1-l) \rfloor$, and the structure of the extremal graphs is obtained.

Keywords: Turán's Theorem, extremal graph theory, edge number

207 Recursive Cube-Connected Cube Networks
Larry Brown
Department of Computer Science and Engineering
Florida Atlantic University

In this paper a recursive extention of the Cube Connected Cube (CCC) network, the Recursive Cube Connected Cube (RCCC) network is defined. Several special types of RCCC are shown to be the same as other sturctures such as hypercubes and binomial trees. Several topological properties of RCCCs are derived, and are used to compare the RCCC to other types of interconnection networks. The RCCC is found to have a low average node degree, and to be suitable for compute, aggregate, and broadcast (CAB) and divide and conquer algorithms.

Index Terms:
Hypercubes, interconnection networks, divide and conquer.

208 Higher Dimensional Representations of Graphs
Andreas Buja, Nathaniel Dean*, Michael L. Littman and Deborah Swayne
Bell Communications Research, Morristown, NJ 07960-1910

Graphs are often used to model complex systems and to visualize relationships, and this often involves drawing a graph in the plane. Actually, it is more natural and more meaningful to view higher dimensional representations of the graph. We present some of the theory and problems associated with constructing such representations, and we describe some computer tools which are now available for experimental research in this area.

213 Existence of 1-factors in double vertex graphs

Y. Alavi*and Jiuqiang Liu, Western Michigan University
Don R. Lick and Biwen Zhu, Eastern Michigan University

The *double vertex graph* $U_2(G)$ of a graph G is defined to be the graph whose vertex set consists of all 2-subsets of $V(G)$ and two vertices of $U_2(G)$ are adjacent if and only if they contain exactly one common element and the other two elements are adjacent in G. Here, we will give sufficient conditions for a double vertex graph to have a 1-factor.

214 Construction of Topological Non–desarguesian Projective Klingenberg Planes

C.A. Baker, Mount Allison University
N.D. Lane, McMaster University
J.W. Lorimer, University of Toronto

P.Y. Bacon (1979), M. Dugas (1979) and D. Keppens (1988) have coordinatized a PK–plane by a planar sexternary ring (PSR), $\mathcal{R} = \langle R, T_1,\ldots, T_6 \rangle$, where $T_1,\ldots, T_6$ are ternary operators defined on certain subsets of R^3. Conversely, Bacon and Keppens have constructed a PK–plane $\mathcal{P}(\mathcal{R})$ from a PSR, $\mathcal{R}$. By using $\mathcal{P}(\mathcal{R})$ the authors have extended the domains of the T_i to R^3 and have altered the assumptions on the T_i appropriately to introduce the notion of a full PSR, $\mathcal{R}' = \langle R, T_1',\ldots, T_6' \rangle$. They have then assumed that R is a topological space in which the set of non–units is closed, each T_i' is continuous and the solutions of certain systems of equations involving pairs of the T_i' are continuous. With such a topological full PSR, $\mathcal{R}'$, they have shown that the PK–plane $\mathcal{P}(\mathcal{R}')$ is a topological PK–plane. By using this technique, they have constructed examples of topological non–desarguesian PK–planes.

215 SUBTREE CATCH GRAPHS

Terry McKee, Wright State University, Dayton OH 45435

Much as intersection graphs come from sets, catch graphs come from pointed sets (sets, each with a distinguished element).

Fred Roberts proved that every interval catch graph is a proper interval graph by showing how every interval catch representation determines a proper interval intersection representation.

We extend this to trees, showing how every subtree catch representation (on host tree T) determines a proper subtree intersection representation (on a host tree with the same number of leaves as T).

216 PARALLEL ALGORITHMS: CABLES AND k-TREES

Gur Saran Adhar*
UNC- Wilmington
adhar@seq.uncwil.edu
Shietung Peng
University of Maryland Baltimore County
peng@algol.cs.umbc.edu

The class of k-trees generalize the notion of trees, maximal outerplanar graphs, and caterpillars. In this paper we present new parallel *NC* algorithms to find a collection of k vertex disjoint paths between specified vertex pair (u, v), called a *uv-cable*, in a k-tree. Parallel algorithms to compute k-path in a k-tree, and k-tree recognition are also presented. The algorithms are based on parallel construction of a representation for k-trees, referred to as *spatial graph* in this paper.

The model of parallel computation used is the CRCW P-RAM (Concurrent Read Concurrent Write Parallel RAM), where more than one processor can concurrently read from or write into the same memory location during the same memory cycle. Writing conflicts are resolved in a non-deterministic fashion. All the algorithms require $O(n^2)$ processors and run in $O(log^2 n)$ time on a CRCW P-RAM.

Keywords: NC algorithm, k-tree, graphs

Friday, February 7, 1992
2:20 p.m.

217 Optimal Transitional Labelings of Connected Graphs

Héctor Hevia
Grand Valley State University, Allendale, MI 49401

A transitional labeling t of a graph G is an assignment of labels to the elements of G (edges and vertices of G), where the labels are taken from the set $\{-1, 0, 1\}$ and such that, for every vertex v and for every edge e incident in G, if $t(v)$ is not equal to $t(e)$, then $t(v) = 0$. If at least one vertex of G is labeled 1 by t and at least one vertex of G is labeled -1 by t then, t is called a polarization of G. Otherwise, the transitional labeling t is called a quasipolarization. The value of t is the minimum between the number of positive elements of G and the number of negative elements of G. The transitional value of a graph G is the maximum possible value that a transitional labeling of G can have. A transitional labeling of G whose value is maximum is called optimal. We give formulas for the transitional values of some families of graphs. It can be proven that for most of the trees and for most of the unicyclic graphs, there exists an optimal transitional labeling that is a polarization. For a given positive integer p, we discuss the existence of connected graphs on p vertices whose optimal transitional labelings are only quasipolarizations.

218 Algebraic Properties of Minkowski Rings

Klaus Fischer & Jay Shapiro, George Mason University

A commutative ring structure called the Minkowski ring may be imposed on the collection of characteristic functions of polytopes in $\mathbb{R}^d$ by defining multiplication via the Minkowski sum of two polytopes. If $\{P_1,\cdots,P_s\}$ is a finite set of polytopes in $\mathbb{R}^d$, then the subring $M^d[P_1,\cdots,P_s]$ generated by these polytopes is isomomorphic to a ring of polynomials $Z[X_1,\cdots,X_s]$ modulo an ideal of relations I. Here, Z is the ring of integers. The purpose of the paper is to describe the ideal I and more generally to study the prime ideals and the Krull dimension of this ring. The ring $M^d[P_1,\cdots,P_s]$ is reduced and the minimal primes arise from relations of rational dependency between vertices $v_1,\cdots,v_s$, one chosen from each P_i, so that respectively they are extremal over an open cone of directions in $\mathbb{R}^d$.

219 ON AGREEMENT SUBTREES OF TWO EVOLUTIONARY TREES

Ewa Kubicka, Grzegorz Kubicki*, F.R. McMorris
University of Louisville

Let S be a set of n objects. An evolutionary tree of S is a binary tree whose all endvertices are labeled without repetition from the study collection S. The operation of pruning of a tree T is removing some endvertices from T and suppressing all inner vertices of degree 2 which are formed by this deletion. Having two or more evolutionary trees, their largest common pruned tree is called an agreement subtree. For two evolutionary trees we discuss how many labeled vertices must be present in their agreement subtree.

Key words: evolutionary trees, pruning, agreement subtree.

220 A New Result on State Minimization in LR Parsers

Manuel E. Bermudez*, University of Florida, Gainesville, FL 32611
George Logothetis, AT&T Bell Laboratories, Summit, NJ 07901.

We consider the following problem: given an LR(1) grammar, i.e. one for which an LR(1) machine is known to exist, construct the smallest possible LR(1)-equivalent machine. Previous work by Pager on this problem has characterized it as a problem of minimizing a partially specified Finite-State Machine. However, partially specified machines are very difficult to work with, and the associated algorithms are very time-consuming. Our approach is different: we have characterized the problem as minimization of a completely specified finite-state machine, given an initial configuration of certain item sets. Standard (efficient) FSA minimization algorithms apply. However, the result is not necessarily minimal in an absolute sense; instead, it is minimal with respect to the initial configuration chosen. In this paper, we present our minimzation algorithm, and we discuss the trade-off we have discovered.

Keywords: LR(1) Machine, Finite-State Machine, State Minimization

Friday, February 7, 1992
2:40 p.m.

221 A CLASS OF GRACEFUL 2-REGULAR GRAPHS CONSISTING OF 4-CYCLES

Jaromir Abrham, Department of Industrial Engineering, University of Toronto, and Anton Kotzig (deceased), département de mathématiques et de statistique, Université de Montréal.

A graceful valuation of a graph G with m vertices and n edges is a one-to-one mapping ψ of the vertex set V(G) into the set $\{0,1,2,\ldots,n\}$ with the following property: If we define, for any edge $e \in E(G)$ with the end vertices u, v, the value $\bar{\psi}(e)$ of e by the equation $\bar{\psi}(e) = |\psi(u) - \psi(v)|$ then $\bar{\psi}$ is a one-to-one mapping of the set E(G) onto the set $\{1,2,\ldots,n\}$. A graph is called graceful if it has a graceful valuation. In some earlier papers, the authors have shown that the graph kC_4 (consisting of k 4-cycles) has an α-valuation (a stronger form of a graceful valuation) for infinitely many values of k; however, the density of those values of k for which kC_4 is graceful appeared to be very low. The main result of the present paper leads to the conclusion that this density is at least 27/128=0.2109.

Key words: graceful valuations, 2-regular graphs.

222 Special Moore Geometries

Frederick J. Fuglister, John Carroll University

In general the diameter, d, of a generalized Moore geometry with parameters a, b and c can be at most 13. Lower bounds have been established in the special cases $c = 0$, $c = a$, $c = b$ and $a = b$.

The formula for the multiplicity of an eigenvalue, λ, of a generalized Moore geometry is a quotient of two cubic polynomials in λ. In the above special cases however, this formula degenerates into a quotient of two quadratic polynomials.

We show that there are exactly two further special cases in which this phenomenon occurs. In the first case, $c^2 = ab$ and there can be no examples. In the second case, $(a(d+1) + c(d-1))\sqrt{ab} = a(b+c)d$ and d must be 1, 2 or 4.

223 CORES OF GRAPHS

Héctor Hevia, Karen S. Novotny and Donald W. VanderJagt*
Grand Valley State University, Allendale, MI 49401

We define a core of a connected graph G as a connected subgraph C of G of minimum size such that whenever C′ is a subgraph of G with C′ ≅ C then C′ = C. For a connected graph G, an induced core C of G is a connected induced subgraph of G such that for every induced subgraph C′ of G such that C′ ≅ C then C′ = C. We discuss the existence of graphs with a core and an induced core of given sizes. In particular, some bounds for the size of an induced core of G are presented. The existence of graphs which are cores or induced cores of some other graphs is also discussed.

224 A NOVEL METHOD OF FUNCTION DECOMPOSITION

Zhonggang Li* and Jie Wu
Department of Computer Science and Engineering
Florida Atlantic University

In this paper we propose a novel function decomposition method which decomposes a logic function into a set of unate functions. Several graph-based algorithms are provided to transform, through several steps, the graph representation of a logic function into a set of specific graphs representing unate functions. Through this process, a logic function can be implemented by several unate function modules and a connection module. It is shown that the union of the test sets for unate function modules constitutes a complete test set for the circuit. The results obtained in this paper could be used to develop a design technique that makes testing logic circuit more economical. Because the processes of sensitization and line justification are easily implementable for unate functions, each implemented by a circuit that is easily testable, we therefore can achieve a reduced size of the test set.

INDEX TERMS:
Complete test sets, unate functions, function decomposition, Hasse diagram, restricted gate networks

225 Eternal Redemption or Damnation? . . . A Four-gon Conclusion?

Yousef Alavi, Jiuqiang Liu, Joseph McCanna, and Allen J. Schwenk*, Western Michigan Univ., Kalamazoo, MI 49008, and Paul Erdős, Hungarian Academy of Sciences.

1. In the beginning God created graphs. And the graphs were void and without labels, and darkness covered the face of the graphs. And God gave Man dominion over the graphs.

2. And the Lord said unto Man, "Seek to name these vertices with nonnegative integers, and thereby induce edge labels. And lo, ye shall find many are filled with grace, the bicliques, the lowly caterpillars, the lobsters, and even the trees. (Well, . . . I forget. Maybe not all trees. Perhaps it is just most trees.) But in others ye shall find no grace."

3. And Man labeled as God had directed, and he found much grace hidden in God's works. But he also found the 5-gon and the 5-clique, totally lacking grace.

4. Wherefore the Lord said, "Do not despair that some lack grace. Let these graphs, one and all, be fruitful and multiply, each bringing forth fruit after his own kind. And if any graph, even one void of grace, shall, in the fullness of time, bear fruit that attains grace, I shall call it 'redeemed'. And if it remains eternally redeemed, it shall be called 'holy'. But if any graph, yea even the graceful Trinity, bears fruit that falls from grace, it shall be 'disgraced'. And if it remains eternally disgraced, I shall call it 'damned' and shall cast it into the abyss."

5. Wherefore Man set out to obey God's word. And he cast his eye upon the lowly 4-gon, and found it to be graceful even unto the 35th generation.

6. But Man still could not discern the eternal state of grace for all the generations.

7. Wherefore Man despaired of ever knowing God's plan for eternal redemption and damnation.

226 Some Functions Related to the van der Waerden Numbers

Bruce M. Landman, UNCG, Greensboro, North Carolina

Numbers similar to the van der Waerden Numbers $w(n)$ are studied, where the class of arithmetic progressions is replaced by certain larger classes. If A' is such a larger class, we define $w'(n)$ to be the least positive integer such that every 2-coloring of $\{1,2,\ldots,w'(n)\}$ will contain a monochromatic member of A'. We first consider sequences of positive integers $\{x_1,\ldots,x_n\}$ which are either arithmetic progressions or for which there exists a polynomial $p(x)$, having integer coefficients, satisfying $p(x_i) = x_{i+1}$, $i=1,\ldots,n-1$, with certain further restrictions placed on the degrees and coefficients of the polynomials allowed. We then consider sequences $\{x_1,\ldots,x_n\}$ which satisfy $x_i=a_ix_{i-1}+b_ix_{i-2}$ for $i=3,\ldots,n$, with various restrictions placed on the a_i and b_i. In both cases, upper bounds are given for the corresponding functions $w'(n)$. Further, it is shown that the existence of somewhat stronger bounds on $w'(n)$ would imply similar bounds for $w(n)$.

Key words: van der Waerden numbers, arithmetic progressions, second order recurrence, polynomial iteration

228 Extended *And-Or* Graph Models and Satisfaction Trees

Renée A. McCauley and W. R. Edwards Jr.*, University of Southwestern Louisiana

Extended *and-or* graphs are used to model the logical structure of Prolog programs. The graphs are "extended" because, in addition to the *and* nodes and *or* nodes found in the well-known search graphs of artificial intelligence, these contain *exclusive-or* nodes. The *exclusive-or* nodes are appropriate for modeling the Prolog *cut* facility. The recursive nature of algorithms due to the recursively defined data structures of Prolog makes the graphs commonly cyclic. Each defined predicate is modeled as a node whose children are the predicates that constitute its definition. Leaf nodes represent facts and system defined predicates. A *satisfaction tree* is defined as a set of constraints/predicates whose satisfaction is sufficient to satisfy a predicate. The *maximum number of satisfaction trees (MST)* for any predicate, the maximum number of ways a predicate might be satisfied, is an indicator of conceptual complexity and can be calculated from the graph. The contributions of this paper include the definition of the extended *and-or* graph model and the development of a computationally efficient algorithm to extract MST counts.

Keywords: *and-or* graphs, extended *and-or* graphs, models of programs, Prolog cut facility, conceptual complexity, satisfaction tree, MST counts