Program and Schedule

# Nineteenth Southeastern International Conference on

# *COMBINATORICS* • *GRAPH THEORY* • *COMPUTING*

**February 15 - 19, 1988**
**Pleasant Hall**
**Louisiana State University**
**Baton Rouge, Louisiana**

Sponsored by:

>The Department of Mathematics

>The Division of Continuing Education

>U.S. Office of Naval Research

**LOUISIANA STATE UNIVERSITY**

MONDAY; FEBRUARY 15, 1988

| | |
|---|---|
| 8:00 | REGISTRATION (PLEASANT HALL LOBBY) |
| 9:00 | WELCOME (ROOM 148) |
| 9:20 | THOMASSEN "CYCLES IN GRAPHS AND DIRECTED GRAPHS" ✓ |
| 10:20 | COFFEE (ROOM 148 - SOUTH) |

| | ROOM 148 | ROOM 130 | ROOM 113 - 115 |
|---|---|---|---|
| 10:40 | 1 J.W. MOON | 2 DE CAEN ✓ | 3 W.F. SMYTH |
| | | | 142 Arasu ✓ |
| 11:00 | 4 D.BIENSTOCK | 5 R.REES | 9 A. HOBBS |
| 11:20 | 7 M. ALBERTSON | 8 G. ZHANG | 12 P. JOHNSON |
| 11:40 | 10 C. COLBOURN | 11 K. PHELPS ✓ | |

| | |
|---|---|
| 12:00 | LUNCH |
| 1:30 | THOMASSEN "CYCLES IN GRAPHS AND DIRECTED GRAPHS" |

| | ROOM 148 | ROOM 130 | ROOM 113 - 115 |
|---|---|---|---|
| 2:30 | 13 G. SIMMONS | 14 S. FAJTLOWICZ | 15 T. BROWN |
| 2:50 | 16 G. PURDY | 17 W. STATON | 18 D. BIENSTOCK |
| 3:10 | 19 L. BATTEN | 20 B. WALLER | 21 J. WISEMAN |
| 3:30 | COFFEE | | |
| 3:50 | 22 W. SCHNYDER | 23 P. CATLIN | 24 D. PRITIKIN |
| 4:10 | 25 H. HARBORTH | 26 R. HEMMINGER | 27 M. TRUSZCZYNSKI |
| 4:30 | 28 J. MANNING | 29 N. DEAN | 30 W.D. WALLIS |
| 4:50 | 31 W. GASARCH | 32 J. OXLEY | 33 K. JONES |
| 5:10 | 34 C. LOVEGROVE | 35 S. AKKARI | 36 W.R. EDWARDS |
| 5:30 | 37 T. TROTTER | 38 T.J. REID | 39 S.H. HUANG |

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|---|---|
| 6:00 | WINE AND CHEESE RECEPTION (LSU FACULTY CLUB) |
| 7:45 | TRANSPORTATION BACK TO MOTELS |

TUESDAY; FEBRUARY 16, 1988

8:15 REGISTRATION (PLEASANT HALL LOBBY)

9:00 KLAWE "FAST ALGORITHMS FOR CONVEX POLYGON PROBLEMS" (ROOM 148)

| | ROOM 148 | ROOM 130 | ROOM 113-115 | ROOM 48A |
|---|---|---|---|---|
| 10:00 | 40 C.T. HOANG | 41 R. SCHELP | 42 D.R. GUICHARD | 43 M. WEISS |
| 10:20 | 44 Y. CRAMA | 45 J. GIMBLE | 46 J. YUCAS | 47 K. BAIK |
| 10:40 | 48 RAYCHONDHURI | ~~49 E. COCKAYNE~~ 179 | 50 I. RUBIO | 51 P.J. CHASE |
| 11:00 | 175 V. LAKSHMANAN | 176 E.O. HARE | 177 M.D. ATKINSON | 178 J.B. KIM |

11:25 "FAST ALGORITHMS FOR CONVEX POLYGON PROBLEMS" (ROOM 148)

12:15 LUNCH; TRANSPORTATION BACK TO MOTELS

P.M. FREE

179 R. Aldred

WEDNESDAY; FEBRUARY 17. 1988

| | | | | | |
|---|---|---|---|---|---|
| 8:30 | REGISTRATION | | | | |
| 9:00 | BRICKELL "CURRENT DEVELOPMENT IN CRYPTOANALYSIS" (ROOM 148) | | | | |
| 9:50 | COFFEE | | | | |
| | ROOM 148 | ROOM 130 | ROOM 113-115 | ROOM 48A | |
| 10:10 | 52 D. STINSON | 53 R. BRIGHAM | 54 C. PEYRAT | 55 M. FELLOWS | |
| 10:30 | 56 S. PAYNE | 57 T. RICE | 58 D. TZVIELI | 59 M. MATTHEWS | |
| 10:50 | 60 C. LIN | 61 G.S. DOMKE | 62 S. MIN LEE | 63 D. FERNANDE | |
| 11:10 | 64 F. BENNETT | 65 D. GRINSTEAD | 66 W.F. SMYTH | 67 L. KOTIN | |
| 11:30 | ~~68 D. KREHER~~ 125 | 69 K. PETERS | 70 Z. MO | 71 J. RISTROPH | |
| 11:50 | 72 M. HALL, JR. | 73 B.L. HARTNELL | 74 A. SUNG | 75 P. GUAN | |
| 12:10 | LUNCH | | | | |
| 1:30 | BRICKELL "CURRENT DEVELOPMENTS IN CRYPTOANAYLSIS" (ROOM 148) | | | | |
| | ROOM 148 | ROOM 130 | ROOM 113-115 | ROOM 48A | |
| | | | | SPECIAL SESS.<br>C. EALY, ORGAN | |
| 2:30 | 76 J.C. BERMOND | 77 D. SHIER | 78 C. BAILEY | 79 R. LIDL | |
| 2:50 | 80 C. COLBOURN | 81 P. HELL | 82 J. HEMMELET | 83 L. FINKELST | |
| 3:10 | 84 D. LEONARD | 85 K. SPITERI | 86 T. DONOVAN | 87 D. ARCHDEAC | |
| 3:30 | COFFEE | | | | |
| 3:50 | 88 R.E. PIPPERT | 89 J. ABRHANMON | 90 A.J. HILTON | 91 P. EADES | |
| 4:10 | 92 P. SLATER | 93 M. BENSON | 94 E.L. LEISS | 95 J. MANNING | |
| 4:30 | 96 K.T. SIEGRIST | 97 B. RICHTER | 98 J. OWINGS | 99 W. CHEROWITZO | |
| 4:50 | 100 T. SONEOKA | 101 N. DEO | 102 VANDER LAAN | 103 J. SOTERO | |
| 5:10 | 104 V. RICE | 105 S. MONSON | 106 H.A. KIERSTEAD | 107 M. LIPMAN | |
| 5:30 | 108 K. WILLIAMS | 109 VESTERGAARD | 110 N. HARTSFIELD | 111 J. STRAIGHT<br>121 M. ATKINSON | |
| 6:00 | TRANSPORTATION TO MOTELS (RETURNING ABOUT 6:40) | | | | |
| 7:00 | CONFERENCE SEAFOOD BUFFET (PLANTATION ROOM; LSU STUDENT UNION) | | | | |
| 9:15 | TRANSPORTATION TO MOTELS | | | | |

THURSDAY; FEBRUARY 18, 1988

8:15 REGISTRATION

9:00 FRANKL "OLD AND NEW PROBLEMS ON FINITE SETS" (ROOM 148)

9:50 COFFEE

| | ROOM 148 | ROOM 130 | ROOM 113-115 |
| --- | --- | --- | --- |
| 10:10 | 112 F. HARARY | 113 N. PIPPENGER | 114 J. GROSSMAN |
| 10:30 | 115 S. STUECKLE | 116 H. LEVINSON | 117 B. PIAZZA |
| 10:50 | 118 R.J. FAUDREE | 119 J. GEORGES | 120 D. BERMAN |
| 11:10 | 121 M. ROSENFELD | 122 W. HARE | 123 C.Q. ZHANG |
| 11:30 | 124 EL-MALLAH | ~~125 D.F. HSU~~ 180 D. Hoffman | 126 A.J. BOALS |
| 11:50 | 127 S.M. VENKATESAN | 128 W. GU | 129 A. SCHWILL |

12:10 LUNCH

| | ROOM 148 | ROOM 130 | ROOM 113-115 |
| --- | --- | --- | --- |
| 1:30 | 130 R. LUNDGREN | 131 F. LAZEBNIK | 132 B. ANDERSON |
| 1:50 | 133 D. BERGSTRAND | 134 J. STRAIGHT | 135 R. STERNFEL |
| 2:10 | 136 T. McKEE | 137 J. ELLIS | 138 J. DiPAOLA |
| 2:30 | 139 A. PELC | 140 A. GYARFAS | 141 C. RYAN |
| 2:50 | ~~142 K.T. ARASU~~ | 143 A. MOISIADIS | 144 A. BAARTMANS |
| 3:10 | 145 F. RUSKEY | 146 C. BAREFOOT | 147 J. BROWN |
| 3:30 | COFFEE | | |
| 3:50 | 148 M. RAMRAS | 149 R. ANSTEE | 150 S. NTAFOS |
| 4:10 | 151 J. ZAKS | 152 R. MADDOX | 153 S.R. DAS |
| 4:30 | 154 C.N. PURDY | 155 B. VARMA | 156 I.G. TOLLIS |
| 4:50 | ~~157 KIM HEFNER~~ 182 Lazergne | 158 P. CHINN | 159 S. NARAYANAN |
| 5:10 | 160 A. HOBBS | ~~161 E. REGENER~~ | 162 D. McINTYRE |
| 5:30 | | | |

6:00 TRANSPORTATION TO MOTELS
7:30 TRANSPORTATION TO PARTY
8:00 SURVIVORS DESSERT PARTY (2626 DALRYMPLE DRIVE)
10:10 TRANSPORTATION TO MOTELS

FRIDAY; FEBRUARY 19, 1988

8:15 REGISTRATION

9:00 RODL "RECENT DEVELOPMENTS IN RAMSEY THEORY" (ROOM 148) ✓

9:50 COFFEE

| | ROOM 148 | ROOM 130 | ROOM 113-115 |
|---|---|---|---|
| 10:10 | 163 L. CUMMINGS | 164 D. MUNTON | 165 F. HADLOCK ✓ |
| 10:30 | 166 M. LEWINTER | 167 ~~T. JOHNSON~~ P. Slater | 168 D. SIMOVICI ✓ |
| 10:50 | 169 M.E. MAYS | 170 M. GILPIN ✓ | 171 S.M. VENKATESAN |
| 11:10 | 172 A.B. GAMBLE | 173 C. SISTAR ✓ | 174 L. KOTLIN |
| 11:30 | | | |
| 11:50 | | | |

12:10 END - - SEE YOU NEXT YEAR!

# SOCIAL EVENTS

SUNDAY, FEBRUARY 14: The Pre-Conference Mixer will be held from 7:00 - 9:00 p.m. in 148 Pleasant Hall. Drinks (both hard and soft) and chips will be served. Note: Registration will begin from 6:00 to 8:00 p.m. in the lobby of Pleasant Hall.

MONDAY, FEBRUARY 15: The Conference Wine and Cheese Reception will be held from 6:00 - 7:30 p.m., in the LSU Faculty Club (next to the Law School at the southeast corner of the Parade Grounds).

TUESDAY, FEBRUARY 16: Mardi Gras - individual activities.

WEDNESDAY, FEBRUARY 17: The Conference Seafood Buffet Banquet will be held from 7:00 - 9:00 p.m., in the Plantation Room of the LSU Student Union Building. Beer, wine, and soft drinks will be available at a cash bar.

THURSDAY, FEBRUARY 18, 1988: A Survivors Dessert Party will be held from 8:00 - 10:00 p.m., at the home of Brooks and Marion Reid, 2626 Dalrymple Drive, a walk of less than a mile from Pleasant Hall. Parking is available on April and June Streets. (Please do not park on the grass.)

# MISCELLANEOUS INFORMATION

Registration will be held in the lobby of Pleasant Hall beginning Sun. 6-8 p.m., Mon. 8-9 a.m., Tues. 8-9 a.m. Late registration will be held Wednesday - Friday in the south end of room 148 in Pleasant Hall.

To leave important messages, phone 388-3255. There will be a bulletin board in the south half of room 148 of Pleasant Hall.

Word processing, photocopying, and blank transparencies are available at Kinko's behind Pleasant Hall on State St.

Coffee and rolls will be available in the south half of room 148 Pleasant Hall beginning Monday morning. A display table will be there also.

Transportation: A bus will arrive at the following hotels at these times:

| | |
|---|---|
| Hilton | 7:50 and 8:30 a.m. |
| Hampton | 8:00 and 8:40 a.m. |
| Sheraton | 8:00 and 8:40 a.m. |
| Ramada | 8:10 and 8:45 a.m. |
| Inn on the Lake | 8:00 and 8:35 a.m. |

Jan Davidson, the Conference Hostess, will be available during all breaks and lunches to assist conference participants. She will be located in the south half of room 148 in Pleasant Hall

# INVITED INSTRUCTIONAL LECTURES

ALL OF THE INVITED LECTURES WILL BE HELD IN 148 PLEASANT HALL

NDAY, FEBRUARY 15, Professor Carsten Thomassen will speak at 9:20 a.m. and 1:30 .m. on "Cycles in Graphs and Directed Graphs."

ESDAY, FEBRUARY 16, Dr. Maria Klawe will speak at 9:00 a.m. and 11:15 a.m. on ast Algorithms for Convex Polygon Problems."

EDNESDAY, FEBRUARY 17, Dr. Ernest Brickell will speak at 9:00 a.m. and 1:30 p.m. "Current Developments in Cryptoanalysis."

URSDAY, FEBRUARY 18, Dr. Peter Frankl will speak at 9:00 a.m. on "Old and New oblems on Finite Sets."

IDAY, FEBRUARY 19, Professor Vojtech Rodl will speak on "Recent Developments in amsey Theory."

# ABSTRACTS OF INVITED LECTURERS

Cycles in Graphs and Directed Graphs.

Carsten Thomassen, The Technical University of Denmark.

Abstract. The lectures will survey sufficient conditions and polynomially bounded algorithms for finding cycles of a specified type in graphs and directed graphs. We consider cycles of length k modulo d and their applications. For example, the problem of finding an even cycle in a directed graph (which is still unsolved) is of interest in connection with sign non-singular matrices. We discuss the existence of various types of cycles implied by a large minimum degree or girth, and we present a polynomially bounded algorithm for finding a shortest cycle in many families of cycles, for example the noncontractible cycles in an embedded graph. We survey some results obtained by cycle space methods, we present a directed analog of Whitney's 2-switching theorem, and we discuss to which extent a graph or digraph is uniquely determined by certain collections of cycles. For example a strongly connected tournament is uniquely determined, up to isomorphism or anti-isomorphism, by its collection of 4-cycles, a result which was conjectured in 1971 by Goldberg and Moon. Also other recent results on cycles in tournaments will be discussed.

Recent Develpoments in Ramsey Theory

Vojtech Rödl, Czech Technical University and Emory University

We will present a survey of some of the recent results in Ramsey theory. The emphasis will be on extistence results dealing with Ramsey type theorems for various structures. A few open problems will be mentioned.

**Current Developments in Cryptanalysis**

Ernest F. Brickell
Bellcore and Sandia National Laboratories

<u>**Abstract**</u>

The last decade has seen explosive growth in unclassified research in all aspects of cryptology. New cryptosystems have been devised that base their security on the difficulty of some easily stated mathematical problems. Many of these problems were new problems that had not been previously studied. There has been a great deal of success at solving these problems and hence breaking these cryptosystems.

In the first hour we will describe the attacks on cryptosystems based on the knapsack problem. Several knapsack cryptosystems have been devised in attempts to avoid these attacks but the attacks can be modified to break these systems. There is, however, a knapsack cryptosystem of Chor and Rivest which has not been broken. There is also an unsolved low density multiplicative knapsack problem that arises from studying the security of an implementation of a discrete exponentiation key exchange.

In the second hour, we will describe what is known about the security of several other cryptosystems. Specifically, we will discuss the remarkable success at cryptanalyzing congruential generators and the Ong-Schnorr-Shamir signature scheme, the only partial breaking of the Okamoto-Shiraishi signature scheme, and the McEliece cryptosystem for which there has been almost no success.

# INDEX OF PAPERS BY AUTHOR

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# INDEX OF PAPERS BY AUTHOR

# ABSTRACTS OF CONTRIBUTED PAPERS

## Recursive Trees With No Nodes of Out-degree One 1.

A. Meir and J.W. Moon*, University of Alberta

The out-degree of a node in a rooted tree is the number of edges incident with the node that lead away from the root. Let $\mu(n)$ denote the expected number of nodes of out-degree one in trees $T_n$ belonging to some family $\mathcal{F}$ of rooted trees and let $p(n)$ denote the probability that a tree $T_n$ in $\mathcal{F}$ has no nodes of out-degree one. If $\mathcal{F}$ is a simply generated family of trees (satisfying some mild conditions), then $\mu(n) \sim \alpha n$ and $p(n) \sim \beta(1-\alpha)^n$ where $\alpha$ and $\beta$ are constants that depend on $\mathcal{F}$. Our object is to show that such a relation does not necessarily hold when $\mathcal{F}$ is not a simply generated family. In particular, if $\mathcal{F}$ is the family of recursive trees, then $\mu(n) \sim n/4$ but $p(n) \sim \gamma^n$ where $\gamma = .73\cdots$.

## Near-factors of finite groups 2.

D. de Caen*, D. Gregory and I. Hughes

Department of Mathematics and Statistics
Queen's University, Kingston, Ontario

Abstract. Let $G$ be a finite group, $A$ and $B$ subsets of $G$. We write $G\backslash 1 = AB$ if every non-identity element $g$ can be written uniquely as $g = ab$ with $a \in A$, $b \in B$. These "near-factorizations" are motivated combinatorially by the problem of partitioning the arc-set of $\vec{K}_n$ into complete directed bipartite subgraphs. We derive some results on the near-factors $A$ and $B$. For example, $A$ and $B$ each generate $G$. Also, if $G$ is abelian then the automorphism $g \to g^{-1}$ is a multiplier of both $A$ and $B$.

## Feedback Arc Sets and Drawing Directed Graphs 3.

Peter Eades
University of Queensland
and
W. F. Smyth*
McMaster University

A *feedback arc set* in a directed graph $G$ is a set of arcs whose reversal makes $G$ acyclic. The determination of a small cardinality feedback arc set is important for the elegant embedding of directed graphs in the plane; unfortunately, the minimization problem is NP-complete. This paper describes several heuristics for determining small feedback arc sets. These heuristics are fast and suitable for directed graph drawing applications. Upper and lower bounds for the minimum size of a feedback arc set are given. The bounds are especially useful for tournaments.

# ABSTRACTS OF CONTRIBUTED PAPERS

**On Embedding Graphs in Trees** 4.

**Dan Bienstock, Bell Communications Research**

We consider the problem of optimally embedding the vertices of a graph into the vertices of a tree, in order to minimise the resulting maximum "overlap" or maximum "stretch" of edges. This may be regarded as a generalization of cutwidth and bandwidth problems, which have important applications in circuit layout. We present a characterization of graphs that have small optimal overlap, and present essentially tight bounds relating the optimal overlap to the optimal stretch. This problem leads us to that of producing a good tree-decomposition of a given graph, which is equivalent to finding a good chordal graph approximation of the grapa.

**An optimization problem in the scheduling of round-robin tournaments** 5.

Rolf Rees, Dept. of Maths. and Comp. Sci., Mount Allison University

It is well-known that the complete graph $K_n$ admits an edge-decomposition into matchings of size $k$ if and only if $0 < k \le \frac{n}{2}$ and $k$ is a divisor of $\binom{n}{2}$. We consider here the problem of constructing such a decomposition with the additional property that the number of orbits (under some automorphism $\sigma$) of matchings is as small as possible.

# ABSTRACTS OF CONTRIBUTED PAPERS

**Graphs with homeomorphically irreducible spanning trees.** 7.

**Michael O. Albertson* and Joan P. Hutchinson, Smith College; David M. Berman, University of New Orleans; and Carsten Thomassen, The Technical University of Denmark**

**It is an NP-complete problem to decide if a graph contains a spanning tree with no vertex of degree 2. We show that these homeomorpically irreducible spanning trees (HISTs) are contained in graphs with minimum degree at least n and in triangulations of the plane. HISTs are nearly present in graphs of diameter 2. Neither r-regular nor r-connected graphs necessarily contain HISTs.**

CLIQUE PARTITION NUMBER OF THE COMPLEMENT OF A TRIANGLE 8.

GUOHUI ZHANG, SOUTHERN ILLINOIS U., CARBONDALE, IL 62901

Given a non-degenerate linear space F(i.e. not a near-pencil) on n vertices and b lines with a line of length >= k >= 3, and if $n >= k^2/2 - k + 2$, then we have $b >= 1 + k^2(n-k)/(n-1)$. Now let $G_n=K_n/K_3$ be the complement of a triangle and $m^2-m+2 <= n <= m^2+m+1$ (where $m>=4$ is such that $|m-\sqrt{n}|<=1/2$). Suppose m is the order of a plane, then we have the following:

THEOREM 1. If $m^2-m+2 <= n <= m^2+3$, then

$$m^2+m-1 <= cp(G_n) <= m^2+m;$$

If $m^2+4 <= n <= m^2+m+1$, then

$$m^2+m+1 <= cp(G_n) <= m^2+m+cp(G_{n-m^2}).$$

Moreover, any minimal clique partition of $G_n$ has a largest clique with m+1 vertices.

**ARC-TOUGHNESS AND FRACTIONAL ARBORICITY IN DIGRAPHS** 9.

by **Arthur M. Hobbs***(Oakland University, Rochester, Michigan, on leave from Texas A&M University) and **Hong-Jian Lai** (Wayne State University, Detroit, Michigan).

A <u>subpartition</u> of a set S is any partition of a subset of S. Given a directed graph D, let U be a subpartition of V(D) with $k \geq 2$ classes $V_1, \ldots, V_k$. For any class $V_i$, let $\delta_D^-(V_i)$ be the number of arcs of D having heads but not tails in $V_i$. Define $i_D(U) = \sum_{i=1}^{k} \delta_D^-(V_i)$. The <u>edge-toughness</u> of a directed graph D is given by $\eta_d(D) = \min_{k \geq 2} \frac{i_D(U)}{k-1}$, where the minimum is taken over all subpartitions U of V(D) with $k \geq 2$ classes. The <u>fractional arboricity</u> of digraph D having n vertices and m arcs is given by $\gamma_d(D) = \max_{k<n} \frac{m - i_d(U)}{n-k}$, where the maximum is taken over all subpartitions U of V(D) having $k < n$ classes. We show that $\gamma_d(D) \geq \eta_d(D)$ and characterize directed graphs D having $\gamma_d(D) = \eta_d(D)$. We relate these measures to sets of branchings and of spanning arborescences in the digraph, and we relate $\gamma_d(D)$ and $\eta_d(D)$ to similar measures defined for undirected graphs.

# ABSTRACTS OF CONTRIBUTED PAPERS

## Unranking and Ranking Spanning Trees of a Graph 10.

*C. J. Colbourn**, *R. P. J. Day, L. D. Nel*

University of Waterloo

*ABSTRACT*

The set $S$ of spanning trees of an $n$-vertex graph $G$ can be placed in one-to-one correspondence with the integers in the interval $[1,s]$, where $s = |S|$. We develop $O(n^3)$ unranking and ranking functions for the spanning trees of an arbitrary graph. The unranking function maps any integer in the interval $[1,s]$ to the corresponding tree, while the ranking function maps a spanning tree to the appropriate index in the interval. The unranking function provides an $O(n^3)$ method for generating uniformly-distributed random spanning trees of a graph.

## CYCLIC BINARY LINEAR CODES 11.

Kevin Phelps
Department of Math - A.C.A.
Auburn University, AL 36849

Two linear binary codes of length n, C, C' are said to be isomorphic if there is a $n \times n$ permutation matrix P such the $C' = CP = \{xP \mid x \in C\}$. In this note we establish that deciding whether two cyclic binary linear codes are isomorphic reduces to deciding the isomorphism of circulant digraphs. We further establish that there exist cyclic codes which are isomorphic but not multipler isomorphic; thus deciding on the isomorphism of cyclic codes is, in a sense, equivalent to deciding the isomorphism of circulant digraphs.

## Unordered Love Theorems for Infinite Directed Graphs 12.

Peter D. Johnson Jr., Auburn University

Following (roughly) the terminology facetiously introduced by J. M. Hammersley, we will say that a digraph $G = (V, A)$ has the Unordered Love Property (ULP) iff for all $u, v \in V$, $u \neq v$, there is exactly one $w = w(u, v) \in V$ such that $(u, w), (v, w) \in A$. A good deal is known about loopless, finite digraphs with this property. We will review what is known, mention some questions that appear to be unanswered in the finite case, and prove some results for infinite digraphs analogous to the known theorems in the finite case. To take one easy instance, it is true in the infinite case, as in the finite, that any projective plane is associable with a loopless digraph with the ULP, in such a way that the points of the plane are the vertices of the digraph, and the lines of the plane are the outsets (sets of out-neighbors) of vertices of the digraph.

**Campaign Graphs** 2:30 148 13.

Gustavus J. Simmons, Sandia National Laboratories, Albuquerque, NM

We define a class of geometrical constructions in the plane in which each line lies on (precisely) k points, and every point is an endpoint of (precisely) one line: A construction is said to be critical if no subset of its points and lines satisfies these conditions, for the same value of k. For k = 3 we show that there is only a single infinite class of such constructions, which can be extended in an easy way to generate an infinite (but non-unique) class of constructions for k = 4. We prove some results concerning the minimum number of edges required for k = 3 and 4 and exhibit a single critical construction for k = 5, which may be all that can be said about the subject.

The title comes from the fact that each point in the construction is by definition the endpoint of an edge, i.e., every point is a new beginning: a phrase that will certainly be familiar to the reader in this election year of 1988.

An update on conjectures of Graffiti. 14.

Siemion Fajtlowicz, University of Houston.

The purpose of this talk is to present over 40 new conjectures of Graffiti and to discuss some of the (considerably fewer) solutions. Among solved conjectures there are three which are related to Turan's Theorem. One of these conjectures, proved by James Shearer, is that for every triangle-free graph the average degree is not more than the Randic Index. The *temperature* of vertex is d/(n-d), where d is the degree of the vertex and n is the number of vertices of the graph. The other two conjectures are that the average temperature is not more than respectively the chromatic number and the rank of the graph.

I shall also discuss some of the counterexamples as well as a few theorems and problems which resulted from conjectures of the program.

**SMALL SETS WHICH MEET ALL THE f(n)-TERM ARITHMETIC PROGRESSIONS IN THE INTERVAL [1, n]** 15.

Tom C. Brown* and Allen R. Freedman, Simon Fraser University

Let n, k be positive integers. We define f(n,k) to be the minimum cardinal of any subset B of [1,n] which meets all of the k-term arithmetic progressions contained in [1,n]. For example, f(9,3) = 4, since the set B = {2,5,6,7} meets every 3-term arithmetic progression contained in [1,9], and no smaller subset B of [1, 9] has this property. We show, answering questions raised by Professor P.Erdos, that $f(n, n^{\varepsilon}) < C n^{1-\varepsilon}$ for some constant C (where C depends on ε), and that f(n, log n) = o(n). We also discuss the behavior of $f(p^2, p)$ when p is a prime, and we give a simple lower bound for the function associated with Szemerédi's theorem. Our main result is this: Let k(n) be any function. Then, whenever k(n) ≥ 4, we have

$$f(n,k(n)) \le \frac{12 n \log n}{k(n) \log k(n)}.$$

# ABSTRACTS OF CONTRIBUTED PAPERS

**Some More Combinatorial Problems in the Plane** 16.

Paul Erdős, Hungarian Academy of Sciences
George Purdy,* University of Cincinnati

2:50 148

**ABSTRACT**

We discuss some more problems and results in the euclidean plane concerning n points and the t lines that they determine. For example, if $t_k$ denotes the number of lines incident with exactly k points, and if $t_3 > t_2$ then we show that $t > cn^2$, where c is a (small) positive constant. This solves a problem raised in our paper "Some Combinatorial Problems in the Plane" (J. Combin. Theory A, 1978, 25, 205-210).

One of the problems we discuss is the following: Given n points, not all on a line, what is the minimum number f(n) of points that are needed so that every induced line contains one, if none of the original points can be used.
There are examples showing that $f(n) < n$, and we conjecture that $f(n) = n-1$.

As many other problems, conjectures and results will be discussed as time permits.

Solutions to Some Conjectures of Graffiti 17.

William Staton, University of Mississippi

Let G be a graph with vertices $V_1, V_2, \ldots, V_n$. Let S be a vertex set in G. Then the coordinate vector generated by S is the vector whose $i^{th}$ entry is the number of neighbors of $V_i$ in S. The computer program Graffiti, developed by Siemion Fajtlowicz at the University of Houston, has recently generated a number of conjectures concerning coordinates of vertex sets in graphs. We discuss solutions to several of these conjectures.

**An extremal problem on sparse 0 – 1 matrices** 18.

Dan Bienstock,* Bell Communications Research, and Ervin Győri, Mathematical Institute of the Hungarian Academy of Sciences

We consider the problem of estimating the number of 1's in a 0-1 square matrix that cannot contain a "trapezoid" with a 1 at each corner. This problem may be viewed as a generalization of Zarankiewicz's problem, but it also arises in computational geometry: a recent algorithm for computing shortest rectilinear paths in the plane, that avoid rectilinear obstacles (due to J. Mitchell) has complexity essentially proportional to this number. We present nearly tight bounds, which imply that Mitchell's algorithm is one of two best for the geometry problem.

# ABSTRACTS OF CONTRIBUTED PAPERS

3:10 148

## TRANSLATION DESIGNS 19.

Lynn Margaret Batten, University of Winnipeg

A <u>translation plane</u> is an affine plane such that the group of all translations is transitive on the points of the plane. A famous theorem of Wagner states that if $A$ is a finite affine plane and $G$ a collineation group of $A$ transitive on the lines of $A$, then $A$ is a translation plane, and $G$ contains the group of all translations of $A$.

We generalize these ideas to Steiner systems.

## On an extremal problem for graphs concerning average distance and independence number. 20.

Bill Waller, University of Houston-Downtown

For a simple connected graph G, let A(G) denote the average distance between distinct vertices of G, and let I(G) denote the independence number of G. The computer program Graffiti of S. Fajtlowicz conjectured in 1985 that $A(G) \le I(G)$ for all simple connected graphs. This conjecture was discussed (but only partially proved) by the author at this conference two years ago. Since then, the conjecture has been fully established by F.R.K. Chung, who showed equality holds only if G is complete. In this paper we address the following more general extremal problem: Let H(n,i) denote the class of all simple connected graphs on n vertices with independence number i. Then what are $\min\{A(G): G \in H(n,i)\}$ and $\max\{A(G): G \in H(n,i)\}$? We solve this problem for all values of n and i, and characterize those graphs in H(n,i) on which these maximum and minimum values are attained.

## <u>"An Upper Bound for T(r+1,r)."</u> 21.

**<u>ABSTRACT</u>**

by: Dominic DeCaen
Queens University
Mathematics Department

Donald L. Kreher
School of Computer Science
RIT

James A. Wiseman - (Speaker)
Mathematics Department
RIT

The Turyn number T(n,m,k) is defined to be the smallest number of subsets of size k chosen from a set of size n so that every subset of size n is represented at least once. The number T(m,k) is: $\lim_{n \to \infty} \frac{T(n,m,k)}{\binom{n}{k}}$

In this paper an upper bound is developed for T(r +1,r) which, for small r, is better than previously known bounds.

# ABSTRACTS OF CONTRIBUTED PAPERS

## Embedding planar graphs on the grid 22.

Walter Schnyder
Louisiana State University

Each planar graph on n vertices has a plane embedding in which vertices are points of the 2n - 4 by 2n - 4 grid and edges are straight line segments.

We present such an embedding in which the coordinates of vertices have a purely combinatorial meaning as counts of elementary triangles in maximal planar graphs.

This embedding is related to our previous work on poset-dimension and planarity of graphs.

## A REDUCTION METHOD FOR GRAPHS 23.

Paul A. Catlin, Wayne State University, Detroit MI 48202

Let $C$ and $S$ be families of multigraphs, where each graph in $C$ is connected. Let $G$ be a graph. Define the $C$-reduction of $G$ to be the contraction of $G$ that is obtained from $G$ by repeated contractions of subgraphs in $C$ until no nontrivial subgraph in $C$ remains. If $C$ is closed under contraction, then the $C$-reduction of any graph is unique. We consider cases where, for any graph $G$,

$$G \in S \iff \text{the } C\text{-reduction of } G \text{ is in } S.$$

For example, this equivalence is trivial if $S$ is the family of graphs with a specified number of cut edges and $C$ is the family of all cycles. A nontrivial instance of this equivalence is that $S$ may be the family of graphs with a spanning closed trail ($K_1$ is regarded as having a spanning closed trail) and $C$ may be the family of graphs having two edge-disjoint spanning trees. Other instances of that equivalence involve double cycle covers; collections of spanning trees in graphs, and edge-arboricity; and edge-connectivity.

3:50 115

## Packing Trees Into Half-complete Bipartite Graphs 24.

Dan Pritikin, Miami University, Oxford, OH 45056

The Gyárfás-Lehel tree-packing conjecture asserts that every list of trees, containing one tree from each of the orders 1 through n, packs into $K_n$. Fishburn stated a pair of subconjectures concerning packing trees into so-called half-complete graphs, these two subconjectures together implying the above conjecture. This report presents a bipartite analogue of these, conjecturing that any list of trees as in the first conjecture packs tightly into a certain half-complete bipartite graph. This new conjecture is proven in the case where the trees being packed are restricted to the class of caterpillars

# ABSTRACTS OF CONTRIBUTED PAPERS

## Drawings of the cycle graph

25.

Heiko Harborth

Technische Universität Braunschweig, West Germany

Realizations of a graph in the plane are called drawings if two lines have at most one point in common, either an endpoint or a simple crossing. For drawings of the cycle graph $C_n$ several questions are discussed, as for example: What is the maximum number of crossings? Which numbers of crossings can occur? What is the number of nonisomorphic drawings of $C_n$ for small values n?

## On contractible edges in longest cycles

26.

Robert L. Hemminger, Vanderbilt University

It follows easily from Tutte's characterization of 3-connected graphs that those with at least five vertices contain contractible edges. Thomassen has given a different, and simpler proof of this result. Using his methods Dean, Hemminger and Toft were able to strengthen this result by showing that every longest cycle contains at least two contractible edges. Moreover, they conjectured that longest cycles in fact contain at least three contractible edges and exhibited an infinite family each having a longest cycle having only three contractible edges.

Since then, and using different techniques, Dean, Hemminger and Ota have shown that the conjecture was correct. In this talk we will try to give some idea of their proof.

4:10 115

## Packing graphs - an on-line variant

27.

Miroslaw Truszczynski, University of Kentucky

Let G, $H_1, H_2, \ldots, H_k$, be graphs. A packing of $H_1, H_2, \ldots, H_k$ into G is a partition $\{E_1, E_2, \ldots, E_k\}$ of the edge set of G such that each set $E_i$, $1 \le i \le k$ induces in G a subgraph isomorphic to $H_i$. The main question is whether a packing exists and if so, how to construct it. In this talk we discuss the following related on-line packing problem. We assume that graphs to be packed are not known a priori, the only information is that they belong to some specified class $F$ of graphs (e.g., class of paths, stars, trees, etc.). These graphs appear one at a time. At each step we have to pack the graph into the "unused portion" of G. We stop when for the first time packing of the next graph becomes impossible. The goal is to find a strategy that ensures that possibly few edges of G remain unused at the time when packing process stops. In the talk we will present several results on this problem. In particular, let $r(G;F)$ be the minimum (over all strategies) of the maximum (over all possible sequences of graphs to be packed) of the number of edges that remain unused. We will show upper bounds on $r(G;F)$ for G being a complete graph and complete bipartite graph and for $F$ being a family of (1) paths, (2) stars and (3) trees. Results concerning on-line packing into graphs with large minimum degree and connections of on-line packing to graph decomposition problems will be discussed, too.

# ABSTRACTS OF CONTRIBUTED PAPERS

28.

## Fast Detection and Display of Symmetry in Embedded Planar Graphs

MIKHAIL ATALLAH AND JOSEPH MANNING*, PURDUE UNIVERSITY

The automatic construction of good drawings of graphs, given only their vertex and edge sets, is an important practical problem. Displaying symmetry emerges as one of the most powerful criteria for achieving goodness. The mere detection of any symmetry in general graphs is shown to be a computationally difficult problem; however, for planar graphs it appears feasible. This paper presents an algorithm to detect all axial and rotational symmetries of an embedded planar graph, and then construct a drawing which displays these symmetries (an embedded planar graph specifies only the topological layout, not the actual positions, of its vertices). The algorithm runs in time and space which are linear in the number of vertices, and hence is optimal. Its application to triconnected planar graphs is of particular interest: these have a unique planar embedding, which may be obtained efficiently, and thus the symmetries may be detected and displayed starting from simply an abstract specification of the graph. Empirical results from the use of this algorithm strongly support the choice of the symmetry heuristic for producing good drawings.

29.

## Contractible Edges in Graphs with Sparsely Distributed Triangles

by

Nathaniel Dean
Bell Communications Research
Morristown, NJ 07960

**Abstract.** Define an edge $e$ of a $k$-connected graph $G$ to be $k$-contractible if the graph $G \cdot e$ obtained from $G$ by contracting $e$ is $k$-connected. It is known that every triangle-free $k$-connected graph $G$ contains a $k$-contractible edge (Thomassen), and these edges form a spanning 2-connected subgraph of $G$ (Dean). In this paper we show that the same conclusion holds even if we allow triangles to be sparsely distributed throughout $G$.

4:30 115

## Clique Covers of Chordal Graphs 30.

W. D. Wallis, Southern Illinois University

A graph is chordal if every cycle greater than a triangle contains a chord. We consider clique covers of chordal graphs. Exact clique covers (clique partitions) of chordal graphs, of graphs with no $K_4$, and of chordal graphs with no $K_4$ are discussed and the complexities of the related problems compared.

# ABSTRACTS OF CONTRIBUTED PAPERS

**Recursive Categericity of Highly Recursive Rooted Graphs** 31.

W. Gasarch,* D. Kueker, D. Mount (Univ. of MD. at College Park)

A highly recursive rooted graph (hrrg) is a triple $G = (V, E, r)$ such that $(V, E)$ is a countable graph that is locally finite, $r \in V$, $V$ is recursive (decidable), and the set of neighbors of a vertex $v$ can be determined recursively from $v$. Two hrrg's $(V_1, E_1, r_1)$ and $(V_2, E_2, r_2)$ are isomorphic if there exists an isomorphism of $(V_1, E_1)$ and $(V_2, E_2)$ that maps $r_1$ to $r_2$. It is possible for two hrrg's to be isomorphic but have no recursive (computable) isomorphism between them. If $G$ is an hrrg such that for all hrrg $H$ that are isomorphic to $G$, $H$ is recursively isomorphic to $G$, then $G$ is *recursively categorical.* We determine *exactly* which hrrg's are recursively categorical: a hrrg $G = (V, E, e)$ is recursively categorical iff given a finite subset of $X$ of $V \times V$ one can recursively decide if there is automorphism of $G$ that extends $X$.

A characterization of a class of non-binary matroids. 32.

James G. Oxley, Louisiana State University

A well-known result of Tutte is that $U_{2,4}$, the 4-point line, is the only non-binary matroid M such that, for every element e, both M\e and M/e, the deletion and contraction of e, are binary. This talk characterizes those non-binary matroids M such that, for every element e, M\e or M/e is binary.

4:50 115

A note on the biclique covering numbers of Kn\Km and complete t-partite graphs 33.

Kathryn F. Jones,* J. Richard Lundgren
University of Colorado at Denver

Norman J. Pullman
Queen's University at Kingston

R. Rees
University of Waterloo

For a graph G, let bp(G) and bc(G) be, respectively, the fewest number of complete bipartite subraphs needed to partition the edges of G and to cover the edges of G. We find bc(G) and bp(G) for certain classes of graphs, such as Kn\ Km and complete t-partite graphs.

# ABSTRACTS OF CONTRIBUTED PAPERS

## CROSSING NUMBERS OF PERMUTATION GRAPHS 34.

Cindy Lovegrove *
R.D. Ringeisen
CLEMSON UNIVERSITY

In 1967 Chartrand and Harary introduced the concept of α-permutation graphs and considered some related topological properties. Ringeisen in 1983 began investigating the crossing number of a specific family of α-permutation graphs, described by Klee as generalized prisms — namely, those whose graphs are cycles. Among the results Ringeisen obtained was the crossing number of such graphs when α is a transposition. Here we extend these results by finding the crossing numbers of generalized prisms when the permutations are 3-cycles.

## A Class of Minimally Connected Matroids 35.

Safwan Akkari, Louisiana State University

This talk will present a characterization of those connected matroids M for which every one- and two-element deletion is disconnected. A bound on the maximum number of elements of such a matroid in terms of its rank will also be given, along with a complete description of the matroids attaining this bound. These results are similar to results of Oxley for minimally connected matroids.

5:10 115

## THE INFORMATION CONTENT OF PARTITIONS 36.

William R. Edwards,* Advanced Computer Studies
Univ. of Sw. Louisiana, Lafayette, LA 70504

Mansur H. Samadzadeh, Computing and Information Sciences
Oklahoma State University, Stillwater, OK 74078

It is generally recognized that the classification and attribute assignment of objects is a major part of cognitive processes, as it is of information processing generally, and that therefore partitions and coverings on sets are fundamental, generally useful abstractions. The information content or computational work of a partition, and the "distance" or work of transformation from one partition to another, is defined from classical information theory, in a way that is consistent and meaningful, particularly in the analysis of software documents, independent of language or type of document. This formal approach is extended to coverings that are not disjoint by the definition of a natural partition generated by a covering.

MONDAY ( 5:30 p.m.)

# ABSTRACTS OF CONTRIBUTED PAPERS

## THE NUMBER OF DEPTH-FIRST SEARCHES OF A POSET 37.

H. A. Kierstead
University of South Carolina

William T. Trotter *
Arizona State University

We show that deciding whether the number of depth-first searches of a finite partially ordered set is at most k is NP-hard. A depth first search of a poset is also called a super-greedy linear extension. This class has been studied by researchers in combinatorial mathematics as a natural generalization of greedy linear extensions. These extensions arise in conjunction with the jump number problem. We also show that the decision problem is NP-hard for greedy linear extensions. However, the decision problem for arbitrary linear extensions remain open.

## On Subsets of 3-Connected Matroids. 38.

Collette R. Coullard, Purdue University, and Talmage James Reid,*
Louisiana State University.

Given a 3-connected matroid M we consider the following question. How small a 3-connected minor of M can we find which both uses a specified subset S, and has an isomorphic copy of a specified minor of M as a minor? If S has one or two elements, then this question was answered by Brylawski and Seymour respectively. If S has at least three elements, then the bound for the case that M is non-binary follows easily from a result of Bixby and Coullard. We show that this bound is best possible and also derive a best possible bound for the case where M is binary.

5:30 115

## On the Construction of Optimal 2-3 Trees 39.

Shou-Hsuan Stephen Huang* and Venkatraman Viswanathan
Department of Computer Science, University of Houston
Houston, Texas 77004

B-trees (and its special case 2-3 trees) has been a very popular data structure since its definition. Its success is due to the fact that the trees are "balanced" with a height of $O(\log n)$. However, given a set of elements and their access frequencies, one can construct many 2-3 trees (possibly with different height). We present algorithms to construct optimal 2-3 trees given a set of keys and their access frequencies. A 2-3 tree is optimal if its node-visit cost $\sum_{i=1}^{h} freq_i \times level_i$ is minimized where $h$ is the height of the tree, $freq_i$ is the access frequency of $key_i$, and $level_i$ is the level of $key_i$. The root of a 2-3 tree is considered to be on level one. A dynamic programming algorithm is used to construct optimal 2-3 trees. The algorithm runs in time $O(n^3 \log n)$ and requires $O(n^2 \log n)$ storage. Generalization of the technique to B-trees of higher order is discussed.

# ABSTRACTS OF CONTRIBUTED PAPERS

## ON THE SIBLING-STRUCTURE OF PERFECT GRAPHS 40.

C. T. HOANG , RUTGERS UNIVERSITY

Two graphs G=(V,E), G'=(V',E') are said to have the same $P_4$-structure if there is a bijection f:V→V' such that a set S of V induces a $P_4$ in G if and only if its image f(S) induces a $P_4$ in G'. Chvatal conjectured and Reed proved that if two graphs have the same $P_4$-structure then either both graphs are perfect (in the sense of Berge) or both graphs are imperfect. Reed's theorem is an extension of Lovasz' Perfect Graph Theorem : a graph is perfect if and only if its complement is.

Reed's theorem can be extended in the following way. We say that two vertices x,y are siblings with respect to a set Z if both Z∪{x} and Z∪{y} induce $P_4$'s. Two graphs G=(V,E) and G'=(V',E') are said to have the same sibling-structure if there is a bijection f:V→V' such that vertices x, y are siblings with respect to a set Z in G if and only if f(x), f(y) are siblings with respect to f(Z) in G'. We prove that if two graphs have the same sibling-structure then either both graphs are perfect or both graphs are imperfect.

## Independence Number of a Difference Graph 41.

A. GYÁRFÁS AND R. H. SCHELP*

Memphis State University

ABSTRACT

Let $n, \ell, k$ be fixed positive integers, $\ell \le k$ and $\ell + k \le 2n-1$. Let $G_n(\ell,k)$ denote the graph with vertex set $V = \{1,2,\ldots,2n\}$ and edge set $E = \{xy | y \ge x, y - x = \ell \text{ or } k \text{ or } \ell + k\}$.
Theorem. The independence number $\beta(G_n(\ell,k)) \le n$ with equality if and only if $n$ is even and $\ell = k = \frac{n}{2}$.

This theorem relates to the following matrix labelling problem. Label the columns and rows of an $n \times m$ matrix with positive integers assigning the sum of the labels given to the $i$th row and the $j$th column to the $ij$th entry. Find the smallest positive integer $N$ such that all labels have value at most $N$ and all matrix entries are distinct.

## Two Theorems on the Addition of Residue Classes 42.

David R. Guichard
Whitman College, Walla Walla WA, 99362

**Abstract.** It is well known that if $a_1, \ldots, a_m$ are residues modulo $n$ and $m \ge n$ then some sum $a_{i_1} + \cdots + a_{i_k}$, $i_1 < \cdots < i_k$, is 0 (mod $n$). In recent related work, Sydney Bulman-Fleming and Edward T. H. Wang have studied what they call *n-divisible* subsequences of a finite sequence $\sigma$, and made a number of conjectures. We confirm two of those conjectures in a more general form. Let $f(a_1, \ldots, a_m; j)$ be the number of sums formed from the $a_i$ which are congruent to $j$ (mod $n$). We prove two main theorems: 1. *If* $f(a_1,\ldots,a_m;0) < 2^{m-1}$ *then* $f(a_1,\ldots,a_m;0) \le 3 \cdot 2^{m-3}$; 2. *Let* $m \ge n$. *There exist* $a_1 \ldots a_m$ *for which* $f(a_1,\ldots,a_m;j)$ *is odd if and only if* $n$ *is not a power of* 2.

## Shellsort and The Frobenius Problem 43.

*Mark Allen Weiss* *
*Alex Pelin*

School of Computer Science
Florida International University
University Park
Miami, FL 33199

*ABSTRACT*

Shellsort is a simple classic algorithm that runs competitively on both mid-sized and nearly sorted files. Moreover, Shellsort is simple to code and can efficiently take advantage of parallel supercomputer architectures with little extra effort. These considerations make Shellsort an attractive algorithm.

Shellsort uses an increment sequence, the choice of which can drastically affect the algorithm's running time. While average case results for Shellsort are confined to only trivial, rather useless increment sequences, Incerpi and Sedgewick recently derived vastly improved upper bounds for Shellsort's worst-case running time by using results of the Frobenius Problem, a classic problem in additive number theory. By using the Frobenius Problem, Weiss and Sedgewick obtained a matching lower bound on Shellsort's worst-case running time. Unfortunately, while the sequences that arise out of these papers have better *provable* worst-case bounds, none seem empirically better on *random* input than a simple sequence suggested by Gonnet.

In this paper, we present an algorithm for converting a given increment sequence to another, presumably better increment sequence by minimizing a function closely related to the Frobenius Problem. This algorithm is pseudo-polynomial in its present form, but has already yielded an increment sequence that is the faster than any other known sequence for file sizes less than 20000.

# ABSTRACTS OF CONTRIBUTED PAPERS

## MORE CHARACTERIZATIONS OF TRIANGULATED GRAPHS 44.

by Yves CRAMA (University of Delaware)

Several new characterizations of triangulated graphs are presented. In particular, the complements of triangulated graphs are shown to constitute a natural subclass of the class of strongly perfect graphs. They are also characterized in terms of the shellability of an associated collection of sets. Finally, the notion of stability function of a graph is introduced, and it is proved that a graph is triangulated if and only if a certain polynomial representing its stability function has all its coefficients equal to 0, +1 or -1.

## Independent Edges in Bipartite Graphs Obtained from Digraphs 45.

John Gimbel* (Department of Mathematical Sciences, University of Alaska, Fairbanks, AK) and K. Brooks Reid (Department of Mathematics, Louisiana State University, Baton Rouge, LA)

Given a digraph D on vertices $v_1$ $v_2$, ... $v_n$ we can associate a bipartite graph B(D) on vertices $s_1$, $s_2$, ... $s_n$, $t_1$, $t_2$ ..., $t_n$ where $s_i t_j$ is an edge in B(D) if $(v_i, v_j)$ is an arc in D. Let $0_G$ be the set of all orientations on a graph G and $S(G) = \{\beta_1(B(D)) | D \in 0_G\}$ where $\beta_1$ is the edge independence number. We will show that for any G the set S(G) is convex and Max $S(G) \leq 2$ Min S(G). Furthermore, we discuss the extremal problem of finding, for given m and n, the minimum number of edges that a graph G on n vertices must have to insure that Max $S(G) \geq m$. We will see that this is related to the concept of graphical closure.

## ON GENERATING LINEAR SPANS OVER GF(p) 46.

Robert Fitzgerald and Joseph Yucas*
Southern Illinois University
Carbondale, IL 62901

ABSTRACT: In this note we introduce the concept of a generating pattern and give constructions of these using primitive polynomials together with the use of special t-spreads also constructed from primitive polynomials. These ideas lead to rather simple algorithms for generating linear spans over GF(p).

## REPRESENTATION OF DATABASE DESIGN PROBLEMS 47.

Ki H. Baik, Dept of EECS, Univ of Wisconsin, Milwaukee, WI 53201

In the area of databases, database design problems are given as a finite set of attributes with the structure of imposed data constraint. The paper introduces the system of representatives to postulate database design problems on databases. It is interesting for mathematicians to represent problems on databases without using terms in databases. The author believes the representation provides formal theory for generic database design system.

# ABSTRACTS OF CONTRIBUTED PAPERS

## 48. *Intersection Number and Edge-Clique Graphs of Chordal Graphs*

*Arundhati Raychaudhuri, College of Staten Island*

The intersection number i(G) of a graph G is the minimum cardinality of a set S so that G is the intersection graph of subsets of S. In this paper we give a polynomial algorithm to calculate i(G), where G is a chordal graph, by utilizing the concept of the edge-clique graph C(G), whose clique cover number is equal to i(G). Also the edge-clique graphs of chordal and strongly chordal graphs are shown to belong to these two classes respectively.

## 49. THE PRODUCT OF INDEPENDENT DOMINATION NUMBERS OF A GRAPH AND ITS COMPLEMENT

E.J. Cockayne *
University of Victoria

G. MacGillivray
Simon Fraser University

C.M. Mynhardt
University of South Africa

ABSTRACT

Let f(p) denote the maximum taken over all p-vertex graphs G of the product of the independent domination numbers of G and its complement. We show that

$$\frac{(p+4)^2}{16} \le f(p) < \frac{(p+3)^2}{8} .$$

The exact value of f(p) remains a mystery.

## 50. Monomial permutations with uniform cycle decomposition

I.J.Dejter and I.Rubio*, University of Puerto Rico, Rio Piedras, PR 00931

It is known that the monomial $x^i$ acts as a permutation on the finite field GF(q) ($q = p^r$, p a prime) whenever (i,q-1) = 1. We study the decomposition of this permutation into disjoint cycles, and in particular when the cycles are of uniform length, excluding fixed points. We prove that if q = 2f+1, where f is an odd prime, then for every i we have this uniform cycle decomposition, with (0,1,-1) as fixed points. Conversely, we prove that if $q = 2f^n+1$, there is a uniform cycle decomposition with (0,1,-1) as fixed points, for a given i, only if n = 1. We further characterize those i for which (0,1,-1) is the set of fixed points.

## 51. A New Combination Generation Method

Phillip J. Chase, Dept. of Defense, Fort Meade, MD 20755-6000

A new combination generation method is presented. It is fast, minimal-change, loopless, and convenient to use. When viewed as a fixed-density binary n-tuple generator, the new algorithm has the additional feature that successive n-tuples differ by a pair of bits which are always either one-apart (adjacent) or two-apart. (Always one-apart is known not to be possible in general.) Complementary procedures are included to realize explicitly the bijection between positions in the new listing and the configurations occupying those positions. Implementations in FORTRAN 77 are included. The new listing is based on a generalization of lexicographic ordering, called graylex ordering, which is explained at some length. Well-known listings which exhibit graylex ordering include the binary reflected Gray code and the Johnson-Trotter permutation listing.

# ABSTRACTS OF CONTRIBUTED PAPERS

## STRONGLY CHORDAL GRAPHS AND DATABASES : CONNECTIONS AND APPLICATIONS 175.

V.S. Lakshmanan

University of Toronto

**Abstract**

The hypergraph model of relational database schemes has been known for some time [3, 1]. It is the aim of this work to report some applications of the theory of strongly chordal graphs to relational database theory, *and* of the latter to the former! Specifically, we will be concerned with the classes of $\beta$-acyclic database schemes, totally balanced matrices, and strongly chordal graphs. We prove that the following statements about a database scheme D are equivalent: (i) D is totally balanced (as a hypergraph); (ii) the 2-section of the *augmentation* of D is strongly chordal; (iii) D admits of a *$\beta$-elimination ordering* of its edges. We relate $\beta$-elimination orderings to simple elimination orderings for strongly chordal graphs. Using the duality property of $\beta$-acyclic hypergraphs, we derive fast recognition algorithms for strongly chordal graphs (equivalently, totally balanced matrices, or $\beta$- acyclic database schemes). We sketch some applications of these results to database theory. For the other direction of the applications, we use the connections observed above to show that a strongly chordal graph on two or more vertices has atleast two simple vertices. This is the counterpart of a similar result for chordal graphs due to Dirac [2]. We also obtain a syntactic characterization of strongly chordal graphs - a graph is strongly chordal if and only if the hypergraph of its maximal cliques has an *I-qual tree* representation.

## SIMPLICIAL GRAPHS 176.

G. A. CHESTON
University of Saskatchewan

E. O. HARE*, S. T. HEDETNIEMI, R. C. LASKAR
Clemson University

A vertex v is simplicial if the graph induced by the set of vertices adjacent to v is complete. A graph G is edge-simplicial if $G = S_1 \cup S_2 \cup \ldots \cup S_k$ , where each $S_i$ is a complete graph containing a simplicial vertex of G. A graph H is vertex-simplicial if every vertex is either a simplicial vertex or adjacent to a simplicial vertex.

If a graph G is (vertex or edge) simplicial it has a unique simplicial decomposition $S_1 \cup S_2 \cup \ldots \cup S_k$, where k is called its simplicial index. We show that for these graphs the vertex independence number, $\beta_0$, the upper domination number, $\Gamma$, the upper fractional domination number, $\Gamma_f$, and the upper irredundance number, IR, are all equal to the simplicial index, k.

## Perfect Addition Tables and Applications to Array Storage 177.

M.D. Atkinson

School of Computer Science, Carleton University, Ottawa, Canada K1S 5B6

Let $a=(a_0,a_1,\ldots)$, $b=(b_0,b_1,\ldots)$, $c=(c_0,c_1,\ldots)$,.... be a finite set of vectors, not necessarily all of the same lengths, with non-negative integer components. Consider the collection of integers of the form $a_i+b_j+c_k+\ldots$. The condition studied is that this collection contains every integer from 0 to t-1, where t is the product of the lengths of the vectors (each integer from 0 to t-1 then has a unique representation as $a_i+b_j+c_k+\ldots$ ). The sets of vectors which fulfil this condition are characterised. The characterisation is then applied to the case of two vectors only and a result about $\mathrm{Max}\{a_i,b_i\}$ is obtained. The implications of the condition to the computing problem of storing extendible multi-dimensional arrays are discussed. Finally, the modular version of the condition is introduced, that all $a_i+b_j+c_k+\ldots$ *mod t* are distinct, and some partial results are given.

## DETERMINANT THEORY FOR FUZZY MATRICES 178.

by Jin Bai Kim, West Virginia University

We study Determinant Theory for Fuzzy square matrices including some Boolean matrices and consider Cayley-Hamilton Theorem for these matrices.

WEDNESDAY -10:10a.m.

# ABSTRACTS OF CONTRIBUTED PAPERS

52

148

## On the existence of designs of block size four having subdesigns

D. R. Stinson*, University of Manitoba, and
Rolf Rees, Mount Allison University

**Abstract** The obvious necessary conditions for the existence of a (v, 4, 1) balanced incomplete block design (BIBD) containing as a subdesign a (w, 4, 1)-BIBD are $v \equiv 1$ or 4 modulo 12, $w \equiv 1$ or 4 modulo 12, and $v \geq 3w + 1$. More generally, we can consider the existence of pairwise balanced designs on v points, having blocks of size 4, except for one block of size w. Such a design can exist only if $v \geq 3w + 1$; and $v \equiv 1$ or 4 modulo 12 and $w \equiv 1$ or 4 modulo 12, or $v \equiv 7$ or 10 modulo 12 and $w \equiv 7$ or 10 modulo 12. We show that these conditions are sufficient for the existence of such a design, with at most 9 ordered pairs (v, w) which are possible exceptions. These possible exceptions are those (v, w) ∈

{(70, 19), (82, 19), (106, 19), (91, 22), (103, 22), (115, 22), (163, 22), (142, 43), (154, 43)}.

53

## Connecting Sets in Graphs--a Domination Related Concept

by

Richard E. Newman-Wolfe, Dept. of Computer Science, University of Florida
Ronald D. Dutton, Dept. of Computer Science, University of Central Florida
Robert C. Brigham*, Dept. of Mathematics, University of Central Florida

Let G=(V,E) be a graph. A path P with terminal vertices u and v is a connecting path through $V' \subseteq V$ if $V(P)-V'=\{u,v\}$ and $V(P) \cap V' \neq \Phi$. The subset $V' \subseteq V$ is a strong connecting set (SCS) if every pair of vertices of V-V' has a connecting path through V'. It is a weak connecting set (WCS) if every pair of nonadjacent vertices of V-V' has a connecting path through V'. The size of a smallest SCS is denoted by $\gamma_S$ and a smallest WCS by $\gamma_W$. It is shown that $\gamma_W \le \gamma_S \le \gamma_C \le \gamma_W + 1$ where $\gamma_C$ is the connected domination number. Results are also given on the structure of graphs having $\gamma_W < \gamma_C$.

54

## Diameter vulnerability of some interconnection networks.

J. Bond and C. Peyrat*

Laboratoire de Recherche en Informatique
U A 410 du CNRS, Batiment 490
Universite Paris-Sud
91405 Orsay France

A good interconnection network may be modeled by a graph with a small maximum degree and a small diameter. For a given degree and a given diameter, some of the largest known graphs are de Bruijn and Kautz graphs. These graphs are underlying graphs of iterated line digraphs.

We study the effect of the deletion of vertices or edges on the diameter of underlying graphs of line digraphs. In particular, we show that the diameter of Kautz or de Bruijn graphs does not increase after deletion of one vertex or one edge and that it increases by one or two after the deletion of several vertices.

55

## Robertson-Seymour Poset Polynomial-Time Complexity Bounds: Making Them Constructive / Making Them Practical

Michael R. Fellows
Computer Science Department
University of Idaho
Moscow, ID 83843

Three approaches are described to the problem of finding constructive and practical algorithms for problems reducible to membership in ideals of RS posets.

(1) A general technique for making algorithmic complexities based on RS posets constructive. Curiously, the method does not depend on identification of the obstruction sets and is non-constructive regarding the polynomial time bound. It does depend on three properties (P-time checkability, P-time self-reducibility and constructive decidability) shared by many applications of RS posets.

(2) A technique for finding constructive and "reasonable" algorithms based on (very) fast self-reduction (and other oracle) algorithms. Among the results of this approach: a linear-time algorithm for k-VERTEX COVER, a quadratic-time algorithm for k-FEEDBACK VERTEX SET and a simple linear-time algorithm for k-DISK DIMENSION.

(3) An approach to removing the astronomical constants involved in many applications of RS posets by proving special cases of tree- and path-width bounds. This is demonstrated for a hypergraph problem of VLSI.

# ABSTRACTS OF CONTRIBUTED PAPERS

56

## What is a Skew Translation Generalized Quadrangle?

Stanley E. Payne, University of Colorado at Denver

The definition of skew translation generalized quadrangle (STGQ) given in the monograph Finite generalized quadrangles (Payne and Thas, Pitman, 1984) is apparently too general to be useful. We offer a new definition in terms of local moufang conditions which embraces all the known nontrivial examples (including all seven infinite families discovered during the 1980's) and yet permits a rather detailed study of the structure of STGQ.

130 57

## K- γ - Insensitive Domination

Teresa Haynes Rice*
Robert C. Brigham
Ronald D. Dutton

University of Central Florida, Orlando, FL 32816

A connected graph is edge domination insensitive if the domination number is unchanged when any single edge is removed. Dutton and Brigham determined extremal edge domination insensitive graphs. We extend the problem by considering removal of more than one edge. We define a connected graph to be *k - edge domination insensitive* if $\gamma(G) = \gamma(G - E')$ where E' is any k edges of G, $k \geq 1$. We present extremal graphs for $k = 2$ and $\gamma = 2$.

## VERY LARGE OPTIMAL FAMILIES OF MINIMAL-DIAMETER CHORDAL NETWORKS 58

DVORA TZVIELI, *Louisiana State University*

**Abstract:** A special class of graphs (double ring networks, circulants) $G(n,h)$ is considered, where each node i of the n nodes on a cycle, is also joined by chords to the nodes $i \pm h \bmod n$. G(n,h) is defined as optimal if its diameter equals some derived lower bound (lb). New infinite families of optimal networks are identified, along with corresponding optimal "hops" h that minimize the diameter. Some of the families are "sparse": given the value k for lb, they contain $\sim c\sqrt{k}$ values of the 4k values of n that correspond to that bound. Other families are "dense": they cover ~90% of all values of n corresponding to the bound k. Sufficient conditions for optimality are proven, where at least one out of several identified relationships between n and k holds. All non-optimal values of n, n≤26500 were numerically found to be "suboptimal" i.e. the minimal diameter of $G(n,h)$ with respect to h exceeds the lower bound by 1. The optimal and suboptimal graphs $G(n,h)$ are applicable in the design of interconnection networks with minimal transmission delay, and to minimal-diameter ILLIAC type computers for parallel processing.

## A Technique for Algorithms in $K_{1,3}$-free Graphs 59

Manton Matthews, University of South Carolina, Computer Science

In this paper we present a technique for algorithms for $K_{1,3}$-free graphs. This technique uses a Breadth First Search Tree and the properties of such trees when constructed in $K_{1,3}$-free graphs. Algorithms, that are linear in the number of edges, are presented for perfect matchings in $K_{1,3}$-free graphs and for a hamiltonian cycle in the squares of $K_{1,3}$-free graphs. This latter algorithm is an improvement on an algorithm of Kleitman in that it applies to a larger class of graphs and is simpler. The algorithm for hamiltonian cycles provides with slight modification an algorithm for finding a cycle of any length l, $3 \leq l \leq |V(G)|$ containing any specified vertex.

60

### Some Results on Hadamard Equivalence

C. Lin, Southern Illinois University

We have been considering profiles of Hadamard matrices, with a view to testing their usefulness as indicators of Hadamard equivalence. In particular, the profiles of matrices of order 24 have been studied. Some results on 4-profiles and 6-profiles will be presented.

61

### Fractional Packings and Coverings of Graphs

130

G. S. Domke *, R. C. Laskar, S. T. Hedetniemi
Clemson University, Clemson, SC 29634

A function $g:V(G)\to[0,1]$ is a **dominating function** if for every $v\in V(G)$, $\sum_{w\in N[v]} g(w) \geq 1$. $\gamma_f(G) = \min_g \sum_{v\in V(G)} g(v)$ where g is a minimal dominating function. Given a minimal dominating function g, a nonempty set $S\subseteq V(G)$ is said to be **g-irredundant** if for every $v\in S$, $\sum_{w\in N[v]\cap S} g(w) = 1$. $ir_f(G) = \min_g \max_S \sum_{v\in S} g(v)$, where S is a maximal g-irredundant set and g is a minimal dominating function. A function $g:V(G)\to[0,1]$ is a **packing function** if for every $v\in V(G)$, $\sum_{w\in N[v]} g(w) \leq 1$. $\rho_f(G) = \min_g \sum_{v\in V(G)} g(v)$ and $P_f(G) = \max_g \sum_{v\in V(G)} g(v)$, where g is a maximal packing function. This paper studies these parameters as well as relates them to each other and to other graph parameters.

62

### Design of diameter e-invariant networks.

Sin-Min Lee, Dept. of Maths. and Computer Science,
San Jose State University, San Jose, CA 95192.

A graph G is said to be diameter e-invariant if its diameter $D(G) = D(G\backslash e)$ for all e in E(G). We consider several constructions of diameter e-invariant networks : Zykov sum, composition, Sabidussi sum, cartesian product and edge expansion. In particular, we show that every connected graph is an induced subgraph of a diameter e-invariant graph of diameter k, $k \geq 2$. The result reflects that it is impossible to have a Kuratowski's type of characterization of diameter e-invariant graphs.

63

### Bad Examples for Maximum Flow Algorithms on Bipartite Networks

David Fernández-Baca, Iowa State University, Ames, IA 50011

The fastest maximum flow algorithms for dense networks known to date run in $O(|V|^3)$ time, where $V$ is the vertex set of the network. Of these methods, all but the Goldberg-Tarjan algorithm work in phases, where each phase finds a blocking flow in a layered network, and all but the MKM algorithm use *preflows*, a technique introduced by Karzanov. A preflow is like an ordinary flow except that, at intermediate stages, the amount of flow entering a vertex may exceed the amount of flow leaving it. A *bipartite flow network* is a flow network where the vertex set of the underlying directed graph can be partitioned into four disjoint sets, $\{s\}$, $S$, $T$, and $\{t\}$. The source, $s$, has arcs directed to every vertex in $S$, every vertex in $T$ has an arc directed to the sink, $t$. The only other arcs allowed are arcs directed from $S$ to $T$. Bipartite networks arise in many applications, and it is common to have networks where $|S| \ll |T|$. It can be shown that all the $O(|V|^3)$ algorithms known to date find a maximum flow in a bipartite network in $O(|S|^2|T|)$ time, where $|S| < |T|$. Here we prove that this bound is tight for several preflow-based maximum flow algorithms, including Tarjan's wave method and the Shiloach-Vishkin algorithm, by presenting a parameterized family of bipartite networks $\mathcal{F} = \{N_{m,n} | n > m > 0\}$, such that $N_{m,n}$ has $|S| = m$ and $|T| = n$ and $N_{m,n}$ forces each algorithm to take $\Theta(|S|^2|T|)$ time.

# ABSTRACTS OF CONTRIBUTED PAPERS

64

## FURTHER RESULTS ON ICOILS

F.E. Bennett,* Mount Saint Vincent University
Lisheng Wu and L. Zhu, Suzhou University

**Abstract**: We shall denote by (i,j,k)-COILS(v) an idempotent Latin square of order v which is orthogonal to its (i,j,k)-conjugate and by (i,j,k)-ICOILS(v,n) an incomplete (i,j,k)-COILS(v) missing a sub-COILS(n), where $\{i,j,k\} = \{1,2,3\}$. A necessary condition for the existence of an (i,j,k)-ICOILS(v,n) is $v \geq 3n + 1$. We show that the condition $v \geq \frac{10n}{3} + 59$ is sufficient for the existence of a (3,1,2) (or(2,3,1))-ICOILS(v,n) for all $n \geq 1$. This result substantially improves a previous bound obtained by two of the authors. Similar results have already been obtained for the other conjugates.

65

## Fractional domination and fractional packing in graphs

D.L. Grinstead* and P.J. Slater
Department of Mathematics and Statistics
University of Alabama in Huntsville
Huntsville, Alabama 35899

The fractional domination number of a graph G = (V,E) has been defined to be $\gamma_f = \min \{\sum_{v \in V} g(v)\}$ where $g(v) \in [0,1]$ and for each $v \in V$ the sum of the weights g(x) for $x \in N[v]$ is at least one. This is a modification of the domination number of G where g(v) would be required to be either 0 or 1. Here we introduce several similarly defined "fractional parameters" including those whose 0-1 versions are efficient domination, strong stability, and redundance.

Concentrating on fractional domination and fractional strong stability, our presentation will include illustrations of these parameters on several examples, their Linear Programming formulations, formulas for certain classes of graphs, and a discussion of bounds.

66

## Graphs of Maximum Diameter

L. Caccetta
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Perth, WA, Australia

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Upper bounds are given for the diameter of an undirected graph of order n which is also constrained to satisfy one of the following conditions:

(a) K-connected;
(b) K-edge-connected;
(c) K-regular.

Constructions are given which show that the upper bounds can always be realized.

48

67

## ON APPROXIMATE ALGORITHMS FOR THE TRAVELING SALESMAN PROBLEM

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W. Richard Stark
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University of South Florida
Tampa, FL 33620

### ABSTRACT

For the traveling salesman problem with a symmetric and positive distance function satisfying the triangle inequality we prove that if P ≠ NP, there is no $\epsilon$-approximate polynomial-time algorithm for arbitrarily small $\epsilon$; i. e., no polynomial-time algorithm exists that will give an approximat solution with relative error at most $\epsilon$ in all instances. Further, for random instances of a restricted size, we find empirically a $k = k(\epsilon) \leq 1$ such that Christofides' 1/2-approximate algorithm (the best known) provides an $\epsilon$-approximate solution 100k% of the time.

# ABSTRACTS OF CONTRIBUTED PAPERS

68

**Combinatorial Toolkit: T-Designs**

**Donald L. Kreher, Rochester Institute of Technology**

**A demonstration of programs developed at R.I.T. for finding $t-(v,k,\lambda)$ designs will be presented. Using these algorithms several new $t-(v,k,\lambda)$ designs were found. These designs include among others two nonisomorphic 6-(14,7,4) designs, and $5-(28,6,\lambda)$ designs, for $2 < \lambda < 21$.**

69

## Complexity Questions for n-Domination and Related Parameters

Michael S. Jacobson
and
Ken Peters *

University of Louisville

A subset $D \subseteq V(G)$ of vertices is n-dominating if every vertex not in D is adjacent to at least n vertices in D. The n-domination number of G, denoted $\gamma_n(G)$, is the minimum cardinality of an n-dominating set of G. A subset $S \subseteq V(G)$ is n-dependent if for every $x \in S$, $\deg_{<S>}(x) \leq n-1$. The n-dependence number of G, denoted $\beta_n(G)$, is the maximum cardinality of an n-dependent set of G.

In this paper we show that both of the problems of determining $\gamma_n(G)$ and $\beta_n(G)$ for an arbitrary graph are in the NP-Complete class. We also exhibit linear-time algorithms (in the number of vertices) for finding the values of these two parameters for the families of trees and series-parallel graphs. The methodology used to construct these algorithms is one developed by T.Wimer.

70

On M-Central Radius R Augmentation Problems for Graphs and Digraphs

Zhuguo Mo, Western Michigan University

Abstract. Let G = (V,E) be a graph, C be a subset of V with cardinality M, and R be a positive integer. C is called an M-central. The M-Central Radius R Augmentation problem for graphs is to determine the minimum number of edges to be added to G such that in the resulting graph every vertex is within distance R from C. This problem and the corresponding problem for digraphs are shown to be NP-complete for any fixed M and R. Both problems can be solved in linear time if the graphs (digraphs) are restricted to acyclic graphs (acyclic digraphs). In addition, the Graph Dominating Set of Radius R problem and the Digraph Dominating Set of Radius R problem are proved to be NP-complete for any fixed R.

71

***ABSTRACT***

***FORECASTING PROCEDURES FOR CRASHING LINEAR PROGRAMS***

John H. Ristroph *
Popanit Poommarapan
Komsan Sirichumsang
Engineering Management Program
University of Southwestern Louisiana

This paper describes the results from a simulation experiment and subsequent regression analysis designed to forecast which constraints in a linear program are binding at the optimum. The procedure used to generate linear programs with randomly chosen coefficients but known optima is presented first, followed by the experimental design. The theoretical basis for the specifications of fitting functions and dependent variables is provided next. Then the adequacy of the regression is tested, and the quality of the forecasts is evaluated. The procedure results in a statistically significant improvement over a chance selection of binding constraints. Directions for future research involving large scale programs are indicated.

# ABSTRACTS OF CONTRIBUTED PAPERS

72

ABSTRACT

## Constructive Methods for Designs

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Emory University

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149

For a design D with parameter $(v, b, r, k, \lambda)$ let us suppose that we have $m$ rows of the incidence matrix $A$ given. Then every further row must have $r$ $1's$ and have exactly $\lambda$ of these in common with each of the given $m$ rows. If these can all be listed and the $v-m$ rows added then the design is constructed. A case in point is a (22, 33, 12, 8, 4) design with 10 rows given with the first 3 columns $0's$. This will be a (10, 30, 12, 4, 4) design which I take to be the 3-design obtained from a Steiner triple systems on 9 points. It has a group of order 1440 which makes the listing practical (less than 1000 further possible rows in 7 orbits). It is hoped that this will lead to a design $D$.

73

## On locating dominating sets and well-covered graphs

A. Finbow and B.L. Hartnell,* Saint Mary's University, Halifax, Canada

Slater has defined a locating dominating set, or beacon set, as a dominating set B of the vertices of a graph G such that for every pair of vertices u & v not in B, $N(u) \cap B \neq N(v) \cap B$. Well-covered graphs, those in which every maximal independent set of vertices is maximum, were introduced by Plummer. We discuss a relationship between the problem of finding beacon sets and the problem of characterizing well-covered graphs.

74

## Subgraphs of Fixed Diameter

Y. Kim, *AT & T, Naperville*
S. Ntafos, *Univ. of Texas at Dallas*
F. Shahrokhi & A. Sung,* *New Mexico Tech*

The problem of enumerating maximal, fixed-diameter subgraphs of a given undirected graph arises in the context of testing parallel software using processor clustering strategies. We show that the problem is NP-hard for all fixed diameter $d \geq 1$ and give some restricted classes of graphs for which the problem can be solved in polynomial time.

75

## Two Methods for Storing Directed Trees

Puhua Guan
Depaprtment of Mathematics
University of Puerto Rico
Rio Piedras, PR 00931

Using "Small bandwidth labelling", in averange, we can save half of the storage needed to store directed trees comparing to the regular linked list method. Using "Combined linked and unlinked list", we can save more than one third of the storage needed to store directed trees with large valencys comparing to the linked listed method.

# ABSTRACTS OF CONTRIBUTED PAPERS

76

**Broadcasting in Bounded Degree Networks**

J-C Bermond
C.N.R.S., University Paris-Sud ,Orsay, FRANCE
Visiting School of Computing Science
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Broadcasting is an information dissemination process in which a message is to be sent from a single originator to all members of a network by placing calls over the communication lines of a network. The broadcast time of a vertex is the minimum number of time units required to complete broadcasting from this vertex. The broadcast time of a graph is the maximum broadcast time of any vertex. In this lecture, we study the broadcasting time of de Bruijn and Kautz networks (which are among the best known families of interconnection networks for given degree and diameter). These networks have a very simple broadcasting scheme and for small degree they have a very good broadcasting time of order log n .

77

**Algorithms for Computing the Chromatic Polynomial**

D. R. Shier* — College of William and Mary
N. Chandrasekharan — Clemson University

The chromatic polynomial of a graph captures a good deal of combinatorial information about a graph, describing its acyclic orientations, its all-terminal reliability, its spanning trees, as well as its colorings. Several methods for computing the chromatic polynomial of a graph G involve producing a computation tree for G whose leaves are "simple" base graphs for which the chromatic polynomial is readily found. Previously studied examples include the cases for which these base graphs are complete graphs, completely disconnected graphs, and trees. This talk discusses the use of chordal graphs as base graphs. Several algorithms for computing the chromatic polynomial based on these concepts (and those of maximal chordal subgraphs) are developed, and computational results are presented.

78

**REGULAR STEINHAUS GRAPHS**

CRAIG BAILEY* U S NAVAL ACADEMY
WAYNE DYMACEK WASHINGTON & LEE UNIVERSITY

A Steinhaus graph is a graph generated from a symmetric 0-1 matrix A whose first row is a string of 0's and 1's and for 1<i<j a(i,j)=a(i-1,j-1)+a(i-1,j) where the addition is mod 2.

We investigate the existence of regular Steinhaus graphs, exhibiting an infinite family and giving evidence to support our conjecture that there are no others.

79

**THE GALOIS ALGEBRA MICROCOMPUTER PACKAGE** 48

R. LIDL (UNIVERSITY OF TASMANIA, AUSTRALIA).

THE GALOIS ALGEBRA PACKAGE FOR IBM PC OR COMPATIBLES CAN BE USED FOR TEACHING OR RESEARCH IN THOSE PARTS OF ALGEBRTA, COMBINATORICS AND NUMBER THEORY AND THEIR APPLICATION WHICH EMPHASISE MODULAR ARITHMETIC AND FINITE FIELDS. A DESCRIPTION OF THE PACKAGE WILL BE GIVEN.

80

## Locating a Broadcast Facility in an Unreliable Network

*Louis D. Nel and Charles J. Colbourn**
University of Waterloo

*ABSTRACT*

A model of a communications network is a probabilistic graph in which each edge has an independent probability of being operational. The Broadcast Facility Location (BFL) problem is to identify a node such that the expected number of nodes connected to it in the presence of random edge failures is as large as possible. This problem can be solved as a special case of the stochastic 1-median problem; however, we show that BFL and stochastic 1-median are both *NP-hard*. Hence, we are interested in approximate solutions which can be determined in polynomial time. We present a location strategy based on efficiently computable two-terminal network reliability bounds. This strategy does not guarantee to find an optimal node but instead finds a set of good nodes by eliminating nodes which cannot be optimal locations. Computational experience is reported for a network with 20 nodes and 31 edges.

81

## Achromatic numbers, generalized colorings, and graph operations

Pavol Hell*, SFU, and Donald J. Miller, UVictoria

We discuss the achromatic numbers of graph products and unions. Best possible upper and lower bounds are given for the unions, and some partial results are mentioned for the products. We also present some recent complexity results (including some joint work with Gary MacGillivray and Joergen Bang-Jensen) on a related notion of generalized coloring. These results suggest that for many classes of digraphs, having two directed cycles is enough to make the colorability problem intractable.

82

## Cliques of Symmetric Matrices

Joe Hemmeter* — University of Delaware
Andrew J. Woldar — Villanova University

Suppose C is a set of n x n symmetric matrices with entries from the field of q elements, and that the rank of x - y is in {0, 1, 2} whenever x and y are in C. What can we say about C? The large sets of matrices with this property have already been characterized. We report on recent progress in studying the smaller sets.

* Presenter

83 48

## New Algorithms For Permutation Groups

Cynthia A. Brown, Gene Cooperman and Larry Finkelstein*
College of Computer Science, Northeastern University, Boston, MA 02115

In this talk, we describe new algorithms for performing fundamental group computations. The algorithms are based, in part, on a new data structure for representing permutation groups invented by M. Jerrum and referred to as a complete labelled branching. This data structure allows the complete coset table for a permutation group $G$ acting on $n$ points to be represented using $O(n^2)$ space, and for a coset representative to be accessed in $O(n)$ time. This data structure was used by Jerrum to give the first algorithm for computing a strong generating set for a permutation group using $O(n^5)$ time and $O(n^2)$ space. In addition, it has led to a new change of basis algorithm for permutation groups (Brown, Finkelstein, Purdom) which generalizes Sims' original method and which runs in time $O(n^3)$, an improvement of two orders of magnitude. We will present an algorithm for testing whether a set $S$ of generators for $G$ is a strong generating set in $O(n^2|S| + n^4)$ time. As a consequence of this test, we are able to construct a presentation for $G$ using at most $n-1$ generators and $(n-1)^2$ relations . This is an improvement on the best known previous bound of $O(n^2(\log(n))^c)$ for the number of relations given by Babai, Luks and Seress. In addition, we will show how our test can be used to give an improved algorithm for constructing a complete labelled branching for $G$ (and hence a strong generating set) which will run substantially faster in practice than Jerrum's original $O(n^5)$ algorithm.

* Presented by L. Finkelstein.

# ABSTRACTS OF CONTRIBUTED PAPERS

148

84

## Affine fault-tolerant parallel processors

Douglas A. Leonard, Auburn University

Abstract The fault-tolerant parallel processing problem dealt with here is that of connecting identical processing elements (a directed graph), and partitioning and repartitioning the elements in such a way that every restricted graph in every non-idle piece of every partition is isomorphic to some fixed graph, while at the same time relegating all known faulty elements to idle pieces of some partition. This is done here by using the affine structure of finite fields.

85

**Title:** Edge-Colorings Among Cartesian Products of Critical Graphs with $K_2$

**Name:** Kenneth P. Spiteri, Auburn University

**Abstract:** Sufficient conditions on critical graphs G for which $G \times K_2$ is class 1 are given. We show that if G is a critical graph that is not an odd cycle, then $G \times K_2$ is either class 1 or critical, and that $G \times K_2 \times K_2$ is class 1.

113 86

## Matrix squares over GF(q), q odd.

T P Donovan, Midwestern State University

This paper considers the matrix equation $X^2 = A$ over GF(q), for q odd. GF(q) denotes the finite field having $q = p^n$ elements (p a prime.) The diagonalizable square matrices are completely characterized and the total number of distinct matrix squares is calculated. The main result is given by the theorem: A non scalar 2x2 diagonalizable matrix over GF(q), q odd, is a square if and only if its eigenvalues (necessarily distinct) are squares in GF(q). Furthermore, such a matrix has 4 square roots over GF(q) unless one of its eigenvalues is zero, in which case, it has 2 square roots over GF(q).

87

## Calculations on the average genus and genus distribution of graphs

*by* Dan Archdeacon, *The University of Vermont*

**Abstract :** A connected cubic graph on $n$ vertices has $2^n$ different embeddings on closed oriented surfaces. To date, most attention has been paid to the minimum and maximum genus of these embeddings. Recently, various authors have studied how these embeddings are distributed among the surfaces lying between the minimum and maximum genus, and studied the average genus of these embeddings. In this paper we present two methods for calculating the genus distribution and average genus. We also give some computer results, including charts of the genus distributions for several cubic graphs on 96 vertices.

# ABSTRACTS OF CONTRIBUTED PAPERS

88

## The Separation Sequence and Network Reliability

K.S.Bagga, L.W.Beineke, M.J.Lipman, *R.E.Pippert
Indiana University - Purdue University at Fort Wayne

The separation sequence (vector) $\bar{\eta}$ of a graph $G(p,q)$ has as its kth entry ($0 < k < p$) the minimum number of edges whose removal reduces the order of the largest remaining component to at most $p - k$. Interest in $\bar{\eta}$ derived from procedures for computing the edge-integrity of a graph, a measure of vulnerability. The sequence has since been found to contain information which can be useful in the computation of reliability parameters. In particular, it is of help in obtaining lower bounds on parameters such as the pair-connected reliability (resilience) and the expected order of a largest component of a graph.

89

## ON THE MISSING VALUE IN A GRACEFUL NUMBERING OF A 2-REGULAR GRAPH

Jaromir Abrham,* Department of Industrial Engineering, University of Toronto, and
Anton Kotzig, département de mathématiques et de statistique, Université de Montréal.

If $\psi$ is a graceful numbering of a 2-regular graph G then there is a unique integer $x$ ($0 \le x \le |V(G)|$) such that $\psi(v) \ne x$ for all $v \in V(G)$. Moreover, if $\psi$ is an $\alpha$-valuation of G, then $|V(G)| = 4k$ (where k is a positive integer) and either $x=k$ or $x=3k$. In the present paper, it is shown that it is always $k \le x \le 3k$, and that, at least for $1 < k \le 6$, $x$ can be any integer satisfying $k \le x \le 3k$. Furthermore, $\psi$ is an $\alpha$-valuation of G if and only if either $x=k$ or $x=3k$. Similar but somewhat weaker results are also obtained for the case when $|V(G)| = 4k-1$.

90

## Some refinements of the total chromatic number conjecture

A.J.W. Hilton (University of Reading)

The total chromatic number $\chi_T(G)$ of a graph G is the least number of colours needed to colour the edges and vertices of G so that no two incident or adjacent elements receive the same colour. In 1965 Behzad conjectured that, if G is a simple graph, then $\Delta(G) + 1 \le \chi_T(G) \le \Delta(G) + 2$ ; Vizing also conjectured this independently slightly later. We present two refinements of this conjecture, together with some recent results which provide evidence for these refinements. This is joint work with A.G. Chetwynd.

91

## SPREMB: a system for manipulating graphs

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Department of Computer Science
University of Queensland

SPREMB provides an environment for researchers in Graph Algorithms and Computational Geometry to practice their art with the aid of Computer Graphics. The system is designed for maximum flexibility: it is very easy to customise SPREMB to a user's particular interests. Unlike microcomputer graph drawing systems such as CABRI, SPREMB does not depend on particular hardware. The system has been implemented on Sun, VAX, and Perkin-Elmer computers, with a variety of graphics devices. This paper explains the philosophy of SPREMB, and describes its various components.

# ABSTRACTS OF CONTRIBUTED PAPERS

92

**Pair-connected reliability of Communication Networks with vertex failures**

A.T. Amin, K.T. Siegrist and P.J. Slater*
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Huntsville, Alabama 35899

We consider probabilistic graphs G = (V,E) in which each vertex is subject to failure while the edges are assumed to be fail-safe. We present results on this vertex version of pair-connected reliability, namely, the expected number of pairs of vertices which remain connected. Comparisons are made to the more widely studied case in which it is the edges that are failure prone.

93

On cordialness of amalgamation of complete graphs.

M. Benson* and Sin-Min Lee, Dept. of Maths. and Computer Science, San Jose State University, San Jose, CA 95192.

A graph G is said to be cordial if its set of vertices can be labeled by 0 and 1 so that the difference of the numbers of vertices labeled with 0 and 1 is less than two and the induced edge labeling which is defined by taking the absolute value of the difference of the verices labeling also has the property that the difference of the number of edges which are label by 0 and 1 is at most one. For $n \geq 3$ and $k \geq 2$, we denote the almagamationof k copies of complete graphs each with n vertices by Amal(n,k). We consider under what conditions on k and n, the graph Amal(n,k) is cordial. The case n = 3 has been considered by Cahit.

DECENTRALIZED LOAD BALANCING: WORST-CASE SCENARIOS 94

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We consider a general undirected graph as a model of a distributed computation system with nodes representing processors (with local memory) and edges communication links. Our decentralized load balancing criterion postulates that the difference in load units among the loads of all nodes within a distance H of any node must be no greater than 1 ($H \geq 1$, integer). We are interested in the maximum load imbalance (i. e., the maximum difference in load between any two nodes in the system) that can occur if the load balancing criterion is satisfied. We derive the following:

Theorem 1: If d is the diameter of the graph G, the maximum load imbalance is given by

$$\left\lceil \frac{d-1}{2H} \right\rceil$$

Theorem 2: If the number of load units to be distributed in the computation system is an exact multiple of the number of nodes, the maximum load imbalance is given by

$$2 \cdot \left\lfloor \frac{\left\lceil \frac{d-1}{2H} \right\rceil}{2} \right\rfloor$$

**A Programming System for Graph Algorithms using Abstract Data Types**

95

JOSEPH MANNING, PURDUE UNIVERSITY

There is often a substantial gap, in notation and even concept, between the abstract formulation of graph algorithms and their detailed realization as computer programs. This can result in a loss of clarity and reliability, as well as a significant increase in the effort of program construction. This paper describes a programming system for graph algorithms, which has been developed in an attempt to alleviate the above problems. It allows graph algorithms to be expressed programatically at a high conceptual level. It implements a graph as an *abstract data type*, whose internal structure is completely hidden and which is accessible only by means of a fixed set of pre-defined operations. This is a recognized modern technique for attaining clarity and reliability. However, the actual selection of the set of operations posed serious design challenges: power, generality, and efficiency competed against simplicity and orthogonality. The final design achieves a successful balance between these factors, and compares very favorably with a number of similar systems recently published. The system supports both directed and undirected graphs, subgraphs and supergraphs, loops and parallel edges, arbitrary properties of vertices and edges such as weights and colors, within a simple and uniform framework. It also incorporates a number of other common programming tools (lists, vectors, queues, stacks, maps), all in a highly-integrated manner. Experience to date has shown this programming system to contribute significantly to the ease and clarity of implementing a wide variety of graph and other combinatorial algorithms.

# ABSTRACTS OF CONTRIBUTED PAPERS

96

**The Distribution of Pair-Connectivity for Network Reliability**

A.T. Amin, K.T. Siegrist* and P.J. Slater
The University of Alabama in Huntsville
Huntsville, Alabama 35899

Consider a probabilistic graph G = (V,E) in which each edge in E fails, independently of the others with probability q. One measure of the reliability of such a network is the pair-connected reliability PC(G;q), the expected number of pairs of connected vertices.

This talk concerns the shape of the distribution of the number of pairs of connected vertices and the quality of PC(G;q) as a measure of the center of this distribution. We also consider the asymptotic behavior of the distribution as the number of vertices grows large.

97

**RECENT WORK ON CROSSING NUMBERS**

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In this talk we survey some recent results and continuing open questions about crossing numbers of graphs. Specifically, let $\nu(G)$ denotes the crossing number of G. Is it true that every graph G with $\nu(G) > 2$ contains a subgraph H such that $\nu(H) = 2$? Are there finitely many minimal graphs G such that $\nu(G) = 2$? What are the graphs G such that any two drawings in G have the same parity of numbers of crossings?

98

**The number of branches in a binary tree**

James C. Owings, Jr., University of Maryland

According to König's Infinity Lemma, if T is a subtree of the full binary tree and each level of T is nonempty, then T has at least one (infinite) branch. One would expect that if each level of T has "lots" of points, T will have "lots" of branches. We show that if the n-th level of T contains at least $A2^n$ points, where A is a positive constant, then T has uncountably many branches. We also show that the result is false if 2 is replaced by any smaller base - in this case, it is possible that T has a unique branch.

99

A MICROCOMPUTER SYSTEM FOR THE INVESTIGATION OF PROJECTIVE PLANES

W. Cherowitzo
U. of Colorado at Denver

## A Design of Reliable Networks using Node Redundancy 100

T. Soneoka,* Y. Manabe and M. Imase
NTT Software Laboratories, Tokyo, Japan

Switching nodes are often duplicated or triplicated to improve the reliability of communications and computer networks. As a model of such redundant networks, this paper proposes a duplex graph $D_2 = (V, E, C)$ (or triplex graph $D_3$), where $V$ is the set of all vertices, $E$ is the edge set, and $C$ is the set of clusters which are disjoint vertex-subsets containing two vertices (or three vertices in the case of $D_3$). The cluster corresponds to each node of the original non-redundant network. The connectivity of the graph is usually used for measuring both network reliability and network flow. As a natural extension of this connectivity, cluster connectivity of duplex or triplex graphs $D_r$ ($r = 2$ or 3) is defined as $\min_{F \subseteq V}\{|F| + b(D_r - F)\}$, where $b(D_r - F)$ is the minimum number of maximal vertex-disjoint paths between any pair of clusters in $D_r - F$. This measurement is shown to be calculated by a polynomial time algorithm.

A node and edge replication method is also proposed for transforming a given graph $G$ with $m$ edges, whose connectivity is equal to its minimum degree, into a maximally cluster-connected duplex or triplex graph $D_r$ ($r = 2$ or 3) with $r \cdot m$ edges.

## MULTIPLE ADJACENCY-PRESERVING GRAPHS OF COMPLETE GRAPHS 101

Narsingh Deo* and Dilip Sarkar
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ABSTRACT: Let $N(v)$ be the set of adjacent nodes of a node $v$ in a graph $G(V, E)$. Let $G(V, E)$ and $H(U, F)$ be two graphs of order $n$ and $rn$, respectively, where $r$ is a positive integer. Let $V = \{v_1, v_2, \ldots v_n\}$ and $U = \{u_1, u_2, \ldots u_m\}$. Graph $H$ is said to be a *Multiple Adjacency-Preserving (MAP)* graph of $G$ if there exist a $r$ to 1 surjection $f: U \rightarrow V$ such that $f(u_i) = v_j \Leftrightarrow f$ is also a bijection from $N(u_i)$ to $N(v_j)$. Given two graphs $G$ and $H$, the problem of finding whether $H$ is a MAP graph of $G$ will be called the MAP problem. The MAP problem is a generalization of the graph isomorphism problem in the sense that when $r = 1$, the MAP problem reduces to the graph isomorphism problem. A planar graph is called $k$-outerplanar if it can be embedded on a plane such that all vertices are on the boundaries of $k$ faces. In this paper it is shown that no 1-outerplanar graph is a MAP graph of a complete graph except $K_3$; and no 2-outerplanar graph is a MAP graph of a complete graph except $K_4$ and $K_5$. It is also shown that some 2-outerplanar graphs are MAP graphs of the 3-cube $Q_3$. We also identify the classes of 1-outerplanar and 2-outerplanar graphs that are MAP graphs of $K_3$, $K_4$, $K_5$ and $Q_3$. Some applications of the MAP problem are also discussed.

## IRREDUNDANCE OF POSETS 102

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Given a poset (X,P) a set $\mathcal{L}$ of linear extensions is a realizer if $\bigcap_{L \in \mathcal{L}} L = P$. The minimum cardinality of $\mathcal{L}$ is the dimension of P, whereas the maximum cardinality of a minimal realizer is the rank of P. A set $\{L_1, \ldots, L_r\}$ of linear extensions (not necessarily a realizer) is irredundant if, for every i, $\bigcap_{i \neq j} L_j - L_i \neq \phi$. The irredundace number of P, denoted ir(P), and the upper irredundance number, IR(P), are respectively the minimum and maximum cardinalities of maximal irredundant sets of realizers.

This paper investigates the following inequalities:

$$ir(P) \leq \dim(P) \leq rank(P) \leq IR(P) \leq |I|/2$$

where I is the set of all incomparable pairs in (X,P).

## Golomb's Conjecture on Primitive Elements in $GF(p^m)$ 103

48

Oscar Moreno and José Sotero*
Department of Mathematics
University of Puerto Rico, Rio Piedras

ABSTRACT

We are dealing with the computational aspects of Golomb's conjecture A: there exist primitive elements α and β in $GF(p^m)$ such that α + β = 1. This is used for constructing Costas arrays. We already proved this assertion for $p > 2^{60}$ using number theory methods. Now we want to prove this conjecture is true for $p < 2^{60}$ using a vector computer Alliant FX/8. In particular, we use a very efficient backtracking algorithm approach that permit us to handle the very large time complexity of the above problem when approached with the sieve method. We also use a sieve method, but for a rather limited value of p, p < 5874197, in order to validate the results of the backtracking method.

# ABSTRACTS OF CONTRIBUTED PAPERS

## Cohesion Stable Edges, Stability, and Cohesion Ratio† 104

Virginia Rice*
Richard D. Ringeisen
Clemson University

Cohesion arose as a way of measuring how close a vertex is to being a cutvertex, an important concept when studying vulnerability in communications networks. The cohesion of a vertex x, denoted $\mu(x)$, is the minimum number of edges whose deletion results in a subgraph for which x is a cutvertex. In particular, we examine changes in cohesion when an edge is deleted. An edge is called stable if, when deleted from the graph, it changes the cohesion of no vertex. Some results identifying stable edges are presented as well as methods which generate graphs with a large proportion of stable edges.

†Research supported by the U.S. Office of Naval Research.

## The Reconstruction of Cacti Revisited 105

Sylvia D. Monson, Queen's University, Kingston, Canada

ABSTRACT: Gaps in an early and important paper on the reconstruction problem are examined and corrected.

## Racing Pawn Games 106

H.A. Kierstead* and P.J. Nyikos, University of South Carolina

Let T be a tree, considered as an ordered set, with no infinite branches. The game G(T,w,b) is played on T by two players White and Black as follows. The players alternate turns, starting with White. The game begins with both pawns on the root of the tree. At White's turn, White moves his own pawn m nodes up the tree and Black's pawn w-m nodes up the tree, where $0 \le m \le w$. At Black's turn, Black moves his own pawn n nodes up the tree and White's pawn b-n nodes up the tree, where $0 \le n \le b$. The winner is the first player to have his pawn reach a leaf of the tree. Galvin proved that for every choice of T, White has a winning strategy for the game G(T,1,1). We answer several questions of Grantham by showing that for every positive integer $m \ge 2$ there exists a finite tree T such that Black has a winning strategy for the game G(T,2m-1,m) and that for all positive integers m and $k \ge 3$ and every tree T with no infinite branches, White has a winning strategy for the game G(T,mk,m).

## A Good Algorithm for the Computation of the Edge-Integrity of Trees 107

K.S. Bagga, L.W. Beineke, M.J. Lipman*,
R.E. Pippert, R.L. Sedlmeyer
Indiana University-Purdue University at Fort Wayne

The edge-integrity of a graph is defined as:
min{ |S| + m(G - S) }, where S is a set of edges in G, and m(G - S) is the maximum number of vertices in a component of G - S. The computation of the edge-integrity for graphs in general is known to be NP-complete. We present an algorithm of order at most $p^3$ for the computation of the edge-integrity of trees. We also present an implementation of the algorithm in Common Lisp on an IBM-AT.

## Notes on $(M, N, R_1, R_2)$-Transitive Directed Graphs  108

Alfred Boals, Kenneth Williams,* Zhuguo Mo - Western Michigan University

In a communications network it may be desirable to enforce some type of redundancy constraint on the paths that exist between nodes. For example, one might require that for every path $P = (a = x_0, x_1, \ldots, x_M = b)$ of length M, a corresponding path from a to b of length N must also exist, using only nodes in P. When representing networks by directed graphs this requirement is equivalent to the directed graph having the (M,N)-transitive property. This type of redundancy constraint may be generalized to the properties represented by $(M, N, R_1, R_2)$-transitivity where M and N are integers, $R_1 \in \{=, \geq\}$ and $R_2 \in \{=, \leq\}$. The $(M, N, \geq, \leq)$-transitive property, for example, then requires that for each path from node a to node b of length $\geq M$, there is a corresponding path from a to b, using the same nodes, of length $\leq N$. We present several characterization results for tounaments including "If a tournament T is $(M, 1, \geq, =)$-transitive then T is $(M, J, \geq, =)$-transitive for $J = 1, 2, \ldots, M-1$." $(M, N, =, =)$-transitive tournaments have been examined in detail elsewhere as have completions of $(M, N, =, =)$-transitive closures. We also present several results for completing additional transitive closures. These include, for an acyclic directed graph G with r nodes, an $O(r^{M+1})$ algorithm to augment G with a nearly minimal set of edges to ensure $(M, N, =, \leq)$-transitivity. In addition, we show that the problems of determining whether adding K or fewer edges to an arbitrary directed graph can produce a result which is $(M, N, R_1, R_2)$-transitive, for each $R_1$ and $R_2$, are all NP-complete for $M \geq N \geq 10$ and $3M \leq 4(N-1)$. Algorithms to complete each of the $(M, N, R_1, R_2)$-transitive closures are NP-hard.

109

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## DELETING ONE EDGE FROM A GRAPH

Abstract: We examine the structure of a graph G which
(1) contains two edges $e_1$ and $e_2$ such that $G-e_1 \not\cong G-e_2$ and which
(2) for any $e \in E(G)$ has either $G-e \cong G-e_1$ or $G-e \cong G-e_2$.

## The frequency of sums when throwing n k-sided dice.  110

Nora Hartsfield  University of California, Santa Cruz

Abstract

A recurrence is presented for computing the number of ways of rolling the sum r when n k-sided dice are thrown.

## Implementation of a Graph Abstract Data Type in Ada  111

Joseph Straight (SUNY College at Fredonia)

Ada is a relatively new programming language that was designed for and under the auspices of the U.S. Department of Defense (DoD). It is intended to support the development and maintenance of large programs by teams of programmers, and is DoD's official language for embedded-systems applications. As such, Ada is expected to gain widespread use, not only for defense-related projects, but for other commercial software as well. Ada has many useful features, including a high degree of portability, support for data abstraction, concurrency, real-time control, and error handling. An abstract data type is implemented as an Ada "package;" once such a package has been written, it can easily be made available for use by a variety of applications programs. In particular, it would be useful to have a package that implements the abstract data type "directed graph," providing a number of basic operations such as arc insertion, vertex deletion, access to the adjacency list of a given vertex, and so on. This talk presents a specification for such a package, discusses several implementation considerations, and illustrates how the package can be used to implement a given graph algorithm, such as finding the strong components of a directed graph.

# ABSTRACTS OF CONTRIBUTED PAPERS

112

## Sum Graphs and Difference Graphs

Frank Harary, New Mexico State University

We introduce the sum graph and the difference graph of a finite set S of positive integers $a_1$ to $a_n$. The sum graph of S, written $G^+(S)$, has S as its node set and $a_i$, $a_j$ are adjacent if $a_i + a_j \in S$. Then a sum graph is the sum graph of some set S. Analogously, the difference graph of S, written $G^-(S)$, also has node set S, but $a_i$, $a_j$ are adjacent if $|a_i - a_j| \in S$. A difference graph is defined similarly. Some properties of these two classes of graphs are derived and several families of sum (difference) graphs are specified. Extensions from the natural numbers N to the set Z of all integers and to finite abelian groups are also formulated and studied.

113

130

## KNOTS IN RANDOM WALKS

Nicholas Pippenger, IBM Almaden Research Center

It is shown that a random self-avoiding polygon of $n$ steps on the three-dimensional cubic lattice is unknotted with a probability $\pi_n$ that tends to 0 at a definite exponential rate, in the sense that $\pi_n^{1/n}$ tends to a constant strictly less than 1 as $n \to \infty$.

114

## Graphs with Specified Edge-toughness and Fractional Arboricity

*Paul A. Catlin, Wayne State University*
*Kurt C. Foster, Oakland University*
**Jerrold W. Grossman, Oakland University*
*Arthur M. Hobbs, Oakland University and Texas A&M University*

We consider the following two measures of the strength of a connected nontrivial undirected graph $G$ (which may have parallel edges):

$$\eta(G) = \min_{E \subseteq E(G)} \frac{|E|}{\omega(G-E)-1} \quad \text{and} \quad \gamma(G) = \max_{H \subseteq G} \frac{|E(H)|}{|V(H)|-1},$$

where $V(\cdot)$, $E(\cdot)$, and $\omega(\cdot)$ stand for vertex set, edge set, and number of components, respectively, and the denominators are not allowed to equal 0. These functions were studied by D. Gusfield ["Connectivity and edge-disjoint spanning trees," *Information Processing Letters* 16 (1983) 87-89] and P. A. Catlin, A. M. Hobbs, and H. J. Lai ["Matroid unions, edge-toughness and fractional arboricity," to appear].

It is easy to see that $\eta(G)$ and $\gamma(G)$ are rational numbers with denominators less than $|V(G)|$, and that $1 \le \eta(G) \le |E(G)| / (|V(G)|-1) \le \gamma(G)$. Catlin, Hobbs, and Lai characterized $\eta(G)$ and $\gamma(G)$ in terms of tree packing and covering, and they obtained necessary and sufficient conditions for $\eta(G) = \gamma(G)$.

We show here that the parameters $\eta(G)$ and $\gamma(G)$ are essentially unrestricted, by giving a constructive proof of the following result.

**Theorem.** *If $x$ and $y$ are rational numbers with $1 \le x \le y$, then there exists a graph $G$ with $\eta(G) = x$ and $\gamma(G) = y$.*

# ABSTRACTS OF CONTRIBUTED PAPERS

115

## The Immersion Order, Forbidden Subgraphs, and the Complexity of Integrity

Michael R. Fellows and Sam Stueckle*
University of Idaho

In this paper the authors show that for each positive integer k the family of finite graphs with edge-integrity at most k is closed under the partial ordering of graphs by immersions. Some information on the obstruction sets for these families is obtained. It is also shown that for every fixed positive integer k it is decidable in time O(n) for any arbitrary graph G of order n whether the integrity of G is at most k and whether the edge-integrity of G is at most k. In addition, it is shown that determining the edge-integrity of a graph is NP-hard.

130

116

## Cylindric Embeddings of P.S and W.P.S. Planar Cayley Diagrams

Hector Bevia S., U. Catolica de Valparaiso, CHILE
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A planar Cayley diagram of a group presentation is called P.S. (point-symmetric) if it has a planar embedding in which the counterclockwise succession of edges is the same at every point. It is called W.P.S. (weakly P.S.) if the succession is the same at each point, sometimes clockwise, other times counterclockwise. An infinite Cayley diagram which has an embedding on the sphere with two essential accumulation points of vertices is called cylindric.

An algorithm is given to decide, from the relators, if the Cayley diagram of a presentation with solvable word problem is P.S. or W.P.S. cylindric. It is also shown (without using group-ends) that any group with a P.S. or W.P.S. cylindric presentation must have an infinite cyclic subgroup of finite index.

117

## Edge-connectivity of Permutation Graphs

Barry L. Piazza U. of Southern Mississippi

For a graph G with vertices labeled 1,2,...,n and a permutation $\alpha$ in $S_n$, the permutation graph $P_\alpha(G)$ consists of two copies of G along with the permutation edges joining vertex i in one copy to vertex $\alpha(i)$ in the other. The purpose of this paper is to examine the edge-connectivity of permutation graphs. In particular, upper and lower bounds for the edge-connectivity of permutation graphs are found. The edge-connectivity is also determined for permutation graphs of trees, cycles, wheels, n-cubes, complete graphs, and complete bipartite graphs. Finally, we show that if $\alpha$ is in the automorphism group of G, $G \neq K_1$, then the edge-connectivity of $P_\alpha(G)$ is equal to the before mentioned lower bound.

# ABSTRACTS OF CONTRIBUTED PAPERS

118

## GRAPHS WITH AN ASCENDING SUBGRAPH DECOMPOSITION

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R.J. GOULD, EMORY UNIVERSITY

M.S. JACOBSON, UNIVERSITY OF LOUISVILLE

L. LESNIAK, DREW UNIVERSITY

ABSTRACT

It has been conjectured that if a graph $G$ has $\binom{n+1}{2}$ edges, then the edge set of $G$ can be partitioned into $n$ Graphs $G_1, G_2, \ldots, G_n$ such that $G_i$ has $i$ edges $(1 \leq i \leq n)$, and $G_i$ is isomorphic to a subgraph of $G_{i+1} (1 \leq i \leq n-1)$. Such a graph $G$ is said to have the an ascending subgraph decomposition (ASD). If the maximum degree of $G$ is at most $7n/12$ or if $G$ is a forest with maximum degree $5n/8$, the $G$ has an ASD. All of the graphs in the decomposition are unions of paths of length at most 3 (for forests at most 2).

119

## Maximal Polygons for Equitransitive Periodic Tilings

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**Abstract** --- It has been shown (Danzer, Grünbaum, and Shepherd: 1987) that in any periodic equitransitive tiling by convex polygonal tiles, the maximum number of sides of any tile is 66. This maximum is achieved in the periodic symmetry group p6m. We extend this result by determining the maximum number of sides in each of the remaining 16 periodic symmetry groups.

120

## THE NUMBER OF CUT-VERTICES IN A GRAPH OF GIVEN MINIMUM DEGREE

Michael O. Albertson, Smith College
David M. Berman*, University of New Orleans

Let G be a graph on n vertices with minimum degree k (at least two). We show that the number of cut-vertices in G is less than

$$(2k-2)n \;/\; (k^2-2).$$

We give a family of examples to show that this bound is best possible.

# ABSTRACTS OF CONTRIBUTED PAPERS

## SPANNING TREES OF PLANAR MAPS

121

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If T is a spanning tree of a planar map G, we say that a country C, is well covered by T, if all edges of C, with exactly one exception, belong to T. A random map with n countries and an arbitrary spanning tree for this map are projected on a wall. A monkey throws a dart at the map. We show that the probability that the dart will land in a well covered country is $e^{-1}$

Other properties of spanning trees of planar maps and maps on other surfaces will be discussed. For instance, we show that if P is a 3-polytope, and all faces of P are even polygons, then the graph of P is the union of two spanning trees.

122

## Some Generalized Helly Type Theorems

by
William R. Hare *
Clemson University
and
G. Gerald Thompson
Augusta College

Helly's well known theorem states that all members of a family $\mathcal{C}$ of compact, convex sets in Euclidean d-space have a point in common provided every d+1 members of $\mathcal{C}$ have a common point. When $\mathcal{C}$ consists of disjoint unions of convex sets (even in the line), there exists no finite Helly type theorem without imposing further conditions. In 1961 Grunbaum and Motzkin proved a Helly type theorem for disjoint unions of two convex sets under further restrictions. Their theorem was extended to the case of three sets in 1968 by Larmen. In this paper a weak extension of their results is proved.

## CIRCUMFERENCE AND GIRTH

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123

ABSTRACT: Let G be a 2-connected graph with girth g and minimum degree k. Then any pair of vertices of G are joined by a path of length at least $\frac{1}{2}g(k-1)$, and the length of a longest cycle of G is at least $\frac{1}{2}gk$ if $g \geq 4$.

# ABSTRACTS OF CONTRIBUTED PAPERS

124

## Partitioning the Edges of a Planar Graph into Two Partial $k$-Trees

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*Charles J. Colbourn*

Department of Combinatorics and Optimization
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### ABSTRACT

In this paper we prove two results on partitioning the edges of a planar graph into two partial $k$-trees, for fixed values of $k$. Interest in this class of partitioning problems arises since many intractable graph and network problems admit polynomial time solutions on $k$-trees and their subgraphs (partial $k$-trees). Note that planar graphs are not partial $k$-trees, for any fixed $k$. In addition, the edges of any planar graph can be partitioned into at most 3 partial 1-trees (forests).

Here, we first prove that every planar graph is a union of two partial 3-trees. Furthermore, such a partitioning can be computed in linear time. Second, we show a recursive procedure to construct an infinite family of planar graphs in which every member does not admit a partitioning into a partial 1-tree (forest) and a partial 2-tree (series-parallel graph).

125

## Bounds on Sphere Partition of Symmetric Group

D. F. Hsu*; Fordham University, Bronx, New York. 10458 and Massachusetts Institute of Technology, Cambridge, MA 02139
Y. D. Lyuu; Harvard University, Cambridge, MA 02138.

Abstract: Let $S_n$ be the symmetric group on $N = \{1,2,...,n\}$. A distance function d based on transposition is defined in $S_n$. A sphere centered at $x \in S_n$ with radius r is the set $S(x,r) = \{y \in S_n \mid d(x,y) <= r\}$. A sphere partition with radius r is defined to be the set of disjoint spheres each with radius r so that their union is $S_n$, It is shown that if there exists a sphere partition of $S_n$ under the distance function then super-exponential numbers of spheres are required. Applications in solution space of combinational optimization problem are also studied.

126

## k-PATH IRREGULAR GRAPHS

Y. Alavi, A. J. Boals,* G. Chartrand, O. R. Oellermann, Western Michigan Unversity; and P. Erdős, Hungarian Academy of Sciences.

A graph G is k-path irregular, $k \geq 1$, if every two vertices of G that are connected by a path of length k have distinct degrees. This extends the concepts of highly irregular graphs (2-path irregular) and totally segregated graphs (1-path irregular). Some results and problems dealing with k-path irregular graphs are presented.

# ABSTRACTS OF CONTRIBUTED PAPERS

127

## SOME RESULTS ON PARTITIONING A PLANAR GRAPH INTO TWO HALVES

S.M.Venkatesan, Dept. of Computer Science, Univ. of Minnesota, MN 55455.

Abstract: New upper bounds are shown on the size of a vertex set that separates a planar graph into two halves. Specifically it is proved that, if arbitrary nonnegative weights summing to 1 are associated with the vertices of a planar graph $G$, then the vertex set of $G$ can be partitioned into three sets $A, B, C$ such that $weight(A), weight(B) \leq 1/2$, $|C| \leq (2\sqrt{7}+72/19)\sqrt{n}$, and no edge from $G$ connects a vertex from $A$ to a vertex from $B$. It is further shown that, if the weights on the vertices are all equal, then the removal from $G$ of a set $C$ of size $\leq 3\sqrt{6n}$ will suffice for such a separation. These two bounds improve on the previously known leading constants of $(8+16/81(\sqrt{6}/(1-\sqrt{2/3})))$ (in case of arbitrary weights) and $(7+1/\sqrt{3})$ (in case of uniform weights) shown by the author. The separation of $G$ is performed in *phases*. In each phase, there is an initial step where a subset of $C$ is identified by splitting a breadth first tree of the target graph, and the components are embedded in a planar graph $G'$ of small radius; the rest of the phase consists of a number of executions of a general step, where the graph $G'$ is split into four parts, three being permanently labeled as subsets of $A$, $B$ and $C$, and the last part embedded as before in a graph $G'$, and split in the next general step or set up as the target graph for the next phase. The subsets of $A, B, C$ so obtained are collected to form the required separation $A, B, C$. The claimed results follow by specifying the radius of $G'$ and the number of general steps per phase. Approaches for further improvements are outlined.

128

## Finding centers of certain tree-like networks

K.Brooks Reid and Weizhen Gu,* LSU

Let $\mathbf{K}^{(4)}$ denote the collection of all edge-weighted $K_4$ so that, in each such $K_4$

D(1) The distance between any two distinct vertices x and y is the weight of the edge incident to both x and y, and

D(2) At least two of the three different matchings have the same sum of edge-weights, and that sum is at least the sum of the edge-weights for the remaining matching.

Let $\mathcal{G}^{(n)}$ denote the collection of connected edge-weighted graphs in which each block is complete. Let $\mathcal{G}_{(4)}$ denote those networks obtained from graphs in $\mathcal{G}^{(4)}$ so that each $K_4$-block is in $\mathbf{K}^{(4)}$ and each $K_3$-block satisfies condition D(1).

In this paper a linear time algorithm is presented for finding an absolute center of a network in $\mathcal{G}_{(4)}$.

129

## A Note on Shortest Edge-disjoint Paths in Geodetically Connected Graphs

*Andreas Schwill, Dept. of Computer Science, University of Oldenburg*

**Abstract**

It is an interesting and mainly unsolved problem how much edge-connectivity of a graph G guarantees the existence of n pairwise edge-disjoint paths $P_1,\dots,P_n$ for any choice of 2n distinct vertices $s_1,t_1,\dots,s_n,t_n$ of G such that $P_i$ connects $s_i$ and $t_i$, $1 \leq i \leq n$. Thomassen conjectured that paths of that kind exist whenever G is k-edge-connected where k=n+1 if n is even and k=n if n is odd.

In this paper we transfer the problem to geodetically connected graphs first investigated by Entringer et al. A graph G is called n-geodetically connected if the removal of any n-1 vertices does not increase the distance between any pair of the remaining vertices. We prove that if G is n-geodetically connected, $n \neq 2$, then for any 2n distinct vertices $s_1,t_1,\dots,s_n,t_n$ there are n pairwise edge-disjoint paths $P_1,\dots,P_n$ such that $P_i$ connects $s_i$ and $t_i$ and the length of $P_i$ equals the distance of $s_i$ and $t_i$ for $1 \leq i \leq n$.

# ABSTRACTS OF CONTRIBUTED PAPERS

130

## TWO-GRAPH INVERSION OF COMPETITION GRAPHS AND BOUND GRAPHS

J. Richard Lundgren*, University of Colorado at Denver
John Maybee, University of Colorado at Boulder
Fred McMorris, University of Louisville

Upper bound graphs and competition graphs have been characterized in terms of certain edge clique coverings. Here we consider these graphs and their duals, namely, lower bound graphs and common enemy graphs. We determine for which pairs of graphs G and H does there exist a poset P such that G is the upper bound graph of P and H is the lower bound graph of P. Similarly we determine when there exists an acyclic digraph D such that G is the competition graph and H is the common enemy graph of D. As corollaries, we determine which graphs are both the upper and lower bound graph of some poset and which graphs are both the competition and common enemy graph of an acyclic digraph.

131

## On the number of proper colorings of graphs.

FELIX LAZEBNIK, Department of Mathematical Sciences,
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Let F denote the family of simple undirected graphs on v vertices having e edges, $P(\lambda; G)$ be the chromatic polynomial of a graph G. For the given integers v, e, $\lambda$, let $f(v,e,\lambda) = \max\{P(\lambda;G) : G \in F\}$. For some values of the parameters $f(v,e,\lambda)$ and all extremal graphs are found. For other values of the parameters several new non-trivial upper and lower bounds for the function $f(v,e,\lambda)$ are obtained and compared. Methods we used depended on the ranges of the parameters. The connection of the results to other problems from extremal graph theory is discussed.

113

132

## SYMMETRIC SEQUENCINGS OF HAMILTONIAN GROUPS

B. A. Anderson Arizona State University

Symmetric sequencings of finite groups G with a unique element of order 2 induce 1-factorizatons of complete graphs such that the automorphism group of the 1-factorization contains G. This class of 1-factorizations may be of interest. A combination of recent results shows that if $Q_{2n}$, $n \geq 3$, is a dicyclic group and A is Abelian of odd order relatively prime to n, then $Q_{2n} \otimes A$ has a symmetric sequencing. If $n = 2$, $Q_{2n} = Q_4$ is the quaternion group. The product result mentioned doesn't include $Q_4$ because $Q_4$ has no symmetric sequencing itself whereas the larger dicyclic groups do. In this note it is shown that the product result can be extended to $Q_4$ if $|A| \geq 3$. Groups of the form $Q_4 \otimes A$ are called Hamiltonian groups. They are non-Abelian groups such that every subgroup is normal. Every such group with a unique element of order 2 has this form.

# ABSTRACTS OF CONTRIBUTED PAPERS

133

## On Upper Bound Graphs of Partially Ordered Sets

Deborah J. Bergstrand*, Williams College
Kathryn F. Jones, University of Colorado at Denver

The upper bound graph of a poset is obtained by considering the poset elements as vertices and then joining any two by an edge if they share an upper bound. Recent work of Lundgren, Maybee and McMorris gives a characterization in terms of ordered edge clique covers of those graphs which arise as the upper bound graph of some poset. In this paper we present two characterizations of those graphs which have an ordered edge cover; we include a polynomial time algorithm. In addition we give several large classes of upper bound graphs. Finally, we begin exploring the class of graphs which are both the upper and lower bound graph of the same poset.

134

**Chromatic Number Versus Cochromatic Number in Graphs with Bounded Clique Number**, P. Erdös (Hungarian Academy of Sciences), J. Gimbel (U. of Alaska), and J. Straight*(SUNY College at Fredonia)

Given a simple graph $G = (V, E)$, a subset $U$ of $V$ is called a clique if it induces a complete subgraph of $G$ and is called an independent set if it induces an empty subgraph of $G$. The cochromatic number $z(G)$ of $G$ is the minimum number of sets into which $V$ can be partitioned so that each set is independent or a clique. Then $z(G)$ is bounded above by the familiar chromatic number $\chi(G)$ of $G$. Let $\omega(G)$ denote the maximum cardinality among the cliques of $G$, and let $n$ be an integer greater than 2. In this paper we explore the following question: is there a function $f(n)$ of $n$ such that, if $G$ has $\omega(G) < n$ and is not the complete graph of order $n-1$, then $\chi(G) \leq z(G) + f(n)$? Some bounds on $f(n)$ are obtained and, in particular, it is shown that $f(3) = 0$ and $f(4) = 1$.

135

## LOCAL MOTION CONNECTEDNESS OF LOW ORDER PLANES

ROBERT STERNFELD, INDIANA STATE UNIVERSITY

Local motion connectedness of the projective plane of order 3 was announced by Killgrove at the 11th British Combinatorial Conference. This paper extends the investigation to projective planes of orders 5 and 7 and discusses the algorithms used in the investigation.

In addition, the question of function equivalence is discussed. Let S be a finite set. Let f and g be two functions mapping from S to S. The functions are equivalent if there exists a permutation, p, of S such that $pfp^{-1} = g$. An invariant that distinguishes the equivalence classes is developed and an algorithm that calculates the invariant is presented. The invariant is closely related to the local motion invariant first discussed in Killgrove, Cates, and Sternfeld, "LOW ORDER SYSTEMS", Congressus Numerantium, vol. 55, pp 221-233.

# ABSTRACTS OF CONTRIBUTED PAPERS

136

## INTERSECTION PROPERTIES OF GRAPHS

Terry McKee, Wright State University, Dayton Ohio

For each graph-theoretic property, a corresponding "intersection property" can be defined. For instance, the intersection property of "is a path" is "is an interval graph." A simple formal language, based on vertices and paths, supports transfer of selected information (e.g., forbidden minor information) about the original property to its intersection property. For instance, simple descriptions of paths and trees produce the standard characterizations of interval and chordal graphs.

137

Title: A Las Vegas Graph Colouring Algorithm

Authors: John A. Ellis* (University of Victoria, British Columbia, Canada)
Pikie Lepolesa (National University of Lesotho, Lesotho)

Abstract

We consider the classic, graph colouring decision problem, i.e., given a graph G and an integer k, can G be properly node coloured with k colours? Some polynomial time, probabilistic node colouring algorithms have been proved to give the correct answer on "almost all", "randomly k-colourable" graphs, where "randomly k-colourable" defines a class of random graphs that is appropriate for evaluating the effectiveness of a probabilistic algorithm. Other authors have suggested that a carefully chosen path down the "Zykov tree" has a high probability of reaching a leaf in the tree that corresponds to a close to optimal colouring.

We put these ideas together to show that there exists a "Las Vegas" colouring algorithm with the properties that it always gives the correct result and "almost always" terminates in polynomial time, on "random k-colourable" graphs. We present the results of experiments that confirm the theoretical analysis.

113

138

## Configurations and their Graphs from Hyperbolic Planes

Jane W. DiPaola* and Harald Gropp
Cheyenne, WY Heidelberg

A hyperbolic plane of type (k,k) is a BIBD with $\lambda=1$ and r=2k. A Martinetti configuration is a partial design with v=b, r=k, $\lambda_1=0,1$. Associated with a Martinetti configuration is a regular graph on v vertices with valency $(k^2-2k)$. When the graph has certain properties a hyperbolic plane H(k,k) can be constructed. Conversely, every H(k,k) yields Martinetti configurations with appropriate graphs. In this paper we examine the graphs from H(k,k) with k=3,4,5. The concept of a clique-coloring is used to derive planes from graphs.

# ABSTRACTS OF CONTRIBUTED PAPERS

139

## DRAWING DIAGRAMS OF ORDERED SETS

Andrzej Pelc, Université du Québec à Hull

An ordered set may have many different diagrams representing it on the plane. Some of these pictorial renderings are easier to draw or to read than others and may thus be considered preferable. We investigate two criteria of quality for diagrams: planarity (avoiding crossings of covering edges) and minimizing the number of slopes used to draw covering edges as straight line segments.

We caracterize those orders of width two which have planar diagrams. We provide an efficient algorithm to test if a planar diagram exists and to draw it. We also study which orders can be drawn using only n slopes to render covering edges, where n is the maximum of all up and down degrees of vertices. Minimizing the number of slopes may be also combined with the planarity requirement.

This is a report of joined work with Jurek Czyzowicz and Ivan Rival.

140

## ON THE WALL PROBLEM

A.Gyárfás* and J.Lehel

Hungarian Academy of Sciences

The wall problem of M.Chrobak and M.Slusarek is equivalent to the question whether the greedy coloring has constant performance ratio on the family of interval graphs. The talk gives a summary of (old and recent) results on the problem.

* Visiting Memphis State University

113

141

## THE WEIGHT DISTRIBUTION OF THE GRASSMANN CODES C(2,N)

CHARLES RYAN*AND KEVIN RYAN    UNIVERSITY OF LOWELL

We consider a class of linear block codes which have as their generator matrix the matrix whose columns are the projective images under the classical Plucker embedding of the points of G(2,n) the grassmann variety of 2-diminsional subspaces of Z(2,n) the n-diminsional vector space over the field with two elements. These codes correspond to strongly normed graphs and have the property that d/n approaches 3/8 as n tends to infinity.

We first establish that there are exactly [n/2] orbits of codewords under the action of Gl(n) the general linear group where codewords of the same weight are in the same orbit. Moreover each of these orbits has a representative which can be expressed as a linear combination of special codewords corresponding to affine charts of G(2,n) viewed as a manifold of local diminsion 2(n-2). Using algebraic results and the above geometry we finally show that the number of codewords in C(2,n) of r(th) lowest weight is equal to the number of n*n symplectic matrices of rank 2r.

# ABSTRACTS OF CONTRIBUTED PAPERS

142

## Affine Difference Sets of Even Order

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Generalizing a result of Ko and Ray-Chaudhuri we show the following: Assume the existence of an affine difference set in $G$ relative to $N$ of even order $n \neq 2$. If $G$ is of the form $G = N \oplus H$, where N is abelian, then $n$ is actually a multiple of 4, say $n = 4k$, and there exists a $(4k-1, 2k-1, k-1)$-Hadamard difference set in $N$. More detailed considerations lead to variations of this result (under appropriate assumptions) which yield even stronger non-existence theorems. In particular, we show the non-existence of abelian affine difference sets of order $n \equiv 4 \bmod 8$ (with the exception $n = 4$) and of nilpotent affine difference sets of order $n \equiv 2 \bmod 4$ ($n \neq 2$). The latter result is the first general non-existence theorem in the non-abelian case.

143

ABSTRACT

## k-SATURATED GRAPHS WITH MINIMAL NUMBER OF EDGES

*Alexandros Moisiadis*, Queen's University, Kingston, Ontario

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An m-partite graph G is said to be k-saturated ($m \geq k$) if it contains no complete subgraph on k vertices, but any graph obtained from G by the addition of an edge incident with vertices in distinct classes does so.

We determine a condition under which an m-partite k-saturated graph on n vertices contains a set of vertices all of which have degree n-1. We completely characterize the m-partite k-saturated graphs on n vertices with minimal number of edges. Finally we fully characterize the minimal k-saturated graphs having minimum degree k-1 ( both arbitrary and m-partite ) and those with minimum degree k ( arbitrary graphs only ).

144

## A Construction of the Steiner System S(4,7,23) using PG(3,2)

113

by

Alphonse Baartmans
West Virginia University

In the Steiner system S(4,7,23) there exist eight (8) distinguished points with the property that any four of the eight points occur on a unique block of S(4,7,23) while no five of the eight points occur on a block. The remaining 23 - 8 = 15 points can be identified with the points of PG(3,2). The blocks of S(4,7,23) can now be partitioned into three types. Namely, 70 blocks ofS(4,7,23) with each block containing a unique set of 4 pts. of the 8 distinguished points; twenty-eight sets of blocks of size 6 each, with each block in a set of size 6 containing a pair of points of the eight distinguished pts.; 15 blocks that contain none of the eight distinguished pts., the planes of PG(3,2). Using the above structure of S(4,7,23) we attempt to construct the design by adjoining 8 additional points to PG(3,2).

# ABSTRACTS OF CONTRIBUTED PAPERS

145

## A Hamilton Path in the Transposition Graph for Multiset Permutations

Frank Ruskey, Dept. of Computer Science, University of Victoria

Let $\mathcal{P}$ be a partially ordered set on the elements $\{1, 2, \ldots, n\}$, and denote by $L(\mathcal{P})$ the set of linear extensions of $\mathcal{P}$. Each element of $L(\mathcal{P})$ can be thought of as a permutation of $\{1, 2, \ldots, n\}$. Now define a graph $G(\mathcal{P})$ with vertex set $L(\mathcal{P})$ and edge set defined by making two vertices adjacent if and only if the two permutations differ by a transposition. The graph $G(\mathcal{P})$ is connected and bipartite. Let $D(\mathcal{P})$ denote the difference in the number of vertices in the two bipartition sets, that is, the difference between the number of even permutations and the number of odd permutations. It has been conjectured that $G(\mathcal{P})$ has a Hamilton path whenever $D(\mathcal{P}) = 0$. Here we show that the conjecture is true if $\mathcal{P}$ consists of disjoint chains of lengths, say, $n_0, n_1, \ldots, n_t$. In this case the linear extensions of $\mathcal{P}$ correspond to permutations of a multiset with $n_k$ occurances of symbol $k$ for $k = 0, 1, \ldots, t$. The Hamilton path can be generated in constant average time.

## GRAPH-THEORETIC MODELING OF CELLULAR DEVELOPMENT II

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ABSTRACT

146

Let $G_0, G_1, G_2, \ldots$ be a sequence of graphs in which $G_{i+1}$ is generated from $G_i$ as follows:

(i) $G = G_0$ is an arbitrary simple graph of order n

(ii) For every vertex v in $G_i$ add a vertex w such that w is adjacent to v and all neighbors of v in $G_i$. The resulting graph H has order 2n.

(iii) $G_{i+1}$ is obtained from H by deleting all vertices of degree at least r

$G_{i+1}$ is called the offspring of $G_i$ and the sequence $G_0, G_1, \ldots$ is called an r-model. This paper is a continuation of work presented at the Fifteenth Southeastern Conference on Combinatorics, Graph Theory and Computir 1984

147

113

## A GEOMETRIC CONSTRUCTION OF NICE DESIGNS FROM CONICS

Julia M. Nowlin Brown, York University

We show for all finite (and for many infinite) fields $\mathfrak{F}$, that there exist in PG(2,$\mathfrak{F}$) at least $\lambda-1$ disjoint families $\mathfrak{S}_1, \ldots, \mathfrak{S}_{\lambda-1}$ of nonempty, irreducible conics such that each of the $\lambda-1$ incidence systems ( point set of PG(2,$\mathfrak{F}$), $\mathfrak{S}_i$, $\in$ ) is isomorphic to PG(2,$\mathfrak{F}$). The union of these $\lambda-1$ families $\mathfrak{S}_i$ together with the line set of PG(2,$\mathfrak{F}$) give a 2-($n^2+n+1$, n+1, $\lambda$) design whose block set partitions into $\lambda$ disjoint 2-($n^2+n+1$, n+1, 1) designs where $n = |\mathfrak{F}|$. In other words, we pack $\lambda$ copies of PG(2,n) into the power set of a set of size $n^2+n+1$. We have tried to do this for $\lambda$ as large as possible. Our present estimates of $\lambda$ appear below:

if char $\mathfrak{F}$ is odd or zero, then

$$\lambda \geq 1 + ( |\mathfrak{F}|^2 \times |\text{largest odd order subgroup of } \mathfrak{F}^*| )$$

if char $\mathfrak{F}$ is two, then

$$\lambda \geq |\mathfrak{F}|$$

Translated into the finite case:

if n is an odd prime power, then

$$\lambda \geq 1 + ( n^2 \times \text{largest odd factor of } n-1 )$$

if n is an even prime power, then

$$\lambda \geq n$$

# ABSTRACTS OF CONTRIBUTED PAPERS

148

## Fundamental subsets of edges of N-cubes

Mark Ramras, Northeastern University

Let E be a subset of the edges of the n-cube $Q_n$. We say that $Q_n$ has an E-partition if there is a subset K of $Aut(Q_n)$ such that the K-translates of E partition $E(Q_n)$. If K can be chosen to be a subgroup G, we say E is a fundamental set for $Q_n$ with group G.

We shall give some necessary conditions for E to be a fundamental set for $Q_n$, and then, by giving a variety of sufficient conditions, indicate what a wealth of fundamental sets $Q_n$ contains.

Sample results: Any set of n mutually non-parallel edges is a fundamental set for $Q_n$. Now let T be any tree with n edges. Since T can be isometrically embedded in $Q_n$, it follows that $Q_n$ is the union of $2^{n-1}$ isomorphic copies of T.

149

## Simplified conditions for degree constrained subgraphs

Richard Anstee, University of British Columbia

A (g,f) factor of a graph G is a subgraph of G whose valencies are bounded between g and f. Let G = (V,E) and let $g = (g_v : v \in V)$, $f = (f_v : v \in V)$ be vectors of integers with $0 \le g_v \le f_v \le degree_G(v)$. Then a (g,f)-factor is a subgraph H of G so that $g_v \le degree_H(V) \le f_v$. The conditions for the existence of a (g,f) factor due to Lovasz involve considering all disjoint pairs of subsets of vertices S,T in contrast to Tutte's conditions for a perfect matching which considers a single subset S of vertices. We seek conditions for the existence of (g,f)-factors of this latter type and extend results of Heinrich, Hell, Liu and Kirpatrick and Hell and Kirpatrick and others. The technique is to consider fractional (g,f)-factors (whose existence conditions are of the desired single subset type) and use alternating walks to seek a (g,f)-factor starting from the fractional factor. Certain conditions on G,g,f restrict the form of a barrier to a (g,f)-factor.

113

150

## PROBLEMS IN DYNAMIC PATH PLANNING

Alex C.-C. Meng, Texas Instruments
Simeon Ntafos*, University of Texas at Dallas

Consider motion planning in 2-d space with a set of fixed polygonal obstacles. To find shortest s-t paths for a robot in this space, the problem is usually reduced to a shortest path problem on a graph. Graphs based on the Voronoi Diagram lead to manouverable (but not shortest) paths, while shortest paths are obtained using the Visibility Graph of the scene.

Consider now a second set of obstacles for which no information is known a priori. We look at problems arising in such situations including path replanning based on restructuring the Voronoi Diagram/Visibility Graph to account for new obstacles and path preprocessing so that good alternative paths around new obstacles can be obtained in real-time.

# ABSTRACTS OF CONTRIBUTED PAPERS

151

## On the connectedness of some metric graphs

Joseph Zaks, University of Haifa, Israel & University of Waterloo, Canada.

Let $F$ be a field, $Q \subseteq F \subseteq R$, and let $F^d$ be the set of all the points of $R^d$, having their coordinates in $F$; let $G(F,d)$ be the graph, obtained from $F^d$ by connecting all pairs of points which are at distance 1 apart.

We have the following:

**Theorem 1:** $G(Q,d)$ has infinitely many connected components for $d \leq 4$, and it is connected for all $d \geq 5$.

**Theorem 2:** If $F = Q[\sqrt{k}]$, $k \equiv 1,2,3,5$ or $6 \pmod 8$, then $G(F,2)$ has infinitely many components, while $G(Q[\sqrt{t}],d)$ is connected for all $t \in N$, $d \geq 5$.

**Theorem 3:** If $a$ is a real algebraic number, satisfying a polynomial over Z of degree $n$, then $G(Q[a],d)$ is connected, for all $d \geq n+4$.

152

## On k-dependent Subsets and Partitions of k-degenerate Graphs

Randall Maddox, University of Mississippi and Harding University

A graph is said to be k-degenerate if the maximum minimum degree of its subgraphs does not exceed k. A vertex set is said to be k-dependent if the subgraph it induces has maximum degree not exceeding k. Given non-negative integers k and r and an r-degenerate graph G, we consider two problems:

1) Find a largest k-dependent vertex set in G;
2) Partition the vertices of G into as few as possible k-dependent sets.

Theorem: If G is 1-degenerate, then G has a k-dependent vertex set with at least the fraction $\frac{k+1}{k+2}$ of the vertices of G.

153

## PERFORMANCE APPRAISAL OF A CLIQUE GENERATION ALGORITHM ON A CYBER SYSTEM 170/720*

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### ABSTRACT

To find *all* cliques of a graph G, an algorithm called *modified cut-set algorithm*, was proposed by Das. The algorithm operates on a list of the edges of the complementary graph $\bar{G}$. Das designates this *list* as derived from the graph $\bar{G}$ an *edge inclusion table*. By performing a binary tree search on this data structure, the algorithm generates set of nodes that Das calls *irredundant solutions* of the edge inclusion table. Each such irredundant solution thus obtained from $\bar{G}$ represents the *complement* of a clique of G. The potential solutions are hence stored as they are generated, and the final step of the algorithm is to make a check through the complete list and delete the redundant ones, if any. Das originally did not quote any experimental results in his paper. However, he felt that his algorithm is *readily programmable on an IBM 360 system* by using APL. This paper attempts to evaluate the performance of the modified cut-set algorithm of Das by programming the algorithm using FORTRAN IV in Cyber system 170/720. A discussion on the efficiency of the algorithm as compared to some of the existing algorithm for both random and Moon-Moser graphs as well as large graphs is also provided.

*This research was supported in part by the Natural Sciences and Engineering Research Council of Canada under Grant A 4750.

S. R. Das, *On a new approach for finding all the modified cut-sets in an incompatibility graph*, IEEE Transactions on Computers, vol. C-22, pp. 187-193, February 1973.

# ABSTRACTS OF CONTRIBUTED PAPERS

154

## Minimal Forbidden Distance One Graphs

C.N. Purdy* and G.B. Purdy, University of Cincinnati

Abstract: A graph has a distance one realization in the plane if it can be placed in the plane in such a way that two vertices are at distance one if and only if the vertices are joined by an edge (with edges being allowed to cross). We discuss graphs which are minimal with respect to the property that they are not distance one realizable. We also discuss possible algorithms for finding such graphs. We describe the production, with the help of a CRAY X-MP/24, of a list of minimal forbidden subgraphs $F_1, F_2, \ldots, F_n$, such that any graph up to a certain size not containing $F_1, F_2, \ldots, F_n$ has a distance one realization. We also attempt to establish a graph-theoretic characterization for distance one realizability. (The Robertson-Seymour Theorem does not apply, since distance one realizability is not a hereditary property.)

155

## BIPARTITE GRAPHS AND THE DEGREE CONDITIONS

K.S. Bagga, Indiana University-Purdue University at Fort Wayne
Badri Varma*, University of Wisconsin-Fox Valley

We investigate the following:

Let G be a graph of order n and let P be a sufficient condition for some property, involving degree of vertices and the order of G. In the case that G is bipartite, for what properties does P remain sufficient when the value of n in P is replaced by n/2 + 1?

To answer this question we consider the hamiltonian and pancyclic properties of graphs.

156

## ON MINIMIZING THE TOTAL-DENSITY OF A CHANNEL

*Ioannis G. Tollis*
The University of Texas at Dallas

In this paper we introduce the concept of *total-density* of a channel and consider the problem of minimizing this new measure by lateral shifting of the channel's shores. The total-density of a channel is the sum of the densities over all the columns of a channel. It is shown that the concept of total-density is very closely related to the channel density and the total wire length, and therefore it is desirable to find a shift that minimizes it.

We also present two algorithms for the total-density minimization problem. The first algorithm runs in $O(l)$ time, where $l$ is the sum of the distances between the extreme terminals on the top and bottom shores, and the second runs in $O(n \log n)$ time, where $n$ is the number of nets in the channel. Experimental results show that the algorithms produce solutions which are better than the theoretical (worst case) upper bounds. Namely, the channel density for the shift $s^*$, where $s^*$ is a shift for which the total density is minimized, is very close to the minimum channel density over all shifts. Furthermore, the total horizontal wire length required is minimized over all shifts. Finally, our algorithms are fast and easy to implement.

157

Abstract

## Sequences of Subsets: The Relationship to Biclique Covers and Partitions of Digraphs

Kim A. S. Hefner, University of Colorado at Denver

A procedure for computing biclique covers and partitions for digraphs using sequences of subsets is examined. A digraph has an adjacency matrix which, when relabeled, can represent the adjacency matrix of a bipartite graph G = (X,Y,E). All possible edges are added to make X a complete graph and Y a complete graph. Then sequences of subsets can be generated which yield biclique covers and biclique partitions when they are reapplied to the digraph problem. The subsets themselves represent the vertices of the bipartite graph, while the elements of the subsets form the numbered cliques and bicliques. Different digraphs will necessarily impose varying constraints on the construction of the sequences of subsets.

130

158

## Some Results on the Number of and the Complexity of Verifying Primal Graphs

Phyllis Z. Chinn,* Mathematics, Humboldt State University, Arcata, CA 95521
R. Bruce Richter, Mathematics, US Navel Academy, Annapolis, MD 21402
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There is a unique set of graphs, the primal graphs, with the property that every graph can be edge-decomposed into non-isomorphic primal graphs, while no primal graph has a non-trivial such décomposition.

The simplest family of primal graphs consists of $P = \{2^i K_2\}$ . We prove here that the problem of decomposing an arbitrary graph into graphs in P is NP-complete, thereby suggesting that the problem of determining whether an arbitrary graph is primal is NP-hard. We also show that the number of primal graphs on at most n vertices is exponential in n.

159

## Consensus Priority: The Merging of Priority And Preference Control Construct

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*Abstract*

Priority control construct in Ada is a useful tool which can be used by calling tasks(clients) to specify the explicit order of rendezvous(execution) with called task(server). A preference based control construct has been suggested [ELRA86] in opposite to the priority control which can be specified by the called tasks(server). Each of the techniques has some reasons and advantages of expressing the urgency and the importance of its own. However each possesses the merits that the other lacks. This paper suggests a new approach termed Consensus Priority to permit the use of both priority and preference control constructs in parallel. The Consensus Priority is a mechanism which takes both priority and preference values and returns an ordered "Consensus" priority mapping. This new approach merges the best future of both priority and preference controls into a single unique scheduling allowing to override the implicit order and the explicit order imposed by priority pragmas.

THURSDAY -5:10p.m.

# ABSTRACTS OF CONTRIBUTED PAPERS

160

148 A NEW APPLICATION OF GRAPH THEORY

by Arthur M. Hobbs, Mathematics Department, Oakland University, Rochester, Michigan 48309 (visiting from Mathematics Department, Texas A&M University, College Station, Texas 77843)

A "square dance" is a dance of four couples directed into various formations by a "caller". It is possible to represent a square dance by a directed graph, and this graph can be used to solve many of the problems faced by professional callers of square dances. In the graph theoretical representation, the formations taken by the dancers are the vertices, and the directions, or "calls", of the caller are the arcs. The momentum and position of the dancers in each formation must be utilized correctly by the caller in choosing his sequence of calls, and achieving this goal is one of the more difficult problems a caller faces. The representation of the dance by a graph immediately solves this problem by incorporating the momentum and position information in the description of the vertices. Many of the remaining significant problems of the experienced caller can be easily represented as trail-finding problems on this graph. In this talk, the graph representation is described in detail and a discussion is given of the means of solving the graph theoretical problems of the caller.

161

## Primal graphs of small order and degree

Eric Regener* and Sara Stairs
Concordia University, Montreal, Canada

The set $\Pi$ of primal graphs is such that every simple graph has an edge-decomposition into non-isomorphic graphs $\in \Pi$ and the elements of $\Pi$ are just those $G$ for which this decomposition is trivial. We report on the results of a computer search for graphs $G \in \Pi$ among all graphs with small $|V(G)|$, with and without the restriction that the maximum degree is 3.

162

## AN EFFICIENT IMPLEMENTATION OF A MESH-MATRIX PLOTTING PACKAGE FOR MODELLING PLANAR FINITE ELEMENT SYSTEMS

David R. McIntyre

Department of Computer Science
Cleveland State University

The use of graph theory has long been useful for both the analysis and comparative study of various ordering for for sparse symmetric Gaussian elimination. We present an efficient implementation of a plotting algorithm which can display both the matrix and the mesh at each stage of the planar mesh model of symmetric Gaussian elimination. The asymptotic time and space complexities of the algorithm are proved.

# ABSTRACTS OF CONTRIBUTED PAPERS

163.

## GRAY PATHS AND SYNCHRONIZABLE CODES IN THE N-CUBE

L.J. Cummings
University of Waterloo

**ABSTRACT**

A binary code has bounded synchronization delay if there exists an integer s such that at most s consecutive bits are required to establish word synchronization in any message. The set of words obtained by choosing those which are lexicographically least in the non-periodic orbits determined by cyclic permutation of all words of length n is called the canonical bounded synchronization delay code and denoted by $\Lambda_n$. It has the maximal number of words possible in a synchronizable code of fixed word length n. Any code of fixed word length n can be represented as a set of vertices in the n-cube. The code $\Lambda_n$ is a connected subset of the n-cube. We give evidence for the conjecture that $\Lambda_n$ is always a Gray path in the n-cube. That is, $\Lambda_n$ forms a path in the n-cube with the property that each vertex of the path differs in only one bit from the adjacent vertices.

164

## The Separation and Edge-Connectivity Vectors of Rectangular Grid Graphs

Phyllis Chinn and Daniel Munton*
Humboldt State University, Arcata, CA 95521

The separation vector of a graph $G(p,q)$ is defined as $\eta(G) = (\eta_1, \eta_2, \ldots, \eta_{p-1})$ where $\eta_k$ is the minimum number of edges whose removal from G leaves components having order at most p-k (Bagga, et al.). The first $\lfloor \frac{p}{2} \rfloor$ entries of $\eta(G)$ are called the edge-connectivity vector $\lambda(G)$ (Pippert and Lipman). $\lambda(G)$ is determined for arbitrary rectangular grid graphs. Some patterns and results for the second half of $\eta(G)$ are also presented. Some possible applications to forest fire management, monitoring of insect pests and irrigation patterns are discussed.

165.

## Algorithms for Subsequence/Supersequence Computation

F. Hadlock - Tennessee Technological University

Given an arbitrary number of strings/sequences over a finite alphabet, it has been shown that both the longest common subsequence (LCS) and the shortest common supersequence (SCS) problems are NP complete. At the same time, several subquadratic (product of lengths) algorithms have been developed for the LCS of 2 strings. In this paper, we give a general algorithm which employs a unified approach to computation of the LCS or SCS for $n > 2$ strings, is subquadratic for LCS, $n = 2$, and linear in this case if one string is a subsequence of the other.

# ABSTRACTS OF CONTRIBUTED PAPERS

166.

TWO-LEGGED CATERPILLARS WHICH SPAN HYPERCUBES

Frank Harary, NMSU, Las Cruces, NM 88003
Martin Lewinter,* SUNY, Purchase, NY 10577
William Widulski, NYU, student

A caterpillar is a tree which becomes a path when its endnodes are removed. A two-legged caterpillar has maximum degree 4. We present classes of two-legged caterpillars which span hypercubes. A characterization of two-legged caterpillars which span hypercubes is at present unknown. Several necessary conditions are presented.

167.

Realization of majority preference digraphs by graphically determined voting patterns

Terri Wilhite Johnson* and P.J. Slater
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The University of Alabama in Huntsville
Huntsville, Alabama 35899

Motivated by the work of N.R. Miller and of K.B. Reid we consider a finite nonempty set V of voters and a finite nonempty set C = $c_1, c_2, \ldots, c_m$ of candidates. Let D be the digraph with vertex set C with $(c_i, c_j)$ an arc in D if and only if more voters in V prefer $c_i$ over $c_j$ than prefer $c_j$ over $c_i$. (Unlike Miller and Reid who considered majority tournaments, we allow ties.)

We examine conditions under which D can be so realized when the voting pattern for each voter is "rationally" determined. Specifically, we seek a graph G in which voters and candidates correspond to vertices and voting patterns are determined by distance constraints.

168.

Several Remarks on the Impact of Design Theory of Relational Databases on Combinatorics

Dan A. Simovici* Corina Reischer

This paper considers the impact of several results of relational database theory on combinatorial set theory. Such concepts like join, functional dependency, decompositions are extended to set systems. We study the behaviour of covers, Sperner systems and matroids from the point of view of relational databases.

# ABSTRACTS OF CONTRIBUTED PAPERS

## CHAINS IN MULTISETS AND FINITE FIELDS 169.

Michael E. Mays, West Virginia University

A chain in a set A is a set of subsets of A ordered by inclusion. Gould and Mays [Utilitas Mathematica, vol. 31 (1987), 227-232] associated a set A and a (restricted) set of subsets of A with every entry in every table of $n$th differences of powers of integers so that the number of chains of a certain fixed length is given by the table entry. These ideas are extended to chains in finite fields by associating the lattice of divisors of $n$ for a finite field of order $p^n$ with the lattice of subsets of a multiset. If the exponent $n$ is square-free then the multiset is in fact a set and the old work applies directly.

## Predicates with Periodic Maximal Length Functions 170.

Michael Gilpin* and Robert Shelton
Michigan Technological University

Let P be a predicate defined on finite sets of positive integers. Let L(n) be the length of the largest subset of [n] = {1, 2, ... ,n} for which P is false. We exhibit conditions on P which force the existence of integers N, M, and K with

$$L(n+M) = L(n) + K \quad \text{whenever } n > N.$$

This extends results related to C. L. Liu's *Chessmaster Problem*, as investigated by Erdős, Hemminger and McKay, Liu and Wagstaff.

## ON EPSILON PARTITIONING OF A PLANAR GRAPH 171.

S.M.Venkatesan, Dept. of Computer Science, Univ. of Minnesota, MN 55455.

Abstract: It is shown that the removal of $4\sqrt{n/\epsilon}$ vertices from a planar graph with non-negative vertex weights adding to $\leq 1$ is sufficient to recursively separate it into pieces of weight $\leq \epsilon$, thus improving on the $2\sqrt{6}\sqrt{n/\epsilon}$ bound shown previously by the author. The construction used in the proof depends on the following procedure to bring the maximum weight of any component (of radius $\sqrt{n\epsilon}/2$) to the range $(2^{i-1}\epsilon, 2^i\epsilon]$, given that the maximum now lies in the range $(2^i\epsilon, 2^{i+1}\epsilon]$: First separate each component in the range $(2^i \cdot 3\epsilon/2, 2^{i+1} \cdot \epsilon]$ into three components each of weight $\leq 2^i \cdot \epsilon$ by applying weighted regular 1/2-separation at the expense of $3/2\sqrt{n\epsilon}$ vertices per component. Next separate each component in the range $(2^i \cdot \epsilon, 2^i \cdot 3\epsilon/2]$ into two components each of weight $\leq 2^i \cdot \epsilon$ at the expense of $\sqrt{n\epsilon}$ vertices per component using a result of Lipton and Tarjan. The increase in separator size due to one application of this procedure is $\sqrt{n/\epsilon}/2^i$, since the total weight of the components in both ranges must be $\leq 1$. By applying this procedure repeatedly, the maximum component weight is brought down to $\leq \epsilon$, giving a total separator size which is the sum of this over all iterations plus the cost of reducing the radius of the original graph to $\sqrt{n\epsilon}/2$, which is a total of $4\sqrt{n/\epsilon}$. Recently, Djidjev has shown how to implement our result in *linear* time, putting perhaps many interesting planar graph problems in linear time.

# ABSTRACTS OF CONTRIBUTED PAPERS

### Forest Covers and a Polyhedral Intersection Theorem 172.

A. B. Gamble* and W. R. Pulleyblank, University of Waterloo

A forest cover of a graph is a spanning forest for which each component has at least two nodes. We consider the convex hull of incidence vectors of forest covers in a graph and show that this polyhedron is the intersection of the forest polytope and the cover polytope. This polytope has both the spanning tree and perfect matching polytopes as faces. Further, the forest cover polytope remains integral with the addition of the constraint requiring that for some integer $k$, exactly $k$ edges be used in the solution.

### On the Number of Ways to Bisect A Graph 173.

Carolyn D. Sister
Eckerd College

We show that there are at least $n$ ways to cut an $n$ vertexed 2-connected graph into two connected pieces with equality if and only if the graph is outerplanar.

### CONJUNCTIONS AND BINARY BOOLEAN FUNCTIONS 174.

Leon Kotin
U. S. Army Communications-Electronics Command
Fort Monmouth, NJ 07703

We introduce the idea that there is a natural relationship between grammatical conjunctions and Boolean functions of two Boolean variables. We examine this relationship, calling upon considerations of syntax, semantics and logic. In particular, we show that to any truly binary Boolean function, there corresponds at least one conjunction (either natural or coined) which is both logically and semantically equivalent to it. On the other hand, although some conjunctions are logically equivalent to a binary Boolean function, they are not so semantically.