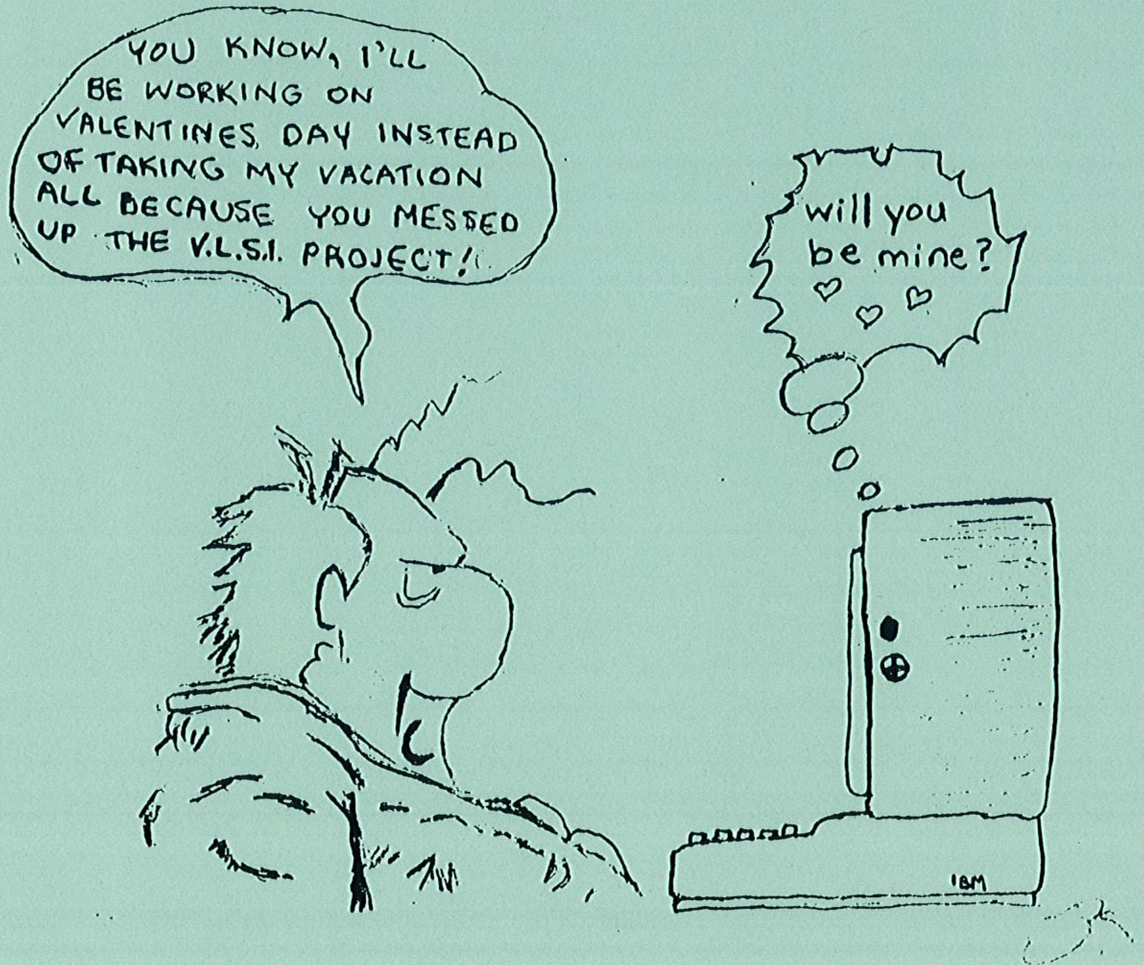
FEBRUARY 11 to 15, 1985



PROGRAM
AND
SCHEDULE

16th Southeastern International Conference on Combinatorics, Graph theory, and Computing

MISCELLANEOUS INFORMATION

The phone number at the Conference desk is 393-3718.
There is a coin-operated xerox machine in the second floor lounge of the University Center.
The Copy Center (behind the police station, which is near the south end of the covered walkway) can xerox onto transparencies. Blank transparencies are available in the bookstore in the University Center and at the copy center.
GCN and GCS are the two halves of the Gold Coast Room of the University Center.
Room 207 is entered from the second floor lounge of the U.C.

COFFEE will be available in Room 219 after Monday Morning, as will the book exhibits and the display of menus from many area restaurants.

TRANSPORTATION: We shall provide van transportation from Day's Inn and University Inn to the University Center at 8:00AM Monday through Friday, making two trips if necessary. There will be van transportation to the motels at 5:30PM Monday through Thursday from the University Center. There will also be van transportation from the reception at the Board of Regents Room on Monday evening. Transportation will be provided at 5:45PM and 6:15PM from the University Center and at 6:30PM from the motels to the Tuesday night and Thursday night parties and at 6:30PM to the Wednesday night banquet. The van will go from the parties back to the University Center and to the motels. Transportation will be provided back to the motels at the close of the banquet. The vans will be available for transportation back to the motels at the close of the Conference on Friday. Although we aim to provide all needed transportation, we encourage car-pooling where possible.

Monday

	GCN	GCS	207
8:40	Registration (from 8:00a.m.) GCR		
9:00	Welcome: Pres. Popovich, VP Michels		
9:30	TARJAN		
10:30	COFFEE (GCR)		
10:50	1 E.Hare	5 Myrvold	9 Wertheimer
11:10	2 Varol	6 Lee	10 Dillon
11:30	3 Pittel	7 Richter	11 Santiago
11:50	4 Huang	8 Fajtlowicz	12 Phelps
12:10	LUNCH		
1:45	TARJAN		
2:45	SCHWENK		
3:00	COFFEE (Room 219)		
3:20	13 Gargano	19 Chinn	25 Monma
3:40	14 Reid	20 Piazza	26 Fowler
4:00	15 Bjorner	21 Jones	27 Grimaldi
4:20	16 Culik	22 Watkins	28 Brecht
4:40	17 Miller	23 Roberts	29 Hsu
5:00	18 Balas	24 Perkel	30 Yen

Tuesday

	GCN	GCS	207
8:40	31 Foulds	37 Marcotte	43 Rucinski
9:00	32 Chung	38 Archdeacon	44 Wagner
9:30	LEIGHTON		
10:30	COFFEE (Room 219)		
10:50	33 Sack	39 Hajela	45 Matthews
11:10	34 Wachs	40 Dutton	46 Bialostocki
11:30	35 Yeh	41 Saks	47 Grinstead
11:50	36 Simion	42 Andrews	48 Regener
12:10	LUNCH		
1:45	LEIGHTON		
2:45	COFFEE (Room 219)		
3:00	49 Whited	56 Entringer	63 McMorris
3:20	50Ramachandran	57 Simmons	64 Maybee
3:40	51 Ramesh	58 Pritikin	65 Grinberg
4:00	52 Slater	59 Lo	66 Canfield
4:20	53 Munson	60 WR Hare	67 Atkinson
4:40	54 Ordman	61 Holliday	68 Skilton
5:00	55 Neufeld	62 Evans	69 Luedeman

Wednesday

	GCN	GCS	207
8:40	70	76 Cockayne	82 Brawley
9:00	71 Rodger	77 BN Clark	83 DeHoff
9:30	SLOANE		
10:30	COFFEE (Room 219)		
10:50	72 Sane	78 Hedetniemi	84 Thomas
11:10	73 Hamm	79 Proskurowski	85 Colijn
11:30	74 Heinrich	80 Stark	86 Zhang
11:50	75 A Hartman	81 Peters	87 Neumann-Lara
12:10	LUNCH		
1:45	SLOANE		
2:45	COFFEE (Room 219)		
3:00	88 Bland	95 Srinivasan	102 Truszczynski
3:20	89 Cherowitzo	96 Buckley	103 Wallis
3:40	90 Batten	97 Koh	104 Lundgren
4:00	91 Lam	98 Leiss	105 Gregory
4:20	92 Ko	99 Sabnani	106 I Hartman
4:40	93 Payne	100 Chen	107 Gimbel
5:00	94 Purdy	101 Deo	108 Goldberg

Thursday

	GCN	GCS	207
8:40	109 Pomerance	114	119
9:00	110 Lynch	115 Ellingham	120 Baildon
9:30	ERDOS		
10:30	SLOANE		
10:50	COFFEE (Room 219)		
11:10	111 Beasley	116 Dewdney	121 Dekster
11:30	112Niederhausen	117 Cull	122 Rouvray
11:50	113 Freeman	118 Bate	123 Robinson
12:10	LUNCH		
1:45	PROBLEM SESSION		
2:45	PROBLEM SESSION (continued)		
3:00	COFFEE (219)		
3:20	124 Abrham	130 Scheinerman	136 Hemminger
3:40	125 Zalcstein	131 West	137 Frick
4:00	126 Freiman	132 Simovici	138 Hartnell
4:20	127 Moreno	133 Ziarko	139
4:40	128 Arasu	134 Dahbura	140
5:00	129	135 El-Mallah	141

Friday

	GCN	GCS
9:00	142 L Clark	149 Livingston
9:20	143 Bauer	150 de Caen
9:40	144 Dejter	151 Dow
10:00	145 Paoli	152 Eades
10:20	COFFEE (Room 219)	
10:50	146 Das	153 Katona
11:10	147 LaPorte	154 Filus
11:30	148 Corneil	155

Monday, February 11, at 9:30am and 1:45pm, Dr. Robert E. Tarjan of AT&T Bell Laboratories will speak on "New Themes in Data Structure Design."

Monday, February 11, at 2:45pm, Dr. Allen Schwenk of the Office of Naval Research will speak on "The Discrete Mathematics Program at ONR."

Tuesday, February 12, at 9:30am and 1:45pm, Prof. Frank Thomson Leighton of M.I.T. will speak on "Networks, Parallel Computation and V.L.S.I."

Wednesday, February 13, at 9:30am and 1:45pm, Dr. Neil J.A. Sloane of AT&T Bell Laboratories will speak on "On the Covering Radius of Codes and Lattices."

Thursday, February 14, at 9:30am, Professor Paul Erdős will speak on "Applications of Graph Theory to Combinatorics and Number Theory."

Thursday, February 14, at 10:30am, Dr. Sloane will give a short talk on "Anti-Hadamard Matrices."

Thursday, February 14, at 1:45pm, there will be a problem session.

Conference participants are invited to the Cocktail Party Reception in the Board of Regents Room of the Administration Building from 6:00PM to 7:30PM on Monday, February 11, 1985. (The drink tickets are for accounting purposes. BLUE TICKETS for a mixed drink, YELLOW TICKETS for beer or wine and no tickets required for soda. Additional tickets may be requested.) Transportation will be available back to the motels after the Reception. The vans going to the motels at 5:30 will return to the University, leaving the motels at about 6:00.

Conference participants are also invited to a Beer Party, Tuesday February 12, from 6:00PM at the home of Dr. and Mrs. John M. Freeman, 741 Azalea Street (but park on Aurelia!). Vans will leave the University Center at 5:45 and 6:15PM for the party, and will leave the motels at 6:30PM. Transportation will be provided back to the motels and the University Center after the party. It is about a one-mile walk from the University to the Freeman home.

The Coference Banquet will be held at the Board of Regents Room, Wednesday February 13. The cash bar opens at 6:30PM. Seating will be at 7:15 and service at 7:30PM. A van will leave for the banquet from the motels at 6:30PM.

There will be a Party 6:00PM to 8:00PM Thursday evening February 14 at the home of Dr. and Mrs. Frederick Hoffman, 4307 N.W. 5th Avenue. It is a walk of less than two miles from the University to the Hoffman home. Vans will leave the University Center at 5:45PM and 6:15PM and the motels at 6:30PM.

Our Conference Proceedings are published by Utilitas Mathematica Publishing Co., P.O. Box 7, University Centre, Winnipeg, Manitoba, Canada, R3T 2N2 in its series Congressus Numerantium. Back issues and additional copies may be ordered from them.

Index of Papers by Author
This index includes all authors of papers to be presented at the conference.
Papers appear with presenter's name on the schedule.

THE CERTIFICATE: A CANONICAL DATA STRUCTURE FOR TREES (1)

E. O. Hare+ and S. T. Hedetniemi, Clemson University

A rooted alias is a canonical representation of a rooted tree. The certificate of a tree is the lexicographically smallest of all of its rooted aliases. Properties of rooted aliases and the certificate of a tree are examined and an algorithm is presented for constructing the certificate of an arbitrary tree.

EXTENDIBLE HASHING WITH DOUBLE DIRECTORIES (2)

H.L. Chou and Y.L. Varol*, Southern Illinois University at Carbondale

For highly dynamic files the performance of conventional hashing schemes is very poor. Extendible Hashing (EH) and Bounded Index Exponential Hashing (BEH) were introduced to handle key to address transformations in growth oriented dynamic files. EH, and to a lesser extent BEH, suffer from poor storage utilization, and the average performance of both schemes deteriorates with the presence of clusters in the key space. This paper proposes a scheme called Extendible Hashing with Double Directories. It maintains high storage utilization at all times, and its performance is affected more linearly by clusters. It incorporates some of the ideas introduced by EH and BEH, as well as additional techniques such as page sharing, use of overflow index, index merger and local hashing. Basic file operations in the new scheme are described and compared with those of EH and BEH.

On search times for a coalesced hashing algorithm. (3)

B. Pittel, Department of Mathematics, OSU, Columbus, Ohio 43210

An allocation model (n balls, m ($\geq n$) cells, at most one ball in a cell) related to a hashing algorithm is studied. A ball x goes into the cell $h(x)$, where $h(\cdot) : \{1,\dots,n\} \to \{1,\dots,m\}$ is random. In case the cell $h(x)$ is already occupied, the ball x is rejected and moved into the rightmost empty cell. This empty cell is found via the sequential search from right to left starting with the cell occupied by the last (before x) rejected ball. Denote $T_1(x)$ the number of necessary probes. In the end, due to a resulting system of references, the n occupied cells form a disjoint union of ordered chains, and to locate a ball x it suffices to search only the cells of a subchain originating at the cell $h(x)$. Denote $T_2(x)$ the total number of these cells. Our main result is: in probability, $\max T_1(x) = \log_\beta n - \log_\beta \log n\,(1 + o(1))$, $\max T_2(x) = \log_\beta n - 2 \log_\beta \log n\,(1 + o(1))$ as $n/m \to \alpha \in (0,1]$, where $\beta = (1 - e^{-\alpha})^{-1}$.

EDGE RECONSTRUCTION OF BIDEGREED GRAPHS (5)

W.J. Myrvold* and M.N. Ellingham, University of Waterloo, and D.G. Hoffman, Auburn University

An *edge-deleted subgraph* of a graph G is a subgraph obtained from G by the deletion of an edge. The Edge Reconstruction Conjecture asserts that every simple finite graph with four or more edges is determined uniquely, up to isomorphism, by its collection of edge-deleted subgraphs. A class of graphs is said to be *edge reconstructible* if there is no graph in the class with four or more edges that is not edge reconstructible. This talk will outline the proof that bidegreed graphs (graphs whose vertices all have one of two possible degrees) are edge reconstructible. This result can be generalized to show that all graphs that do not have three consecutive integers in their degree sequence are also edge reconstructible.

GRACEFULLNESS OF SOME BIPARTITE AND TRIPARTITE GRAPHS (6)

S. M. Lee* and Grace Wang, San Jose State University

For $m, n \geq 1$, let $K(m,n)$ be the complete bipartite graph with partition of m and n nodes. We show that the join $K(m,n) + K(1) = K(m,n,1)$ is graceful. It is also shown that the corona of $K(m,n)$ is graceful.

ADDITIVITY THEOREMS FOR THE GENUS OF A GRAPH (7)

Bruce Richter, Ohio State University

The orientable genus $\gamma(G)$ of a graph G is the least number h of handles such that G embeds in the sphere with h handles. Recently, Decker et al have proved that if H and K have exactly two vertices in common, then for some $\varepsilon \in \{-1,0,1\}$, $\gamma(H \cup K) = \gamma(H) + \gamma(K) + \varepsilon$. In this talk, analogous results are presented for the non-orientable genus $\tilde{\gamma}(G)$ and the generalized genus $\varepsilon(G) = \min(\tilde{\gamma}(G), 2\gamma(G))$.

ON A PROGRAM MAKING CONJECTURES IN GRAPH THEORY (8)

S. Fajtlowicz, University of Houston

Graffiti is a program making conjectures in Graph Theory. It consists essentially of three parts.

The first part allows a user to define certain graphs. The second computes values of selected invariants for a given graph. The third part verifies whether a formula in terms of invariants is valid for all graphs for which the invariants were computed. Formulas for which Graffiti finds no counter-examples are reported as conjectures.

A program based on the same principles was written before by Shui-Tain Chen. It produced hundreds of conjectures but almost all of them were trivial and the burden was on the user to select a few which were difficult to solve.

Graffiti is equipped with a heuristic procedure Echo which selects some conjectures as nontrivial. Using Echo I found a number of conjectures which I can neither prove nor disprove.

Balanced Ternary Designs Derived From Other Combinatorial Designs (9-10)

J.F. Dillon and M.A. Wertheimer, Dept. of Defense

A BTD differs from a BIBD only in that its "blocks" are multisets with multiplicities 1and 2. In these two talks we show that BTDs arise from better known designs in very natural ways. PART I reviews the basics of BTDs and presents some general quotient constructions and an application to a covering problem. PART II presents some general constructions based on difference sets, weighing matrices and other designs. Special cases provide answers to some existence questions raised in a recent survey paper by E.J. Billington and P.J. Robinson.

ON A THEOREM OF WELCH (11)

Moreno, Oscar and Santiago, Nilda*, University of Puerto Rico

In 1980 Moreno conjectured a generalization of a Theorem of Welch, and proved some cases of this conjecture. This gives the number of solutions to the equation $T_r(y^l) = 0$ for every l dividing $2^a + 1$ for some a, and the solutions being over a finite field.

Presently we will prove this conjecture using "uniform cyclotomy" as introduced by Baumert, Mills and Ward in 1980. Actually our result as well as uniform cyclotomy can be also obtained as a consequence of R.J. Mc Ellece work on irreducible cyclic codes.

EVERY FINITE GROUP IS THE AUTOMORPHISM GROUP OF SOME LINEAR CODE (12)

K. T. Phelps, Georgia Institute of Technology

We prove that every finite group is isomorphic to the full automorphism group of some linear binary code. This result remains true even if you specify the minimum distance, d, of the linear code.

FRINGE ANALYSIS AND FRINGE-BALANCED TREES (4)

Shou-Hsuan Stephen Huang, University of Houston

Fringe analysis is a technique to analyze search trees based on subtrees near the fringe of trees. It was first formulated by A. Yao to compute the expected number of nodes in 2-3 trees and B-trees. The analysis is based on a set (called base) of subtrees. By studying the steady state solution of the recurrence relation derived from transition after random insertions, we will be able to find various properties about the search trees. However previous research failed to present a formal way of constructing the base needed in such analysis. We first present a formal definition of a parameterized (d-)fringe. Then we present an algorithm to find the base for the analysis. In some cases (such as AVL trees), the base may not exist for d > 1. Thus the accuracy of the analysis may be limited. We also define a parameterized fringe-balanced trees d-T for each class of binary search trees T, so that we can always find a base for fringe analysis. These fringe-balanced trees d-T converge to T as d increases.

(13) A DIGRAPH GENERALIZATION OF BALANCED SIGNED GRAPHS

MICHAEL GARGANO* and LOUIS V. QUINTAS, Pace University, New York, NY 10038

The concept of balance (every circuit has positive weight) in a signed graph (each line is considered either positive or negative) is extended to symmetric digraphs in which each arc is weighted with an element of an abelian group. A theorem analogous to Harary's Structure Theorem for Signed Graphs is obtained which characterizes balanced symmetric digraphs in this general setting. In addition, some properties of such graphs are derived and observations are made concerning other extensions of balance to signed digraphs and to the group valued graphs of Sampathkumar and Bhave.

Problems on the number of cycles in complements of trees

(14)

K.B. Reid, Louisiana State University

A decade ago the author proved that the unique tree with n vertices whose complement contains the least number of cycles has exactly two non-endvertices one of which is adjacent to exactly two endvertices (Discrete Math 15 (1976) 163-174). Problems of this sort, including some recent problems of Holroyd and Wingate (preprint), will be discussed.

COMPLEXITY AND EVASIVENESS IN GREEDOIDS. (15)

Anders Björner, Dept. of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02146

Let E be a finite set of cardinality n and suppose that $P \subseteq 2^E$. The complexity $c(P)$ is the minimum number of entries of the incidence vector χ_A that the best P-testing algorithm needs to inspect in the worst case $A \subseteq E$. Let $p(t) = \sum_{F \in P} t^{|F|}$. It is known that $c(P) \leq k$ implies $(1+t)^{n-k} | p(t)$. Here we prove that the converse is true in case P is the family of spanning sets in a greedoid $G = (E, \mathcal{L})$ (a certain generalization of a matroid). Thus the complexity of the spanning property is combinatorially tractable.

With each greedoid $G = (E, \mathcal{L})$ is canonically associated an invariant polynomial $\lambda_G(t)$, which can be recursively computed by a deletion-contraction type algorithm. A reformulation of the result is: if the lowest-degree nonzero term of $\lambda_G(t)$ has degree k, then the complexity of the spanning property is $n-k$.

Some results on primal graphs. (17)

PHYLLIS ZWEIG CHINN* and PAUL A. THOELECKE, Humboldt State University, Arcata, CA 95521 and BRUCE RICHTER, Ohio State University, Columbus, OH 43210.

The notion of primal graphs was introduced by A.K. Dewdney in 1970.

Definition. Let $\underline{S}$ be a set of graphs and $\underline{P}$ be a subset of $\underline{S}$. Then $\underline{P}$ is primal relative to $\underline{S}$ if every graph in $\underline{S}$ can be factored into a sum of distinct (non-isomorphic) graphs in $\underline{P}$ and graphs in $\underline{P}$ have only a trivial such factorization.

Dewdney proved that there exists a unique set of graphs primal relative to the set of all graphs on n vertices and that the union of these, for all n, forms a unique set of graphs primal relative to the set of all graphs. The authors extend his result to show the existence of a unique set of primal graphs relative to any set of graphs.

Lemma. For any set $\underline{H}$ of graphs which is hereditary, i.e. all subgraphs of graphs in $\underline{H}$ are also in $\underline{H}$, the graphs which are primal relative to $\underline{H}$ are also primal relative to the set of all graphs.

This lemma is used to prove several types of graphs are primal. Some families of primal and non-primal graphs are discussed, and it is shown that there exist primal graphs not in the families described. Some open questions and conjectures will be posed.

On the Cut Frequency Vector of Permutation Graphs

Barry Piazza*, Clemson University
Sam Stueckle, Clemson University (20)

An edge cut is a set of edges whose removal creates a disconnected graph. The i^{th} component of the cut frequency vector is the total number of i^{th} order cuts. We consider the problem, introduced to us by Boesch in a talk at the Fifth International Conference in Kalamazoo, of finding graphs with the smallest possible cut frequency vector over the class of regular graphs with a given number of points and given degree.

We show that the permutation graph of an r-regular graph with p vertices and $\lambda = r$ minimizes the first $k = \min(2r-1, p-1)$ out of the $p(r+1)$ components of the cut frequency vector.

(21) MAXIMAL BIPARTITE SUBGRAPHS AND INDEPENDENCE

Kathryn Fraughnaugh Jones, University of Colorado at Denver

In a graph with n vertices and q edges, it is proved that the number of edges b in a maximal bipartite subgraph and the independence i are related by the inequality

(1) $b \leq q + 2i - n$.

For a k-regular graph the bipartite density b/q and independence ratio i/n are shown to be related by

$$\frac{i}{n} \geq \frac{k(b/q) - k + 2}{4}$$

Several classes of graphs for which (1) is best possible are investigated.

INTERSECTION GRAPHS OF PATHS IN A TREE (25)

Clyde L. Monma* and Victor K. Wei, Bell Communications Research

Let P be a finite family of nonempty sets. The intersection graph of P is obtained by associating each set in P with a vertex and connecting two vertices with an edge exactly when their corresponding sets have a nonempty intersection.

We present a unified framework for studying certain classes of intersection graphs arising from families of paths in a tree. The main results are a characterization of these graphs in terms of their "clique tree" representations and a unified recognition algorithm which constructs such a representation for a graph or shows that the graph is not in the class. The algorithm decomposes an arbitrary graph by (maximal) clique separators, checks the nondecomposable "atoms" (by the Atom Theorem), and checks to see if the pieces can be "glued" together properly (by the Separator Theorem). The classes of graphs considered here include the (vertex) path, directed (vertex) path, and chordal graphs studied by Gavril, the (edge) path graphs studied by Golumbic and Jamison, Lobb, Syslo, and Tarjan, as well as other new classes of graphs.

A Note on Complete (k,μ)-Arcs (26)

Joel C. Fowler, Georgia Institute of Technology*

We give two new lower bounds for the size of a complete (k,μ)-arc. These improve already known bounds in many cases and are applicable under more general circumstances.

*Work done while at the California Institute of Technology.

Properties of Integer-Interval Graphs

(27) Ralph P. Grimaldi - Rose-Hulman Institute of Technology and Applied Computing Devices

For $n \in \mathbb{Z}^+$, the integer-interval graph I_n is constructed as follows: for each closed interval $[a,b]$, $0 \leq a < b \leq n$, $a,b \in \mathbb{Z}$, there is a vertex; and, if $0 \leq a,b,c,d \leq n$ with $a,b,c,d \in \mathbb{Z}$, and $[a,b] \cap [c,d] \neq \emptyset$, then there is an edge connecting the vertices that correspond to $[a,b]$, $[c,d]$.

Here we consider degree patterns of the vertices and use these results to investigate the existence of Euler and Hamilton cycles, and the number of edges and the clique number of such graphs.

(16) A Constructive Proof of a Stronger Structured Theorem

Karel Culik, Computer Sci. Dept, Wayne State University Detroit, MI 48202

A control graph (flow diagram) CG=⟨V,E,r,S⟩ the vertices, edges of which are labeled by instructions $\Phi(v)$, by truth values $\Gamma(v,w)$, resp., is a language independent representation of a control-flow algorithm. A control graph CG'=⟨V', E', r', S'⟩ is called a control subgraph of CG if $V' \subseteq V$, and an execution block if, in addition, each $(v,w) \in E-E'$ such that $w \in V'$ satisfies either $w=r'$ or $w \in S'$. A CG is called execution structured if each of its control subgraphs is an execution block, and structured, if, in addition, each fine strong component H=⟨W,F⟩ of CG satisfies: there is one single entry vertex $e \in W$ and one single exit vertex $x \in W$ such that either $e=x$ (W-FSC), or $e \neq x$ and $(x,e) \in F$ (U-FSC).

CG and CG*, having the same input and output variables, are called execution equivalent if for each initialization Init of input variables of CG and of CG* the same execution sequence is determined, thus ExSeq(CG,Init)=ExSeq(CG*,Init). A CG* is called almost execution equivalent with CG if after omitting all instructions containing PIV-variables from ExSeq(CG*,Init) the ExSeq(CG,Init) is obtained.

Theorem: To each CG=⟨V,E,r,S⟩ with $|S|=1$ there exists, and can be effectively constructed, a structured CG* which is almost execution equivalent with CG. The double induction with respect to $M=|E|\geq 1$ and the number $N \geq 0$ of FSCs is used. The constructions of splitting vertices (and multiplying FSCs), and of PIV-transformation are used, where the process identifying variables, PIVs, are used to encode various processes.

(17) A Parallel Algorithm for Bisection Width in Trees

Mark Goldberg, Clarkson University

Zevi Miller*, Miami University

The bisection width b(G) of a graph G is the minimum number of edges necessary in an edge cut of G so that the two sides of the cut have equal (or nearly equal) size. Our main result is a $O(n^3)$ processor $O(\log^3 n)$ time parallel algorithm for determining b(T) when T is a tree. We make use of a generalization of MacGregor's one processor algorithm for trees and of Tarjan and Vishkin's results on computing certain tree functions in logarithmic parallel time.

(18) T-DECOMPOSABLE GRAPHS

Egon Balas, Carnegie-Mellon University

For a graph G = (V,E) and some $F \subset E$, let G(F):= (V,F). If there exists $F \subset E$ such that G(F) is triangulated and $G(E \setminus F)$ is triangle-free, we say that E has a T-partition and G is T-decomposable. Given a graph G and a T-partition of its edge set, there is a polynomial time algorithm for finding a maximum-weight clique in G. We conjecture that recognizing T-decomposability is an NP-complete problem. We call G totally T-decomposable, if for any edge-maximal triangulated subgraph G(F) of G, the subgraph $G(E \setminus F)$ is triangle-free. There is a polynomial-time algorithm whose application to an arbitrary graph G either finds a maximum-weight clique, or establishes that G is not totally T-decomposable.

COMPUTING THE CONNECTIVITY OF CIRCULANT GRAPHS (22)

Mark E. Watkins, Syracuse University

The connectivities of a locally finite, infinite circulant graph Γ are determined: $\kappa_f(\Gamma)$ equals the valence of Γ (i.e. the atoms are trivial), while $\kappa_\infty(\Gamma)$, the cardinality of a smallest separating set leaving at least two infinite components, equals the last term in the symbol for Γ.

For circulant graphs on n vertices, we use some elementary properties of their atoms to facilitate a procedure that outputs their connectivity in $n^{3/2}$ time. An APL function that realizes this procedure is included. A formula obtained independently by E.A. van Doorn is thus readily computable.

On Degree-Pair Sets and Sequences for Graphs (23)

Michael S. Jacobson, University of Louisville
Lael F. Kinch, University of Louisville
John Roberts*, University of Louisville

With each edge xy in a graph G, we associate the unordered pair [deg x, deg y] called the degree-pair of xy. The degree-pair set of G is the set of all such pairs for edges of G and the degree-pair sequence of G is the (unordered) listing of all such pairs. In the following paper, we consider:

(1) What sets are degree-pair sets?
(2) What sequences are degree-pair sequences?
(3) What is minimum order (minimum size) for graphs realizing given degree-pair sets?

(24) TRIVALENT POLYGONAL GRAPHS OF GIRTH 6 AND 7

Manley Perkel, Wright State University, Dayton, Ohio

A (strict) trivalent polygonal graph is an undirected, connected graph (finite, no loops or multiple edges) of valency 3 and girth $m \geq 3$, such that every path of length 2 lies in a unique circuit of length m. Related to these are the triangular graphs which are undirected, connected graphs (of valency $m \geq 3$) such that the neighborhood of every vertex is a cycle of length m. In previous work, questions concerning the duality between members of these two families of graphs have been partially answered in the case where the polygonal graphs arise from reflexible maps on orientable surfaces. In this paper we give more complete answers to these questions in the case m=6. We also consider the case m=7 where we outline constructions of infinite families of these graphs.

LOWER BOUNDS FOR TWO-TERMINAL NETWORK RELIABILITY USING EDGE-DISJOINT PATHS

Timothy B. Brecht*, and Charles J. Colbourn, University of Waterloo

(28)

The problem of computing two-terminal network reliability is simply to compute the probability that two specified communication centers can communicate. Exact calculation of this reliability has been shown to be NP-hard. It is therefore desirable to find efficient means to obtain bounds for reliability measures.

Efficient techniques are discussed for computing lower bounds using edge-disjoint paths between the specified communication centers. The performance of the bounds is related to the method used to compute the edge-disjoint paths. Various methods for finding edge-disjoint paths and the performance of the resulting bound is discussed.

A COMBINATORIAL PROBLEM RELATED TO MULTIPROCESSOR COMPUTING AND COMMUNICATION SYSTEMS (29)

D. Frank Hsu, Fordham University

Let C (s1, s2,...,sk), where 0<s1 <s2 <···<sk, be a directed graph which has arcs from each point i to i + s1, i + s2,..., i + sk (mod n). We study a minimization problem on this class of graphs arising from studies on multimodule memory organizations, distributed multi-loop computer networks and concurrent processings.

Hamiltonian Elimination Orderings of Interval Graphs

Shwu-Huey Yen* and Margaret B. Cozzens, Northeastern University (30)

Jamison and Laskar in 1983 discussed five possible different elimination orderings for chordal graphs. These elimination orderings are the following: perfect elimination ordering (PEO), strong elimination ordering (SEO), interval elimination ordering (IEO), hamiltonian elimination ordering (HEO), and bicompatible elimination ordering (BCO). Those chordal graphs corresponding to each of the five orderings except HEO have been characterized: PEO - chordal graphs, SEO - strongly chordal graphs, IEO - interval graphs, and BCO - indifference graphs.

In this paper we characterize those chordal graphs that have an HEO. A hamiltonian elimination ordering (HEO) of a graph G = (V,E) is an ordering $v_1, v_2, \ldots, v_n$ of V such that for each i, $\{v_i, v_{i+1}\} \in E$ and for each v_i, v_j, v_k, if $i < j < k$ and $\{v_i, v_j\} \in E$ and $\{v_i, v_k\} \in E$ then $\{v_j, v_k\} \in E$. Since BCO→HEO→IEO→SEO→PEO, and $v_1 - v_2 - \ldots - v_n$ is a path, the class of graphs that have an HEO must be contained in the class of connected interval graphs.

Analogous to Dirac's theorem regarding the existence of simplicial vertices in chordal graphs, we show that graphs with an HEO contain at least one strongly simplicial vertex. A simplicial vertex a of a graph G is strongly simplicial if $G - \bar{N}(a)$ is connected.

ENUMERATION OF GRAPH THEORETIC SOLUTIONS FOR FACILITIES LAYOUT (31)

L. R. Foulds, University of Florida

One of the important problems in industrial engineering is the design of a system of physical facilities such as buildings on a plane site or machines on a manufacturing shop floor. There lurks within the significant subproblem of specifying which facilities should be located adjacently to each other. This problem insolves a relationship chart of ratings which summarizes the desirability of siting each pair of facilities adjacently. The problem is to find the layout which maximizes the sum of the ratings corresponding to adjacent facilities. In this paper the problem is formulated in graph theoretic terms and all possible feasible solutions are enumerated.

ON FAULT-TOLERANT GRAPHS (32)

F. R. K. Chung, Bell Communications Research, Morristown, New Jersey

We study several graph-embedding problems related to the design of fault-tolerant VLSI circuits. For example, one wants to construct a graph with the minimum number of edges having the property that after removing half of its edges or vertices the remaining graph still contains a "large" specified subgraph. We will discuss results and problems on these fault-tolerant graphs. Both probabilistic and constructive aspects will be considered.

Heaps in Heaps (33)

Thomas Strothotte and Jörg-Rüdiger Sack*
University of Waterloo, Waterloo; Carleton University, Ottawa

In this paper we present a new view of the *heap* data structure. We view a heap on n elements as an ordered collection of log n sub-structures of sizes 2^i, i in {0,...,log n}. We use the view of the heap structure in the design of an algorithm for splitting a heap on n elements into two heaps of sizes k and n-k, respectively. The algorithm requires $O(\log^2(\max(k, n-k))$ comparisons, improving, for $k > \log^2(n)$, the previous bound of $O(k)$ comparisons. We also present a new algorithm for merging heaps of sizes n and k into one heap of size n+k in $O(\log n * \log k)$ comparisons.

BALANCED GRAPHS AND THE PROBLEM OF SUBGRAPHS OF RANDOM GRAPHS

(43) Andrzej Ruciński*, University of Florida and
Adam Mickiewicz University, Poznań
Andrew Vince, University of Florida

Let $K_{n,p}$ be a random graph on n vertices where each eadge is chosen independently with probability $p=p(n)$. We give a new elementary proof of the fact that $n^{-1/m(G)}$ is the threshold $(n \to \infty)$ for the existence of a given graph G as a subgraph of $K_{n,p}$, where

$$m(G) = \max_{H \subseteq G} d(H) \quad \text{and} \quad d(H) = |E(H)|/|V(H)|.$$

This result was originally proved for balanced graphs, i.e. $m(G)=d(G)$, in a classic paper of Erdös and Rényi [1960] and subsequently generalized by Bollobás [1981]. Let p be of the same order of magnitude as the threshold. It is known that if $d(H) < d(G)$ for all proper subgraphs H of G, then the number of copies of G in $K_{n,p}$ has an asymptotic Poisson distribution. We prove the necessity of this later condition.

ON THE CHROMATIC INDEX OF MULTIGRAPHS AND A CONJECTURE OF SEYMOUR

Odile Marcotte, Universite de Sherbrooke, Quebec, Canada (37)

Let $G = (V, E, w)$ be a multigraph, where V is a set of vertices, E is a set of edges and w is a vector of edge multiplicities. It is well known that ρ, the maximum degree of G, is a lower bound on the cardinality of a proper edge colouring of G. Another lower bound is given by

$$\kappa = \max\left\{\frac{w(E(S))}{\left(\frac{|S|-1}{2}\right)} \,\middle|\, S \subseteq V,\ |S| \text{ odd and } |S| \neq 1\right\}$$

where $w(E(S))$ is the number of edges both ends of which belong to S. Seymour has made the conjecture that the minimum number of colours in a proper edge colouring of G is less than or equal to $\max\{\rho+1, \lceil\kappa\rceil\}$, where $\lceil\kappa\rceil$ denotes the least integer greater than or equal to κ. In this talk we show that Seymour's conjecture can be reduced to a conjecture about critical nonseparable graphs (in the sense of matching theory). We also show that the latter conjecture is verified in the case of outerplanar graphs, thus proving that Seymour's conjecture holds for outerplanar graphs.

ORTHOGONAL EDGE COLORINGS OF GRAPHS (38)

Dan Archdeacon* and Jeff Dinitz, University of Vermont, and Frank Harary, University of Michigan

We study proper edge colorings of simple graphs. Two colorings are orthogonal if any two edges which receive the same color in one coloring receive distinct colors in the other coloring. Let G be a graph which admits a pair of orthogonal colorings using n and m colors. We examine some necessary and some sufficient conditions on G, n and m. A relationship with certain combinatorial designs is discussed, in partucular Room Squares, Howell Designs and orthogonal Latin Squares are special cases of orthogonal colorings. Some open problems are presented.

ON THE DISCREPANCY OF COLORING FINITE SETS (39)

D. Hajela, Bell Communications Research, Murray Hill, New Jersey

Given a set $S \subseteq \{1, \ldots, n\}$ and a map $\chi: \{1, \ldots, n\} \to \{-1, 1\}$, (i.e. a coloring of $\{1, \ldots, n\}$ with two colors, say red and blue) define the discrepancy of S with respect to χ to be $d_\chi(S) = \left|\sum_{i \in S} \chi(i)\right|$ (the difference between the reds and blues on S). We show that given n subsets of $\{1, \ldots, n\}$ the average discrepancy over all colorings of each of the n subsets is as small as $(n \log n)^{1/2}$ in magnitude.

PROBABILITY CONSENSUS AND INDEPENDENCE PRESERVATION (44)

Christian Genest (Waterloo) and Carl Wagner*(Tennessee)

A method of aggregating n subjective probability distributions into a single consensual distribution is independence-preserving if, whenever events are independent for each of the subjective distributions, they are independent for the consensual distribution. We show, under a mild regularity condition, that if $n \geq 5$, the only independence-preserving aggregation methods are dictatorial (adopt one of the distributions as the consensus). If $n = 4$, however, a wide variety of non-dictatorial independence-preserving aggregation methods exist. We give a complete characterization of these, using recent results of Abou-Zaid.

Networks, Parallel Computation and VLSI

Tom Leighton
M.I.T.

Abstract

Recent advances in VLSI fabrication technology have made it possible to construct large numbers of identical processors using surprisingly small amounts of chip area. As a result, it has now become feasible to integrate large numbers (potentially millions) of simple processors into a single network for the purposes of parallel computation. The potential savings in computation time gained by using such large scale networks to solve problems involving integer or polynomial arithmetic, signal processing, packet routing, linear algebra and/or numerical analysis is dramatic. For example, many problems such as sorting and matrix multiplication which require more than linear time on traditional sequential machines can be solved in logarithmic time on an appropriate parallel machine.

As might be expected, the variety of combinatorial and computational problems raised by the developing technology is quite broad. Of course, one area of primary interest (and the subject of our talks) is the design of network architectures which are well suited for fast parallel computation. In the talk, we will describe the most commonly used and the best known architectures, and we will show how they can be used to solve a variety of problems efficiently. The class of networks to be discussed will include the mesh, mesh of trees, hypercube, shuffle-exchange graph, de Bruijn graph, butterfly and the cube-connected cycles. A special emphasis will be placed on understanding the structure of the networks, and hence on the reasons for their usefulness in parallel computation. We will also describe the construction of a universal network which is capable of simulating any other network of the same size with at most a logarithmic factor delay in time. Such universal networks might well be good candidates for the architecture of future supercomputers.

The lectures will be introductory in nature and no previous familiarity with networks, parallel computation or VLSI will be assumed..

EXTREMAL RESULTS FOR $K_{1,3}$-FREE GRAPHS (45)

Manton Matthews, University of South Carolina

In this paper we present several extremal results for $K_{1,3}$-free graphs. The first of these deal with results similar to a result of Posa on the minimum number of edges in a graph necessary to guarantee two disjoint cycles. Also, we define the $K_{1,3}$-free ramsey number to be the minimum integer rc(m,n) such that every $K_{1,3}$-free grapg on this number of vertices must have either a K_n or an independent set of size m. An explicit formula is obtained for rc(3,n) and the value of rc(n,3) is shown to be the same as r(n,3). General upper and lower bounds for rc(n,m) are established and sharper bounds for the case n = 4.

BINARY SEARCH ON A TAPE

(34)

T. C. Hu, University of California, San Diego
Michelle Wachs*, University of Miami

Comparison search procedures for sorted files are represented by binary search trees. The usual cost of such trees is the average number of comparisons needed in the search procedure. Here we consider search procedures for sorted files stored on a tape. In addition to the number of comparisons, we are concerned with the number of movements needed to locate a record. The complete binary tree (usual binary search) is optimal with respect to comparisons and the linear binary tree (sequential search) is optimal with respect to movements. A tape-optimal tree, i.e. a tree which is optimal with respect to both comparisons and movements, is a hybrid of the complete binary tree and the linear binary tree. We present a characterization of tape-optimal trees.

AN EFFICIENT TREE MACHINE FOR THE DIVIDE-AND-CONQUER ALGORITHMS

(35)

Der-Yun Yen*, Cleveland State University
Barrett R. Bryant, The University of Alabama in Birmingham

A modified tree machine architecture is proposed on which parallel divide-and-conquer algorithms can be implemented without incurring any data routing time. This modification gives more time-efficient solutions to problems which can be solved by a divide-and-conquer algorithm. The modified tree machine eliminates data routing time by communicating between processors through direct connections between processing elements and data memory modules. Therefore, in addition to being accessed by its own processing element, each memory module can also be accessed by other processing elements at different times in order to exchange data. This modification requires an additional switching network comprised of $\log_2(n)$ processing elements, where n is the number of processing elements required by an ordinary tree machine to solve the problem. However, the elimination of data routing time from the complexity of the divide-and-conquer algorithms significantly outweighs this minor addition of processing elements required for the switching network.

Results on the mathematics of prefix compression

(36)

Rodica Simion*, Southern Illinois University and Bryn Mawr College
Herbert S. Wilf, University of Pennsylvania

Let A be a (linearly ordered) finite alphabet and $m_1, m_2, \ldots, m_n \geq 0$ fixed integers. Consider all dictionaries over A consisting of m_i words of length i, i=1, 2, ..., n. We evaluate the probability that two lexicographically consecutive words randomly selected from such dictionaries have a common prefix of ℓ letters, for $\ell = 1, 2, \ldots$. The average length of the maximum common prefix of consecutive words is calculated. We also discuss extremal dictionaries for which storing using common prefix compression is most economical, and illustrate the effect of this method with examples of lists of combinatorial objects, such as integer partitions and compositions, permutations, k-subsets.

STRONG, WEAK, AND OTHER COLORINGS OF UNIFORM HYPERGRAPHS

(40)

Ronald D. Dutton* and Robert C. Brigham, University of Central Florida

An r-uniform hypergraph is one in which every edge contains exactly r nodes. A common extension of the concept of graph coloring to such hypergraphs is to define an i-coloring as an assignment of colors to the nodes in such a way that no edge contains i nodes of the same color. $\chi^i(H)$ is the minimum number of colors needed to achieve an i-coloring. χ^2 and χ^r are sometimes called the strong and weak chromatic numbers, respectively. A second extension defines an $\hat{i}$-coloring as one in which each edge receives at least r - i + 2 different colors, and $\hat{\chi}^i(H)$ denotes the minimum numbers of colors necessary. $\hat{\chi}^2 = \chi^2$ and $\hat{\chi}^r = \chi^r$, but equality does not hold for all i. Several relationships involving these chromatic numbers are developed, including their values for complete uniform hypergrapsh. A collory is the following sharp inequality for strong and weak chromatic numbers: $\chi^r \leq \lceil \frac{\chi^2}{r-1} \rceil$.

On Balanced Colorings of Subset Collections

(41)

Noga Alon and D. J. Kleitman, MIT, Michael Saks*
and Paul Seymour, Bell Communications Research.

Let X be a finite set and S a collection of subsets of X. A two coloring of X is balanced for S if for each edge $S \in S$ the number of elements of each color is the same (in particular the cardinality of every set must be even). For positive integers n, let f(n) be the size of the smallest collection of n element sets for which no balanced two-coloring exists (trivially f(n)=1 if n is odd). Erdös and Sös asked whether f(n) is bounded for all n. We give two proofs that f(n) is unbounded; one is a limit argument, the other is a construction based on a theorem of Huckeman, Jurgat and Shapeley.

On a Generalization of Chromatic Number

(43)

James A. Andrews* and Michael S. Jacobson, Univ of Louisville

In a graph G, a set of vertices $S \subseteq V(G)$ is n-dependent if $\Delta(\langle S \rangle) \leq n$. The n-chromatic number of G, $\chi_n(G)$, is the smallest t such that V(G) can be partitioned into t sets each of which is n-dependent. Various results relating $\chi_n(G)$ to other parameters are presented. Additionally, we study the following property, related to Ramsey theory: We say that G "χ_t arrows" $(n_1, \ldots, n_k)$, denoted

$$G \xrightarrow{\chi_t} (n_1, \ldots, n_k),$$

if for every factorization of $G = F_1 \oplus \ldots \oplus F_k$ We have $\chi_t(F_i) \geq n_i$ for some $i \in \{1, \ldots, k\}$.

Some Ramsey Numbers for Tournaments

(46)

A. Bialostocki* and P. Dierker
University of Idaho, Moscow, Idaho

A. Notation: Let TT_k, P_k, and S_k denote the transitive tournament on k vertices, the directed path with k edges and the directed outgoing star with k edges, respectively.

B. Definition: Let D_1 and D_2 be two acyclic directed graphs. The number $RT(D_1,D_2)$ will denote the smallest integer n satisfying the following property: If T is any tournament on n vertices whose edges are colored by red and blue, then T contains either a copy of D_1 all of whose edges are red, or a copy of D_2 all of whose edges are blue.

C. Results:

1. $RT(TT_3,TT_3) = 14$
2. $*? \leq RT(TT_3,TT_4) \leq 42$
3. $RT(TT_3,S_k) = 3(2k-1)+1$
4. $RT(TT_4,S_k) = 7(2k-1)+1$
5. $13(2k-1)+1 \leq RT(TT_5,S_k) \leq 30k-14$
6. $RT(TT_3,P_k) = 3k+1$
7. $RT(TT_4,P_k) = 7k+1$
8. $RT(TT_5,P_k) = 13k+1$

D. If time allows, we shall review some known results and suggest some directions in Ramsey theory for tournaments.

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*Presently a computer search is being conducted for a lower bound.

On a conjecture of Paul Erdös

(47)

*Charles M. Grinstead and Richard Hughey

Given a positive integer n, let $\mathcal{C}_n$ be the collection of all graphs with chromatic number n. For any graph G, let R(G) be the standard two-color Ramsey number for G. Paul Erdös conjectured that for all G in $\mathcal{C}_n$,

$$R(G) \geq R(K_n).$$

We will give results in this talk which will lead to the settling of the case n=4 in the near future (o(1) year).

Ramsey Theory by Computer

(48)

*E. Regener, C.W.H. Lam, J. Opatrny
Concordia University, Montreal, Canada

Given two graphs J and K without isolated vertices, the generalized Ramsey number r(J, K) is the smallest n for which any graph G with n vertices must have $J < G$ or $K < \bar{G}$, where "<" denotes edge-containment. We have developed general programs to search for maximal G with $J \nless G$ and $K \nless \bar{G}$, for given J and K. Since the latter property is transitive on subgraphs, such G can be built by adjoining one vertex at a time in a breadth-first search. We shrink and prune the search tree using various invariants and heuristics. In applying the program to try to determine $r(W_6, W_6)$, where W_6 is the wheel on 6 points, we have found so far that a graph on 17 points excluding W_6 must contain the complete graph K_5.

ALGORITHMS FOR GENERATING ALL MINIMAL CUTSETS IN A GRAPH

D.R. Shier and D. E. Whited*, Clemson University (49)

The problem of generating all minimal cutsets separating two specified vertices in a graph arises in several types of network reliability calculations. One approach to finding such s-t cutsets in either directed or undirected graphs is based on "inverting" the set of all simple s-t paths in the graph. (It turns out that simple paths are generally much easier to produce than cutsets.) We present several results that reduce the effort necessary to perform the path inversion and discuss several strategies for implementing the approach. Computational results are given to illustrate the efficacy of this approach compared to existing approaches for generating minimal s-t cutsets.

SEPARATOR- AND BIFURCATOR- BASED LOWER BOUNDS FOR MAXIMUM EDGELENGTH IN VARIABLE ASPECT-RATIO VLSI LAYOUTS

Paul Czerwinski and Vijaya Ramachandran*, University of Illinois (50)

Under the Thompson model, it is well-known that bounded-degree n-node graphs with $o(\sqrt{n})$ separator can be laid out in linear area with maximum edgelength $O(\sqrt{n}/\log n)$, and n-node bounded-degree graphs with $F=\Omega(\sqrt{n})$ bifurcator can be laid out in $O(F^2\log^2(n/F))$ area with maximum edgelength $O(F\log(n/F)/\log\log(n/F))$. These results are also existentially optimal (to within a constant factor) in the sense that, for each class, there exists a graph in the class that requires the specified amount of area, and has the specified maximum edgelength.

We generalize and unify these results by considering embeddings in variable aspect-ratio bounding rectangles. We show that if we consider minimum-area embeddings of classes of graphs with separator $F=o(\sqrt{n})$ or bifurcator $F=\Omega(\sqrt{n})$, then, if the longer side of the bounding rectangle is a and the shorter side is b, there exists a graph in the class for which the maximum edgelength in the embedding is $\Omega(a/\log(b/F))$. For the special case when $a=\Theta(b)=\Theta$ (squareroot of the area), this gives the previously known results. Further, our proof is simpler than the earlier proof for the lower bound on the maximum edgelength for bifurcator-based embeddings.

Our results are also existentially optimal, since we have shown (in a separate paper) layout strategies that achieve this bound on the maximum edgelength.

BOUNDS FOR ALL-TERMINAL RELIABILITY IN PLANAR NETWORKS

Aparna Ramesh* and Charles J. Colbourn, University of Waterloo (51)

A computer network can be modeled as a graph where the nodes of the graph represent sites and the edges represent links between sites. The edges of the graph fail randomly with equal probability p. The all terminal reliability of the network is the probability that the network is connected. The reliability of the network with m edges and n vertices can be written as $R=\sum_{i=0}^{i=m} f_i\, p^i (1-p)^{m-i}$ where f_i is the number of sets of i edges whose complement contains a spanning tree. The probiem of calculating the exact value of R in the all-terminal case is open for planar networks. The ideal situation in such a case is to get upper and lower bounds on R, by giving upper and lower bounds on f_i for which the exact values are not known. All known bounds for the all-terminal reliability use exact values of $f_0, \ldots, f_c$ and f_d and use approximations for the remaining f_i where c is the cardinality of a minimum cut and d = m-n+1. We get upper and lower bounds on R by using exact values of $f_{d-1}, f_{d-2}, f_{d-3}$ in addition to $f_0, \ldots, f_c$ and f_d. The effect of using the exact values of f_{d-1}, f_{d-2} and f_{d-3} in Ball-Provan and Kruskal-Katona bounds is studied.

EXTREMAL SPANNING TREES OF CUBIC GRAPHS

(56) Lane Clark and Roger Entringer*, University of New Mexico

Given a connected cubic graph G of order n, Payan, Tchuente and Xuong have shown that G contains a spanning tree T in which at least $\frac{1}{4}n$ vertices have degree 3. They conjecture that $\frac{1}{4}$ can be replaced by $\frac{1}{3}$ if G contains no K_4-e. We prove the conjecture for graphs with sufficiently large girth.

A CHARACTERIZATION OF CRITICAL k-OCHROMATIC GRAPHS (57)

Gustavus J. Simmons, Sandia National Laboratories, Albuquerque

Given a graph G and an ordering ϕ of the vertices in V(G), let $\chi_\phi(G)$ denote the fewest number of colors with which V(G) can be properly colored in the order ϕ; where a new color is introduced only when a vertex cannot be properly colored in its order with any of the colors already used and where if some vertex(ices) could be properly colored with more than one color it is assumed that a choice will be made that results in the minimum number of colors being required overall. The usual chromatic number, $\chi(G)$, is simply the minimum of $\chi_\phi(G)$ over all possible orderings of V(G) and the ochromatic number, $\chi^0(G)$, is defined to be the corresponding maximum.

The ochromatic number exhibits several counter intuitive properties; for example, the ochromatic number of a subgraph can be greater than, less than or equal to the ochromatic number of a containing graph. In this paper, we construct arbitrarily long chains of subgraphs with monotone increasing ochromatic numbers! The significance of this remark is that it is much more difficult to determine whether a graph is critically k-ochromatic than to determine whether it is critically k-chromatic for example. The main result reported here is: given that G is critically k-ochromatic, which means that there exists at least one ordering ϕ of V(G) that requires k colors to properly color V(G) in the order ϕ, that the resulting partitioning of V(G) is such that V_1 is a maximal independent dominating set in G, V_2 is a maximal independent dominating set in G/V_1, etc. V_i the subset of vertices assigned color i in coloring V(G) under ϕ. Criticality says that this descending chain condition holds, irrespective of which ordering is used (so long as k colors are required) or of choices of colors that might be made (again so long as k colors are required overall).

* This work performed at Sandia National Laboratories supported by the U. S. Department of Energy under contract no. DE-AC04-76DP00789.

Bichromaticity of Bipartite Graphs

(58) Dan Pritikin, Miami University, Oxford, Ohio 45056

Let B be a bipartite graph with edge set E and vertex bipartition M, N. The bichromaticity $\beta(B)$ is defined as the maximum number β such that a complete bipartite graph on β vertices is obtainable from B by a sequence of identifications of vertices of M or vertices of N. Let $\mu=\max\{|M|,|N|\}$. Harary, Hsu and Miller proved that $\beta(B)\geq\mu+1$ and that $\beta(T)=\mu+1$ for T an arbitrary tree. We prove that $\beta(B)\leq\mu+\frac{|E|}{\mu}$ yielding a simpler proof that $\beta(T)=\mu+1$. We also characterize graphs for which $K_{\mu,2}$ is obtainable by such identifications. For Q_K, the graph of the K-dimensional cube, we obtain the inequality

$$2^{K-1}+2^{\lfloor\log_2 K\rfloor}\leq\beta(Q_K)\leq 2^{K-1}+K,$$

Double bound sequences and sets for partially ordered sets

John K. Luedeman, Clemson University (63)
*F. R. McMorris, University of Louisville
John Roberts, University of Louisville

For a poset $P=\{x_1,\ldots,x_n\}$, let $D(x_i)=\{y: y\leq x_i$ or $x_i\leq y\}$, the sequence $|D(x_1)|,\ldots,|D(x_n)|$ arranged in ascending order is the DB-sequence of P and the set $\{|D(x_1)|,\ldots,|D(x_n)|\}$ the DB-set of P. It is shown that every set of positive integers is the DB-set of some poset and the minimum number of elements in a realizing poset is determined. Sufficient conditions for a sequence to be a DB-sequence are given.

The B-set of P is the set $\{|L(x_1)|,\ldots,|L(x_n)|\}\cup\{|U(x_1)|,\ldots,|U(x_n)|\}$ where $L(x_i)=\{y: y\leq x_i\}$ and $U(x_i)=\{y: x_i\leq y\}$. A set of positive integers S is a B-set if and only if $1\in S$. The minimum number of elements in posets realizing certain B-sets are determined.

BOUND GRAPHS, COMPETITION GRAPHS, AND 2-GRAPH INVERSES

J. Richard Lundgren, University of Colorado at Denver
John S. Maybee*, University of Colorado at Boulder (64)

Certain graphs such as upper bound graphs and consanguinity graphs and competition graphs and common enemy graphs naturally occur as duals when characterized using row and column graphs of a matrix. In this paper we investigate when a pair of graphs is such a dual. That is we determine when two graphs are the upper bound and lower bound graphs respectively of a poset, and when two graphs are the competition and common enemy graphs respectively of an acyclic digraph.

On length, width and the size of cutsets in ordered sets

John Ginsburg, University of Winnipeg, Winnipeg, Canada. (65)

This work has been done jointly with Bill Sands, University of Calgary. Let P be a finite ordered set. If x is an element of P then a subset C of P is called a cutset for x in P if every element of C is noncomparable to x and every maximal chain in P meets $\{x\}\cup C$. If every element of P has a cutset having n or fewer elements we say that P has the n-cutset property. We denote the length and width of P by ℓ and w. **Theorem.** If P has the 2-cutset property then $w<\ell+2$. More generally, if P has the n-cutset property then $w<\ell^{n-2}(\ell+2)$.

MINIMUM AVERAGE DISTANCE PARTITIONS OF THE VERTICES ON A LINE NETWORK

Peter J. Slater, University of Alabama (52)

For a graph or network with vertex set V and with d(u,v) denoting the distance between vertices u and v, a (vertex) m-median is a set $S \subseteq V$ with $|S| = m$ which minimizes $\sum \{d(u,S) : u \in V\}$ where $d(u,S) = \min \{d(u,v) : v \in S\}$. Much work has been done for this facility location problem. Here the modified problem where there are m different facilities to locate, and generalizations of this, will be discussed.

Minimum-Weight Two-Connected Spanning Networks

(53)

Clyde L. Monma, Bell Communications Research
Beth Spellman Munson*, AT&T Bell Laboratories
W. R. Pulleyblank, University of Waterloo

Consider a set of vertices V with a non-negative, symmetric distance function defined on V x V which satisfies the triangle inequality. We consider the problem of constructing a minimum-weight, two-connected network spanning V. We obtain a number of structural properties and use these to study the worst-case behavior of several heuristics.

Resource Allocation and Threshold Coverings

Edward T. Ordman, Memphis State University. (54)

In the generalized dining philosophers problem, a collection of processes (e.g., in a distributed computer system) cannot all proceed at once because they compete for one or more limited resources. Consider each process a node; a minimal set of processes that cannot run at once form a (hyper)edge. Every (hyper)graph arises in this way. We explore the number of resource units and types needed to give rise to an arbitrary graph; this involves covering the graph with threshold graphs. The amount of shared memory or number of messages needed to schedule processes in a distributed system with a certain graph of conflicts thus relates to threshold covering numbers and clique covering numbers. There are implications for relative strength of synchronization primitives (e.g. PV-chunk versus test-and-set).

OPTIMAL EXPANSION OF EXISTING NETWORKS

Eric M. Neufeld, University of Waterloo (55)

The design problem in communication networks is placed in a combinatorial setting by modelling a network as a graph where vertices represent processors and edges represent links. Given this model and some constraints, we seek the best network with respect to some measure of reliability. One widely-known measure is the probability that a network remains connected in an environment of fault-free processors and (statistically independent) link failures. When all edges fail with uniform probability p, reliability is directly related to connected spanning subgraph counts for $p = ½$.

Since networks are subject to change, we address the network expansion problem: how may we optimally add a new link to an existing network? The design problem becomes: how may we design reliable networks with future expansion in mind? The latter problem is #P-Complete in general; therefore we consider series-parallel graphs and a related class of graphs, a generalization of wheels. In particular, we show the best generalized wheel is the wheel with maximum diameter.

CHARACTERIZATION OF k-FOLDING MINIMAL GRAPHS FOR k = 2,3 (59)

Sheng-Ping Lo, AT&T Bell Laboratories

A labeling of a graph G is a one-to-one function mapping from V(G), the vertex set of G, to the set of nonnegative integers. To represent a labeling π of G on the real line, we associate the point $\pi(u)$ with vertex u in G and draw an interval $[\pi(u),\pi(v)]$ for each edge $\{u,v\}$ in G with $\pi(u)<\pi(v)$. The stack number of G with respect to a labeling π of G is the maximum number of intervals stacked at a point in the above representation of the labeling π. The folding number of a graph G is the minimum value of the stack numbers of G over all labelings of G. A graph is said to be k-folding minimal if its folding number is k and the elimination of any edge will result a graph of folding number less than k. In this paper we will prove the characterizations for k-folding minimal graphs in the cases of k be 2 and 3.

*COMPLETE COLORING PARAMETERS OF GRAPHS (60)

W. R. Hare, S. T. Hedetniemi, R. Laskar, Clemson University
J. Pfaff, Bell Laboratories

A coloring of a graph is called complete if for every pair of distinct colors there exist two adjacent vertices which are assigned these two colors. The chromatic number $\chi(G)$ and the achromatic number $\psi(G)$ are the minimum and maximum orders of a complete coloring of G, respectively. The Grundy number $\Gamma(G)$ and the ochromatic number $\chi^o(G)$ of a graph correspond to the maximum number of colors which can be assigned to the vertices of a graph in complete colorings which satisfy certain restrictions on the order in which the vertices are colored. It can be shown that these parameters satisfy: $\chi(G) \leq \Gamma(G) \leq \chi^o(G) \leq \psi(G)$. This paper establishes several new lower and upper bounds for these parameters and introduces a new complete coloring number, $\Gamma^1(G)$, which satisfies the inequalities $\chi(G) \leq \Gamma^1(G) \leq \Gamma(G)$.

(61) Quasisymmetric Block Designs with y = λ

Robert L. Holliday, Lake Forest College

One way to make quasisymmetric block designs more like symmetric block designs is to require that one of the intersection numbers be λ. In this paper, we consider possible parameter sets for block designs with y (the larger of the two intersection numbers) equal to λ. When x (the smaller intersection number) is 1, we obtain a single family of parameter sets for possible block designs. We show, with elementary number theoretic arguments, that if $x = 2$ or $x = 3$ then no quasisymmetric block designs exist with $y = \lambda$. A computer search shows that such designs with $x \geq 4$ are likely to be scarce.

ORTHOMORPHISM GRAPHS OF GROUPS (62)

Anthony B. Evans, Wright State University

Let (G, +) be a finite, not necessarily abelian, group and let $\theta: G \to G$ be a permutation with $0\theta = 0$. θ is a left orthomorphism of G if the mapping $x \to -x\theta + x$ is a permutation. Two left orthomorphisms θ, ϕ of G are left adjacent if the mapping $x \to -x\theta + x\phi$ is a permutation.

We discuss the construction of affine planes, nets and mutually orthogonal latin squares using left orthomorphisms. We also discuss some constructions of left orthomorphisms.

Antichains in the partition lattice.

E. Rodney Canfield, University of Georgia. (66)

The partition lattice is the lattice of all partitions of the set {1,2, ..., n} into pairwise-disjoint, non empty blocks, two partitions being related if one is a refinement of the other. A ranking function is obtained by letting the rank of a partition be the number of blocks in the partition. The number of partitions with rank k is S(n,k), the Stirling number of the second kind. An antichain is a collection of partitions no two of which are related, examples of which are all partitons of a given rank. It is known that for some K(n) one has S(n,1) < ... < S(n,K(n)) > S(n,K(n)+1) > ... > S(n,n). A question of Rota, answered in the negative by the author, was: is S(n,K(n)) the largest possible size of an antichain. In this paper we report on current progress for these problems: how large is the largest antichain relative to S(n,K(n)); and, with how wide a range of ranks must the largest antichain have non empty intersection.

PARTIAL ORDERS AND COMPARISON PROBLEMS

M. D. ATKINSON, Carleton University, Ottawa (67)

Let $P = \{x_1, x_2, \ldots, x_n\}$ be a finite partially ordered set. Let z(P) denote the number of total orderings of the set which are consistent with the partial order constraints. This number is of interest as a measure of how constraining the partial order is, and it is connected with comparison problems in the theory of algorithms. An algorithm, valid for any partially ordered set, is given for calculating z(P). The algorithm is inefficient in the worst case and, for certain special partially ordered sets, polynomial time algorithms are given. In particular it is shown that z(P) can be calculated in $O(n^5)$ operations when the Hasse diagram of P is a tree. From a special case of the algorithms a generating function is derived which connects the theory with older results on 'up-down' permutations and secant and tangent numbers.

IMBEDDING POSETS IN THE INTEGERS

D.K. Skilton, Simon Fraser University (68)

An *imbedding* of a poset P in the integers is a one-to-one order preserving map from P into the integers. Such a map always exists when P is finite, and moreover, certain imbeddings of subsets of finite P can be extended to imbeddings of the whole of P. Daykin has asked when an imbedding in the integers of a finite subset of a countable poset can be extended to the whole poset. This paper answers Daykin's question and some related questions.

(69) POSETS AND INTEGER PAIR SEQUENCES

John K. Luedeman, Clemson University

Let $(P,\leq)$ be a finite poset. For $p \in P$, let $ub(p) = \{x \in P: x \geq p\}$ be the set of upper bounds of p. Let S_P denote the sequence $(a_1, b_1)(a_2, b_2) \ldots (a_k, b_k)$ where (a_i, b_i) is in S if there exist x, $y \in P$, x covers y and $a_i = |ub(x)|$, $b_i = |ub(y)|$. To make S_P well defined, we order S_P reverse lexicographically. S_P is the integer pair upper bound sequence of $(P,\leq)$. A sequence S of positive integer pairs is called an integer pair upper bound sequence if S is ordered reverse lexicographically and there exists a poset $(P,\leq)$ with $S_P = S$. In this case we say that $(P,\leq)$ realizes S. We characterize integer pair upper bound sequences realized by tree posets, and linear orders.

Let T_P denote the set of ordered pairs of S_P. T_P is the integer pair upper bound set of $(P,\leq)$. Similarly, a set T of pairs of positive integers is called an integer pair upper bound set if there is a poset $(P,\leq)$ with $T_P = T$. We characterize integer pair upper bound sets T, and determine the minimum cardinality of a poset realizing T.

C. A. Rodger, Auburn University (71)

A partial Mendelsohn triple system of order m can be embedded in a Mendelsohn triple system of order t, for all admissable $t \geq 4m$, where if t is even then $m \geq 14$.

AN EXTREMAL PROPERTY OF THE 4 - (23, 7, 1) DESIGN

Sharad S. Sane, University of Bombay, Bombay, India

(72)

The following result is proved: The collection of blocks of the Witt-Lüneberg design on 23 points is maximal when considered as a collection of mutually interesting sets of equal cardinality.

EMBEDDING, IMMERSING, AND ENCLOSING (73)

Charles J. Colbourn, University of Waterloo, Rose C. Hamm*College of Charleston, Alexander Rosa, McMaster University

Many researchers have studied operations which produce block designs containing a specified partial block design. One operation of this type is completion, in which all parameters (t,v,k,λ) are the same in the partial block design and the containing design. Another is embedding, in which v is allowed in increase but t,k,λ are the same in the partial and containing systems. A third is nesting, in which k,λ increase while t,v do not. A fourth is extending, in which v,k,t each are one larger in the containing system. We consider two further operations. Immersing is an operation in which t,v,k are the same in the containing and partial systems, but λ is allowed to be larger. Enclosing is an operation combining embedding and immersing; that is, v,λ can increase while t,k cannot. We describe two main research directions. First, we note that every partial triple system can be finitely immersed. Second, we establish the existence of small enclosings for certain classes of partial triple systems.

BOUNDS FOR THE DOMINATION NUMBER OF GRID GRAPHS

(76)

E. J. Cockayne*, University of Victoria;
E. O. Hare, S. T. Hedetniemi and T. V. Wimer, Clemson University

A grid $G_{m,n}$ is a graph having m x n vertices connected by edges to form a rectangular lattice. It is shown that $\gamma(G_{n,n})$, the domination number of $G_{n,n}$, has lower and upper bounds $1/5(n^2 + n - 3)$ and $1/5(n^2 + 4n - 2)$, respectively.

A CONJECTURE ON DOMINATING CYCLES (77)

Brent N. Clark* and Charles J. Colbourn, University of Waterloo, Paul Erdos, Hungarian Academy of Sciences

A *dominating cycle* in a graph is a cycle for which each vertex lies at distance at most one from the cycle. We conjecture that for each c there is a constant k_c such that every c-connected graph with minimum degree $\delta \geq \frac{n}{c+1} + k_c$ has a dominating cycle. We show that this conjecture, if true, is best possible. We further prove the conjecture for graphs of connectivity 1, 2, and 3.

IRREDUNDANCE IN GRAPHS: A SURVEY

(78)

S. T. Hedetniemi* and R. Laskar, Clemson University
J. Pfaff, Bell Laboratories

In 1976, while studying methods for finding minimal dominating sets in graphs, Cockayne and Hedetniemi were led to consider sets of vertices S which have the property that every vertex in S 'contributes' at least one vertex (either itself or an adjacent vertex) to the set of vertices dominated by S that no other vertex in S can 'contribute'. It was thought that maximal sets having this property were minimal dominating sets, until it was realized that such sets need not dominate every vertex in a graph. Hence, the term irredundance was created. A set of vertices S is irredundant if for no vertex v in S is the closed neighborhood of v contained in the union of the closed neighborhoods of the vertices in S-v.

Since its definition, a steady stream of results has appeared relating irredundant sets to dominating sets and questions of NP-completeness. This paper surveys the literature on irredundant sets of graphs and provides the first proof that the irredundance problem (finding the size of a smallest maximal irredundant set in a graph) is NP-complete, even for bipartite graphs.

Recognition of partial k-trees (79)

Stefan Arnborg (The Royal Institute of Technology, Stockholm) and Andrzej Proskurowski* (Creighton University, Omaha, on leave from University of Oregon, Eugene)

Our interest in the class of k-trees and their partial subgraphs is motivated by practical applications in areas such as network reliability, concurrent broadcasting, and complexity of querries in relational data-base systems. Because of their bounded decomposability, partial k-trees are efficiently tractable by dynamic programming type of algorithms. Here, we present some partial results leading to recognition of these graphs through application of a set of confluent reductions (rewriting rules) in a manner more efficient than a general "brute force" recognition method.

(82) Computing in an Infinite Algebraic Extension of GF(q)

J.V. Brawley*(Clemson University) and
G.E. Schnibben (Frances Marion College)

Each infinite algebraic extension E of the finite field GF(q) can be described as the union of a tower of finite fields containing GF(q). Such a characterization, however, does not readily lend itself to the problem of producing an actual algorithm which will generate random elements from E and compute sums and products of these elements. In this paper we discuss certain infinite algebraic extensions of GF(q) for which such an algorithm can be described, and we describe the algorithm. Thus, in a computational sense, we know these fields just as we know the ring of integers or the field of rational numbers.

USING THE HEURISTIC ALGORITHM Min TO FIND A HAMILTONIAN PATH

Ruth Ann DeHoff, University of Houston (83)

It is a well known fact that for some degree sequences one can determine if a graph will have a Hamiltonian cycle. In 1823, J. C. Warnsdorff, a German mathematician, developed the heuristic algorithm Min to solve the "Knight's Tour" problem. Since the Knight's Tour problem is one of finding a Hamiltonian path, S. Fajtlowicz and M. Rusinkiewicz investigated using Min to find Hamiltonian paths in random graphs. They found Min to work well in a high percentage of cases tested. S. Fajtlowicz suggested that we investigate under what conditions Min will find a Hamiltonian path, when it is known that the graph contains a Hamiltonian cycle. To investigate in special graphs, we wrote a program that verifies whether Min finds a Hamiltonian path in every graph with a given degree sequence. The program produces graphs, known to have Hamiltonian cycles, for which Min does not find a Hamiltonian path.

AN APPROXIMATE GRAPH PARTITIONING AND COLORING METHOD (84)

Gomer Thomas* and Jane Cameron, Clarkson University, Potsdam, New York

This paper describes an approximate algorithm for the problem of partitioning a graph into an arbitrary number of cells so as to minimize the number of internal edges, and it presents some results achieved by an approximate graph coloring method based on this algorithm. The partitioning algorithm utilizes a hill-climbing strategy, supplemented with some techniques designed to ameliorate the local maximum problem. Although no theoretical performance predictions are yet available, the coloring method has been tested experimentally on randomly generated graphs of various sizes (up to 1000 vertices) and various edge densities. It appears in each case tested to produce colorings with fewer average colors than any other approximate method known to the authors. The partitioning algorithm is easily seen to have polynomial worst-case execution time. Experimental results indicate that if implemented properly its average time on random graphs would be O(n**2).

(85) Computer-Drawn Genograms

A.W. Colijn, University of Calgary

Genograms are an extended form of family trees, and they are widely used by psychiatrists and family therapists to represent family relationships. A set of computer programs has been developed to draw genograms from computer data about a family These programs, which are recursive in nature are able to handle complicated cases such as multiple marriages. The advantages of computer-generated genograms over hand-drawn ones include the ability to quickly re-draw the genogram, focussing on a different person as the central one.

INCOMPLETE SELF-ORTHOGONAL LATIN SQUARES (74)

K. Heinrich*, Simon Fraser University, and L. Zhu, University of Waterloo

There exists a self-orthogonal latin square of order n with a self-orthogonal subsquare of order k for all $n \geq 3k + 1$ except for $(n;k) \in \{(8;2),(6;0)\}$ and perhaps for $(n;k) \in \{(6m+2;\ 2m),(6m+6;\ 2m)\}$.

MULTI-SET DESIGNS (75)

A.Assaf, Technion, Israel Institute of Technology, A. Hartman*, IBM Isreal Scientific Centre, E. Mendelsohn, University of Toronto

A multi-subset T of the set $\{1,2,\ldots,v\}$ may be represented by its characteristic vector $(t_i : i = 1,2,\ldots,v)$ where t_i is the multiplicity of i in T. The size of a multi-set T, denoted by $\|T\|$, is defined by $\|T\| = \sum_{i=1} t_i$. We say that $T = (t_i)$ is contained m times in the multi-set $B = (b_i)$ where $m = \prod_{i=1} \binom{b_i}{t_i}$. For example the multi-set $\{x,x,y\}$ is contained 12 times in the multi-set $\{x,x,x,x,y,y,z\}$, and contained zero times in the multi-set $\{x,y,y,z\}$.

A *multi-set design* of *order* v, denoted $MB_t(v,k,\lambda)$, is an ordered pair (V,B) where V is the set $\{1,2,\ldots,v\}$ and B is a collection of multi-subsets of V of size k (called *blocks*) with the property that every multi-subset of V of size t is contained a total of λ times in the blocks of B.

In this paper we derive some neccessary conditions on the parameters t,v,k and λ for the existence of $MB_t(v,k,\lambda)$. We show that these conditions are sufficient when $t = 2$, $k = 3$ and all v and λ. We also exhibit some constructions in the cases when $k = 4$ and $t = 2$ or 3.

On Balanced Configurations of Points (88)

Robert G. Bland*and Dall-Hoon Cho, Cornell University

We will discuss several examples of problems of the following general form. Given a set $\mathcal{P}$ of configurations of points in Euclidean d-space, a map μ from $\mathcal{P}$ to R_+ that measures "balance" ($P \in \mathcal{P}$ is balanced if $\mu(P)$ is small), and a map φ that takes pairs ($P \in \mathcal{P}$, $x \in P$) to $\mathcal{P}$, determine $\beta(\mathcal{P},\mu,\varphi) \equiv \max\{\min\{\mu(P') : P' \in \Phi(P)\} : P \in \mathcal{P}\}$. Here $\Phi(P)$ is the subset of $\mathcal{P}$ generated by P and φ. For example we might take $\mathcal{P}_1$ to be the set of finite subsets of the unit ball, $\varphi_1(P,x) \equiv (P \setminus x) \cup (-x)$, and $\mu_1 \equiv \|\Sigma\{x : x \in P\}\|_2$; then $\beta(\mathcal{P}_1,\varphi_1,\mu_1) = d^{1/2}$. Or we might take $\mathcal{P}_2$ to be the restriction of $\mathcal{P}_1$ to subsets P such that every hyperplane H through the origin misses at least two points of P, and take μ_2 to be the maximum over all such hyperplanes H of the ratio of the numbers of points of P on the two sides of H; then $d < \beta(\mathcal{P}_2,\varphi_1,\mu_2) < 2d$. It follows from well-known results that the chromatic number problem and the nowhere-zero flow problem fit into this general framework as further specializations of the second example above.

TRANSLATION B-OVALS (89)

William Cherowitzo, University of Colorado at Denver

All translation ovals in finite Desarguesian planes have been determined by Stapley Payne. Translation ovals have also been shown to exist in finite non-Desarguesian planes. In this paper we examine the type of abstract oval (*a la Euckenhout*) that corresponds to a translation oval in any finite plane. These *translation B-ovals* are investigated by means of a ternary algebraic structure associated with any abstract oval.

A PARALLEL MINIMUM SPANNING TREE ALGORITHM IN LISP (80)

Leon Kotin, U.S. Army Communications-Electronics Command, and W. Richard Stark*, University of South Florida

The classical Kruskal and Prim algorithms for generating minimum spanning trees of a weighted connected graph are well known. In contrast to the sequential character of these algorithms, we discuss a minimum spanning tree algorithm which is largely parallel. It consists of selecting a shortest arc from each node, resulting in a forest. (Cycles are essentially automatically avoided in the process.) Shrinking each tree of the forest to a point now yields a multigraph, for which the process is repeated, if necessary. The algorithm is implemented in LISP.

(81) VERTEX AND EDGE DOMINATION PARAMETERS IN GRAPHS

Renu Laskar and Ken Peters*, Clemson University

The vertex-domination number $\gamma(G)$ and the edge-domination number $\gamma_1(G)$ are the minimum numbers of vertices that dominate V(G) and the minimum number of edges that dominate E(G), respectively. Sets of vertices that dominate E(G) and sets of edges that dominate V(G) are studied. A vertex v and an edge e=xy dominate each other if either (i) v=x or v=y, or (ii) both vx and vy are in E(G). The parameters $\gamma_{01}(G)$ and $\gamma_{10}(G)$ are introduced and studied, where the former is the minimum number of vertices dominating the edge set and the latter is the minimum number of edges dominating the vertex set. The parameter $\gamma_{10}(G)$ was first introduced by Sampathkumar and Neeralagi and called the neighborhood number of G.

BIGEODETIC GRAPHS (95)

N. Srinivasan*(on leave from Madras University), J.Opatrny, and V.S. Alagar, Concordia University, Montreal, Quebec, Canada

The non existence of graphs with exactly two paths of minimum length between each pair of nonadjacent vertices and with diameter $d \geq 3$ is proved. Hence bigeodetic graphs are defined as graphs in which each pair of nonadjacent vertices has at most two paths of minimum length between them. The block cut-vertex incidence pattern of bigeodetic separable graphs are discussed. Two characterizations of bigeodetic graphs are given and some properties of these graphs are studied.

The extremal problem of finding maximum number of edges in a bigeodetic block of diameter d, on $p \geq 2d$ vertices, construction of planar bigeodetic blocks with given girth $g \neq 4k$ and diameter $d \geq 3k$ and construction of hamiltonian (eulerian) / non hamiltonian (non eulerian), perfect bigeodetic blocks are discussed. A general procedure to construct bigeodetic blocks from an arbitrary geodetic block and containing a given geodetic block as induced subgraph is also given.

CLOSED GEODETIC GAMES FOR GRAPHS (96)

Fred Buckley*, Baruch College (CUNY), New York, N.Y. 10010, and Frank Harary, University of Michigan, Ann Arbor

Let S be a subset of the vertex set V(G) of a nontrivial connected graph G. The geodetic closure (S) of S is the set of all vertices on geodesics between two vertices in S. The first player A chooses a vertex v_1 of G. The second player B then picks $v_2 \neq v_1$ and forms the geodetic closure $(S_2) = (\{v_1,v_2\})$. Now A selects $v_3 \in V - (S_2)$ and forms $(S_3) = (\{v_1,v_2,v_3\})$, etc. The player who first selects a vertex v_n such that $(S_n) = V$ wins the achievement game, but loses the avoidance game. These geodetic achievement and avoidance games are solved for several families of graphs by determining which player is the winner.

HAMILTON PATHS IN MULTIPARTITE ORIENTED GRAPHS (86)

Cun-Quan Zhang, Simon Fraser University

An oriented graph $T = (V, A)$ is called t-partite tournament if $V = V_1 \cup V_2 \ldots V_t$, each V_i is an independent set and any pair of vertices in different V_i and V_j are joined by exactly one arc.

W. Jackson (1981) proved that the regular bipartite tournament contains a Hamilton cycle. In this paper, we show that if T is a regular multipartite tournament, T contains a cycle of length at least $|V| - 1$. Hence, T contains a Hamilton path.

ACYCLIC IN-CONNECTION OF DIRECTED GRAPHS (87)

Victor Neumann-Lara UNAM

The acyclic in-connection of a digraph D is defined as the maximal number of weak components which can be obtained by removing an acyclic set of arcs in D. In this paper, some results concerning the acyclic in-connection of tournaments are presented.

ON DECOMPOSITIONS AND COVERINGS OF GRAPHS (102)

Miroslaw Truszczynski, Dept. of Computer Science, University of Kentucky

Let *H* be a family of graphs. By an *H*-decomposition of a graph G we mean a partition π of the edge set of G such that every $M \in \pi$ spans in G a subgraph isomorphic to a graph in *H*. The problem is to characterize graphs having an *H*-decomposition for a given family *H*. A related problem is the one of finding an *H*-covering of the edge set of a graph with graphs isomorphic to graphs in *H* which minimizes the sum of sizes of graphs used. This minimum is called the *H*-covering number of a graph. The talk will survey known results about *H*-decompositions for *H* s consisiting of graphs with 2 or 3 edges. These results will be used to obtain results on *H*-covering numbers. Existence of *H*-decompositions and bounds for *H*-covering numbers for some other families *H* will also be studied.

One-factorizations of $G \times K_3$ (103)

W.D. Wallis* and Wang Zhi-jian, University of Newcastle and Soochow Railway Teachers College

We discuss the following problem: if G is a bridgeless cubic graph and $\times$ denotes Cartesian product, does $G \times K_3$ necessarily have a one-factorization? Some partial results are obtained.

THE GEOMETRY OF $\overset{L}{\cdot\cdot\cdot}$ (90)

L.M. Batten, University of Winnipeg

The classification of geometries with the diagram of the title is still not settled. In fact, we are far away from a solution. In this talk, we present the history of the problem, give a number of properties of geometries having this diagram, and present necessary and sufficient conditions under which geometries with this diagram are embeddable in projective 3-space.

Estimates of a computer search for a projective plane of order 10 (91)

C. Lam,* S. Crossfield and L. Thiel
Concordia University, Montreal, Quebec

The weight enumerator of the binary error correcting code generated by the rows of a projective plane of order 10 is completely determined once the numbers of code-words of weight 12 and 16 are known. In 1983, we finished a computer search which shows that there does not exist any code-words of weight 12. We are close to finishing another computer search which, most likely, will shows that there are no code-words of weight 16. If the result is as expected, then it implies that 24,675 code-words of weight 19 must exist. Such a code-word induces a 19-point configuration with 6 lines and 16 points such that each line contains exactly 5 of the 19 points. A computer program finds 66 non-isomorphic 19-point configuration. By ad hoc arguments, we can show that 18 of these cases cannot be completed to a plane. We will present estimates which shows that the remaining cases can be solved in about 2 years of computing time on a VAX-11/780.

A STUDY OF WU'S METHOD - A METHOD TO PROVE CERTAIN THEOREMS IN ELEMENTARY GEOMETRY (92)

Hai-Ping Ko* and Moayyed A. Hussain, Corporate Research & Development, General Electric Company, Schenectady, New York

Wu [1] introduced a method to prove certain kinds of theorems in Euclidean Geometry. He treated these problems as problems of proving if any given algebraic variety is a subset of another given algebraic variety. Therefore the method is also applicable to prove certain theorems in algebraic projective geometry. The theoretical background of the method is profound in algebra and computing. We studied the method, its algorithm developed by Chou [2], and the method of solving a system of polynomial equations developed in MACSYMA. We experimented these methods onto theorems in Euclidean geometry and algebraic projective geometry. We found that (1) Chou's algorithm was not a complete representation of Wu's method but the algorithm worked for all practical theorems we selected so far; and (2) to complete the theorem proving process by Wu's method, it seemed necessary to deal with problems of solving a system of polynomial equations. We shall also give a formulation of those properties of geometric theorems which are dictated in Wu's method.

[1] Wu, Wen-tsun, "On the Decision Problem and the Mechanization of Theorem Proving in Elementary Geometry," Scientia Sinica 21 (1978), 159-172

[2] Chou, Shang-Ching, "Proving Elementary Geometry Theorems Using Wu's Algorithm," Automated Theorem Proving: after 25 years, Contemporary Mathematics, Vol. 29, 243-286 (1983)

Simulation Study of Dynamic Memory Allocation Strategies. (97)

Hikyoo Kon, Lamar University, Department of Computer Science

There are many methods for dynamic memory allocation where allocation requests of differing sizes for computer main memory are dynamically made. Three most commonly used methods are "First-fit", "Best-fit", and "Buddy system." While these methods are relatively simple to implement and perform better than others in terms of overall space utilization and computational overhead involved, it is not that clear about the trade-offs among them.
In this study, we attempted to find the tradeoffs among these three methods, by simulation and statistical significance test. Emphases were placed on the effects of external factors. These included:

- distribution of input parameters such as
  - request memory size (relative to the fixed total size)
  - request memory lifetime
  - request arrival time, and
- overall workload level of the allocation system.

As performance measures, we chose

- the memory utilization ratio
- total number of searches,
- mean available memory size,
- mean rejected memory size, and
- frequency of rejections over acceptance.

THE SIZE OF THE INACCESSIBLE SET: A QUERY TYPE SPECIFIC MEASURE OF SECURITY IN STATISTICAL DATABASES (98)

Ernst L. Leiss, University of Houston

The inaccessible set is that set of elements in a statistical database which can not be compromised if queries of a certain fixed type are used. We define the size of the inaccessible set as a measure of the security of statistical databases; this measure has the property that it reflects changes in the type of the queries in a very direct way. We then derive several of these measures for various types of queries.

This research was supported in part under NSF grant ECS-8303579.

CHARACTERIZATION OF THE TESTING PROBLEM IN PROTOCOLS (99)

Krishan Sabnani, AT&T Bell Laboratories, Murray Hill, New Jersey

In computer communications, the conformance testing of protocols is an important problem. This problem is precisely defined here in graph-theoretic framework. A protocol is modeled as a finite state machine (FSM). The FSM in turn is specified as a directed graph with edge labels. The nodes of the graph correspond to the states and the edge labels correspond to the input (output) events that cause (are caused by) the state transition. The FSM has a well-defined initial state. It can be returned to its initial state by a *reset* input. A protocol implementation is a black box with input/output ports.

We want to check whether the protocol implementation has a subgraph same as the protocol specification with the same initial state. This can be done by applying a properly designed input sequence and obseving the outputs. A solution to this problem is a procedure to design such a sequence of inputs and outputs. But this problem is at least as difficult as a graph subisomorphism problem.

On the other hand, if we assume that the number of states in the FSM is smaller than or equal to that in the specification, then it is a simple problem. It has a simple solution which involves computing a unique input/output sequence for each state.

CLIQUE COVERS OF DIGRAPHS (104)

J. Richard Lundgren,* University of Colorado at Denver
John S. Maybee, University of Colorado at Boulder

In recent year there has been a considerable amount of interest in clique covers of graphs. Here we propose a notion of clique for digraphs which permits us to extend the notion of clique covers and clique partitions to digraphs. We discuss clique cover numbers, clique partition numbers, and claw cover numbers for certain types of digraphs. In particular, we obtain some general upper bounds for these numbers for complete digraphs, certain types of tournaments, and m-regular digraphs. While the ideas used are entirely analogous to those used in graphs, the results obtained are quite different.

PARTITIONS OF THE EDGE-SET OF A MULTIGRAPH BY COMPLETE SUBGRAPHS (105)

David A. Gregory*, Queen's University, and D. deCaen, Northeastern Univ.

Let G be a loopless multigraph on v vertices and suppose that the edge-set of G is partitioned by b complete subgraphs. We give conditions that are sufficient to imply that $b \geq v$.

The result implies Majumdar's generalization of Fisher's inequality for block designs, and also yields a theorem of H. Ryser on set families.

An algorithm for path partitions in acyclic graphs (106)

R. Aharoni I. Ben-Arroyo Hartman*

Technion - Israel Institute of Technology

A *path partition* in a digraph $G = (V,E)$ is a partition of V into disjoint paths. A *partial k-colouring* is a collection of at most k disjoint independent sets called "colour classes". A path partition $\Pi = \{P_1, P_2, \ldots, P_m\}$ and a partial k-colouring $\Omega^k = \{C_1, C_2, \ldots, C_k\}$ are *orthogonal* if each $P_i \in \Pi$ meets exactly $\min\{|P_i|, k\}$ different colour classes of Ω^k. We give an algorithm for finding, for each k, an orthogonal path partition and partial k-colouring in acyclic digraphs. The algorithm implies a result proved by Aharoni, Hartman, and Hoffman, and independently proved by Cameron, and Saks, and extends some results proved by A. Frank for posets. For $k = 1$, it extends to all digraphs, providing an alternative proof of the Gallai-Milgram theorem.

An infinite family of generalized quadrangles (93)

Stanley E. Payne, University of Colorado at Denver

For each positive odd integer e, let $t = 2^e$, $s = t^2$. We construct a generalized quadrangle S of order (s,t) which is new if e is greater than or equal to 5. In some sense these are the even characteristic analogues of a family of quadrangles of odd characteristic recently discovered by W. M. Kantor.

A Proof of the Points-Lines-Planes Conjecture (94)

George Purdy, Texas A & M University

We will give a proof of the Points-Lines-Planes Conjecture which states that given n points in E_3 which determine L lines and P planes $L^2 > Cnp$ where C is a positive constant.

(109) On the Shnirelmann and asymptotic densities of certain sets of numbers

Carl Pomerance, Bell Communications Research and University of Georgia

If A is a set of natural numbers and A(n) is the number of members of A up to n, then d(A), the asymptotic density of A, is lim A(n)/n if this limit exists. The Shnirelmann density, D(A), is inf A(n)/n where the infimum is over all natural numbers. Sometimes these two concepts agree as when A is the set of odd numbers and both densities are 1/2. Other times they disagree as when A is the set of even numbers. In this note we are primarily concerned with the situation when A is of the form A_S, the set of natural numbers not divisible by any member of S, where S is some set of prime numbers. So, for example, if S={2,3}, then $d(A_S) = 1/3$ and $D(A_S) = 1/4$. Among other results, we show that for any set of primes S, there is a set of primes $S' \supset S$ with $D(A_S) = D(A_{S'}) = d(A_{S'})$. We also investigate the largest possible value of $d(A_S) - D(A_S)$ where S ranges over all subsets of the primes. (This is joint work with Paul Erdös and John Selfridge.)

Taxonomic Classes of Sets (110)

James F. Lynch, Clarkson University

Let $C = \{S_1,\ldots,S_m\}$ where each S_i is a finite set. C is said to be a taxonomic class of sets if $S_i \subseteq S_j$, $S_j \subseteq S_i$ or $S_i \cap S_j = \emptyset$ for $1 \leq i, j \leq m$. Let t_n be the number of taxonomic classes of subsets of $\{1,\ldots,n\}$ and t(x) be the exponential generating function of $t_1, t_2, \ldots$ We show that

$$n!n^{n-1} < t_n < (2n)^{3n}$$

and t(x) satisfies the functional equation

$$t(x) = e^{e^x + t(x)} .$$

K-DEFINITE GROUP TESTING AND ITS APPLICATION TO POLLING IN COMPUTER NETWORKS (100)

Robert W. Chen*, University of Miami, and Frank K. Hwang, AT&T Bell Laboratories

The idea of group testing is to correctly identify all defectives in a given set of items by using tests on groups of items. The outcome of a group test is either that the group contains no defective, or that the group contains one or more defectives, with no information on which ones or how many. Recently the idea of group testing has been found to be applicable to polling techniques for computer communications networks. However, an additional type of group test arises where the outcome of the test is either that the group contains no defective, the group contains exactly one defective (and the defective is revealed), or that the group contains two or more defectives. In the last case there is no information about how many defectives there are or which ones are the defectives. In this paper we make the further generalization that a test can have k+1 possible outcomes: no defective, one defective, ..., k-1 defectives, or k or more defectives. The defectives are revealed except in the last case. We call this the k-definite model and give an efficient algorithm for it. We also give a lower bound on the mimimax number of tests required which differs to our construction by a factor $\log_2 d$ (d is the number of defectives in the given set).

(101) TOEPLITZ GRAPHS: ANOTHER INTERCONNECTION NETWORK

Narsingh Deo, Washington State University, Pullman, WA

We derive a new class of graphs from Toeplitz matrices and show that it has most of the properties desirable in computer interconnection networks.

A Toeplitz graph TG(s,w) of order $n = s \cdot w + 2 \geq 3$ is defined by its adjacency matrix M, whose (i,j)th entry $m_{ij} = 1$, when $|i-j| = 1 + w \cdot k$, for $k = 0,1,2,\ldots,s$; and $m_{ij} = 0$, otherwise. Thus, M is a binary, symmetric Toeplitz matrix, with diagonal stripes of 1's and 0's. Parameter w is the width of each stripe, and s is the number of stripes above (and hence below) the main diagonal. Toeplitz graphs are simple (without self-loops or parallel edges), undirected, connected and both vertex-symmetric and edge-symmetric. Properties of these graphs are explored here. For example, we show that every cycle C_n, every complete graph K_n, every complete bipartite graph $K_{\frac{n}{2},\frac{n}{2}}$ (of even order), and every Möbius ladder is a Toeplitz graph. In fact, $TG(n-2,1) = C_n$; $TG(1,n-2) = K_n$; $TG(2,\frac{n-2}{2}) = K_{\frac{n}{2},\frac{n}{2}}$; and $TG(\frac{n-2}{2},2) =$ Möbius ladder of order n.

Studies of other properties such as the eigen values, diameter, girth, and reliability of Toeplitz graphs and their enumeration are also reported.

COMPUTER TESTING FOR ISOMORPHIC FACTORIZATIONS (115)

M. N. Ellingham, University of Waterloo

An isomorphic factorization of a graph into t parts is a partition of its edge set into t subsets such that all the subgraphs induced by the subsets are isomorphic. We discuss the problems involved in testing for the existence of isomorphic factorizations. We also give some computer-obtained results for small (order 12 or less) regular graphs, and relate these to known theoretical results.

COMPLETION NUMBERS OF GRAPHS (107)

John Gimbel, Colby College

A vertex partition $v_1, v_2, \ldots, v_n$ of a graph G is <u>complete</u> if for all distinct i and j between 1 and n there is an edge e which has an endpoint in V_i and an endpoint in V_j. The <u>completion number</u> of a graph is the maximum order of all its complete vertex partitions. In this paper we examine several results on completion numbers. This includes several bounds and remarks of a topological nature.

LOWER AND UPPER BOUNDS FOR THE BISECTION WIDTH OF A RANDOM GRAPH (108)

Mark K. Goldberg* and James F. Lynch, Clarkson University

Given a graph G, the bisection width, bw(G), of G is the minimal number of edges cut by a partition of the vertex set into two subsets of the same size (within one element).

We prove that for almost all graphs (≈ for a random graph) with n vertices and λn edges (λ is a positive constant) the following holds true

(i) if $0 < \lambda \leq 1$ then $bw(G) = o(n)$;

(ii) if $1 \leq \lambda$ then $\delta n \leq bw(G) \leq \frac{\lambda-1}{2} n + o(n)$,

where δ is the minimal root of the equation $2^{1-\lambda} \cdot \lambda^{\lambda} = (\lambda-x)^{\lambda-x} \cdot x^x$.
We conjecture that if $\lambda \geq 1$ then $bw(G) = \frac{\lambda-\sqrt{\lambda}}{2} n + o(n)$.

FINITELY STARLIKE SETS AND REFINEMENTS OF HELLY'S THEOREM (120)

John D. Baildon, Worthington Scranton Campus, Pennsylvania State University, Dunmore, PA 18512.

A set is finitely starlike if every finite set of points in it can see a common point via the set. It is shown that the closure of a bounded finitely starlike set is starshaped, and an example is given demonstrating that the boundedness condition cannot be dropped. Refinements are made in Helly's theorem to show that if $\mathcal{F}$ is a family of compact convex sets in E^n and S is a subset of E^n (or if $\mathcal{F}$ is a family of convex sets in E^n and S is a bounded subset of E^n) such that every n + 1 sets in $\mathcal{F}$ contain a common point of S, then $\mathcal{F}$ has an intersection point in the closed convex hull of S. Moreover, if S is compact, the point is in the convex hull of S, and if S is closed and convex, it is in S itself. Similar modifications of Krasnoselskii's theorem are also obtained.

Term Rank, Permanent and Rook Polynomial Preservers: (111)

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We characterize those linear operators on the $m \times n$ matrices (over various semirings) that preserve the term rank (of each matrix). Similar characterizations are obtained for linear operators preserving the permanent, and for linear operators preserving the rook polynomial. The results depend to some extent on the nature of the underlying semiring.

(112) A Formula for Explicit Solutions of Certain Linear Recursions on Polynomial Sequences

Heinrich Niederhausen, Florida Atlantic University, Boca Raton

Let $(b_n(x))_{n \geq 0}$ be a polynomial sequence, satisfying $Bb_n(x) = b_{n-1}(x)$ $(n \geq 1)$ for some linear operator B. Whereas this recursive relationship usually proves to be very effective in calculating either B or (b_n), in practical applications other recursions of the same polynomials may occur, which are not trivially reducible to the above form. Such recursions define B implicitly, like $b_n'(x) = b_{n-2}'(x) - b_{n-2}(x) - b_{n-1}(x)$ (Laguerre polynomials), i.e., $D = B^2(D-I) - B$. In general, let $R, A_1, A_2, \ldots$ be members of a certain class of linear operators. If the recursion can be expressed as the formal power series $R = \sum_{i \geq 1} A_i B^i$ (where R is degree reducing, and A_1 is invertible), then B can be expressed in terms of R using generalized Lagrange inversion. Furthermore, (b_n) can be expressed in terms of the associated polynomial sequence (r_n) of R. To obtain explicit formulas, however, we must drastically restrict the number of non-zero A_i's. The case $R = A_1 B + A_2 B^p + A_3 B^n$ leads already to a double sum for B. As an example, we show an application of the formula to a gambler's ruin problem.

(113) Transforms of operators relative to the generating function of a polynomial sequence

J.M. Freeman, Florida Atlantic University, Boca Raton

Take a field K and $P = K[x]$. For subrings A, B, C of $P[[t]]$, let $L(A:B,C)$ be the set of continuous A-linear operators T on $P[[t]]$ with $TB \subset C$. Consider the rings $L_t = L(P:K[[t]], K[[t]])$ and $L_x = L(K[[t]]:P,P)$.

<u>Key observation</u>: Suppose $g(x,t) = \sum p_n(x) f_n(t)$ where degree $p_n(x) = n$ = codegree $f_n(t)$. Then for any $h(x,t)$ in $P[[t]]$, there exist unique T in L_t and X in L_x satisfying

$$Tg(x,t) = h(x,t) = Xg(x,t).$$

This gives the invertible transform $L_x \xrightarrow{\ \hat{}\ } L_t$ defined $\hat{X} = T$, and satisfying $(XY)^{\hat{}} = \hat{Y}\hat{X}$. Fourier and Laplace transforms arise from $g(x,t) = \exp(axt)$ with $a = -i, -1$.

We illustrate with the transforms and inverse transforms of operators associated with polynomial sequences and generating functions (delta operators, multiplication and composition operators, etc.). We also discuss the transforms of functionals on P interpreted as elements of $L(K[[t]]:P,K)$.

A methodology for average-time testing of graph algorithms (116)

A.K. Dewdney* and T.R.S. Walsh, The University of Western Ontario

A straightforward extension of the definition for deterministic polynomial-time complexity to low-order polynomial time complexity suggests a definition for instance lengths of graphs. It also suggests a methodology for average-time testing based on a randomly drawn sequence of graphs whose instance lengths increase by a fixed increment. This methodology is also implicit in Karp's notion of convergence a.e. of an algorithm. It turns out to be especially easy to select random unlabelled graphs for each instance length as the distribution is very sharply peaked.

The methodology is demonstrated on a variety of well-known graph algorithms including the isomorphism algorithm of Babai and Kucera and the matching algorithm of Karp and Sipser.

(117) IS TOWERS OF HANOI REALLY HARD ?

Paul Cull* and Colin Gerety, Oregon State University

Although the standard Towers of Hanoi problem requires time which is doubly exponential in the number of bits in the input, we claim that the problem is really easy but is I/O (input/output) bound. We give examples of several questions about Towers of Hanoi and show that all these questions (as well as the standard problem) can be answered in time O(max(|INPUT|,|OUTPUT|)). We conjecture that all questions about Towers of Hanoi are I/O bound.

Fault Detection in CMOS Circuits and an Algorithm for Generating Eulerian Circuits in Directed Hypercubes. (118)

J. A. Bate,* D. M. Miller
University of Manitoba

In order to detect stuck-open faults in CMOS circuits using spectral techniques, it is necessary to generate a sequence of test vectors which correspond to an Eulerian circuit in a directed hypercube. An algorithm is required which will uniquely determine the next edge to be followed using only knowledge of the current edge with <u>no</u> additional information. (For example, an Eulerian circuit may not be generated and stored in advance using conventional methods.) Such an algorithm is presented, and some related problems are presented.

Non-isometric distance 1 preserving mapping E2 → E6 (121)

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In 1978, Zaks posed the following problem.
Let $f : E^n \to E^m$, $2 \leq n < m$, be a function (not necessarily continuous) satisfying the following "distance 1 preserving" property: for any $x, y \in E^n$, the condition $d(x,y) = 1$ implies that $d(f(x),f(y)) = 1$ where $d(.,.)$ is the distance. Does it follow that f is an isometry (onto its image)? The corresponding case $n = m = 1$ is false, and the case $n = m \geq 2$ has been shown. Even the case $n = 2$, $m = 3$ is open. We give an example of a mapping $f: E^2 \to E^6$ which is distance 1 preserving but not isometric.

Topological Indices as Chemical Behavior Descriptors (122)

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Graph theorists have contributed steadily to the development of chemistry since the pioneering work of Arthur Cayley on isomer enumeration in the mid-1870s. Their contributions, however, have been largely confined to one or two applications only. It is pointed out here that there are numerous interesting mathematical problems still to be tackled involving graphical invariants, usually referred to by chemists as topological indices (TIs). Differing TIs have been widely used to characterize chemical behavior dependent upon (a) molecular size, (b) molecular shape, and (c) specific molecular sites. Three comparatively simple indices based on the distance matrix are exhibited. The first index has been used to predict the physicochemical properties of hydrocarbon molecules; the second, properties depending on the degree of branching present in molecules, e.g. the octane number of fuels; and the third, the occurrence of carcinogenicity in polycyclic aromatic species (polyhexes). Current work on TIs and some of the mathematical problems associated with their further evolution are briefly discussed.

TREE IDENTITIES FROM RANDOM GROWTH (123)

Robert W. Robinson, University of Georgia

With any rooted tree is associated an identity, the <u>tree identity</u>, which arises in certain cluster expansions in theoretical physics and was first observed by Paul Federbush. Every Pattern η for growing T is assigned a weight $w(\eta)$, and the tree identity for T asserts that these weights sum to 1. In the present paper a proof is obtained by interpreting $w(\eta)$ as the probability that the pattern η is followed when T is grown at random.

Using Cayley's theorem that there are exactly N^{N-1} rooted labeled trees on N vertices, it is then possible to abstract the patterns and sum over all such trees to derive the identity

$$\sum_{|\eta| = N} w(\eta) = \frac{N^{N-1}}{N!} .$$

Related identities are obtained when the class of rooted trees is replaced by one of the following classes: rooted connected graphs; initially connected acyclic digraphs; intially connected digraphs.

(124)

Minimizing the Number of Components in Perfect Systems of Difference Sets

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A. Kotzig, Université de Montréal

Let c, m, $p_1,\ldots,p_m$ be positive intergers. Let $S_i = \{x_{0i} < x_{1i} <\ldots< x_{p_i,i}\}$, $i = 1,\ldots, m$, be sequences of intergers, and let $D_i = \{x_{ji} - x_{ki} \mid 0 \le k < j \le p_i\}$, $i = 1,\ldots, m$ be their difference sets. Then $\{D_1,\ldots,D_m\}$ is a perfect system of difference sets (PSDS) starting with c if $\bigcup_{i=1}^{m} D_i = \{c, c+1,\ldots, c-1+\frac{1}{2}\sum_{i=1}^{m} p_i(p_i+1)\}$. Each D_i is called a component of the given PSDS; the size of D_i is p_i. Only PSDS with $p_i > 1$, $i = 1,\ldots, m$, are considered. It is known that, for some classes of PSDS, $m \ge 2c - 1$; however, this inequality does not hold in general. A lower bound for ratios $m/(2c - 1)$ for all PSDS with m components and start c is obtained. Examples of PSDS are given which indicate that the lower bound obtained is the greatest lower bound. It is also shown that only components of size two and three have to be used in the construction of such examples.

Complexity of Homeomorphism Testing. (125)

S.P. Franklin and Y. Zalcstein*, Memphis State University.

Isomorphism testing in several classes of groups, as well as in classes of algebras such as semigroups and automata has been shown to be polynomially equivalent to graph isomorphism. We consider this question from a category theoretic perspective, introducing the notion of polynomial time equivalence of categories. With a suitable notion of graph homomorphism, allowing a homomorphism to collapse edges, it turns out that many (but not all) of the known reductions are polynomial time equivalences of categories. Next we show that the category of finite topological spaces is polynomial time equivalent to the category of transitive digraphs. Furthermore, the subcategory of T_0 spaces is polynomial-time equivalent to the category of transitive oriented digraphs. Finally, topological connectivity is shown to correspond to weak connectivity of digraphs yielding connected spaces versions of both results. In particular, testing homeomorphism of finite topological spaces is polynomially equivalent to graph isomorphism.

CHARACTERIZATION AND COMPUTATIONAL COMPLEXITY QUESTIONS FOR REPRESENTATION CLASSES OF GRAPHS

Edward R. Scheinerman, The Johns Hopkins University (130)

Many classes of graphs are defined via a representation scheme such as intersection of intervals, overlap of paths in trees, disjointedness of function diagrams, etc. We present a general framework for studying classes of this type including a characterization. We also show that the problem of determining membership in any such class can have arbitrary computational complexity.

Partial matching in degree-restricted bipartite graphs (131)

Douglas B. West*and Prithviraj Banerjee, University of Illinois

Given a set of n "input" nodes, let $f(n,s,d)$ be the minimum number of "output" nodes in a bipartite graph such that every set of at most s inputs is joined to at least s outputs, subject to the restriction that every output node has degree at most d. Let $\alpha_n(s,d)=f(n,s,d)/n$. We provide lower bounds and constructive upper bounds. In general, $\alpha_n(s,d)\le s/(s+d-1)+O(1/n)$, which is optimal when $s=2$ or $d=2$. This can be improved when $s>2$ and d equals or greatly exceeds s. If $s=d$ and t is the smallest even integer at least $\sqrt{s}$, then $\alpha_n(s,d)\le t/s+O(1/n)$. If $d\ge(s-1)\binom{r-1}{s-2}$, then $\alpha_n(s,d)\le r/n+O(1/n)$. The trivial lower bound of $\alpha_n(s,d)\ge \max\{s/n,1/d\}$ can be approximately doubled; $\alpha_n(s,d)\ge 2/(d+1)$. This problem is related to the construction of superconcentrators and connectors.

GRAPH THEORY AS A SOURCE OF DIFFICULT PROBLEMS FOR RELATIONAL DATABASES

Dan A. Simovici*, University of Massachusetts, and Corina Reischer, University of Quebec (132)

We study the use of known intractable problems from Graph Theory for proving the NP-completeness of certain problems arising from the study of relational databases. We deal here with two types of problems: intensional problems (referring to the intensions of relations, that is, to their relational schemes) and extensional problems (concerning the sets of tuples constituting the relations). Among other results, we prove the NP-completeness of the existence of canonical systems of functional dependencies in which functional dependencies ({x}, {y}) have unique RAP- derivations by transformation from the unconnected subgraph problem.

Γ-Regularity: Multigraphs and Digraphs

(136)

*Robert L. Hemminger, Vanderbilt University
Eric L. Wilson, Wittenberg University

Plonka defined a graph to be m-Γ-regular if the degrees of the neighbors of each vecter sum to m. He shows that simple Γ-regular graphs are either regular or bidegree bipartite (and conversely). We give a very simple proof of his theorem that holds for graphs in general. Using similar methods we also characterize three classes of digraphs with analogous properties.

CRITICAL GRAPHS WITH RESPECT TO GENERALIZED COLORINGS (137)

M. Frick, Rand Afrikaans University, Johannesburg, South Africa

The K_m-chromatic number χ_m of a graph G is defined as the smallest positive integer k such that the vertex set V(G) of G can be partitioned in k subsets, each inducing a K_m-free subgraph. A graph G is called (k, χ_m)-critical if $\chi_m(G) = k$ and $\chi_m(G') < k$ for each proper subgraph G' of G. We discuss various constructions of critical graphs.

ON PACKING WITH DISJOING STARS

B. L. Hartnell, Saint Mary's University (138)

Prompted by the study of designing certain types of specialized communication networks in which whenever a vertex is selected to be removed all of its neighbours are deleted as well, we are led to consider the following 2 person game. Two players alternate colouring a vertex in a given graph (whenever a vertex is chosen to be coloured all of its neighbours are automatically coloured as well). A player can select a vertex to be coloured (directly) only if none of its neighbours are presently coloured. Some preliminary results are discussed.

On a Conjecture of R. Graham (126)

*G. Freiman, A. Haimovick, J. Schonheim
Tel Aviv University

Let $K = \{a_i | a_i \in \mathbb{Z}, \quad a_i < a_{i+1}, \quad i = 1,2,\ldots,n\}$,

$$C_{i,j} = \frac{a_i}{(a_i, a_j)}$$

Conjecture:

$$\max_{1 \le i,j \le n} C_{i,j} \ge n$$

Let p_j be a prime number dividing one of a_i, let $P = \{p_1 < p_2 < \ldots < p_r\}$ be the set of all such primes.

Theorem. Conjecture of R. Graham is true if $1 \le r \le 5$ or $p_r \ge \frac{n}{8}$ or $n \le 185$.

ON GOLOMB'S CONJECTURE G (127)

Oscar Moreno, University of Puerto Rico

Recently Moreno has given a proof of Golomb's conjecture D. This is a conjecture on the existence of primitive quadratics over a finite field.

In this paper we would see how similarly we can give a proof of a limited case of his conjecture C for the assymptotic situation and also enumerate a finite number of exceptions. We will suggest a method of attack for his conjecture A.

DIVISIBLE QUOTIENT LISTS AND THEIR MULTIPLIERS (128)

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Divisible quotient sets in a given group G yield divisible quotient lists in a homomorphic image of G. A multiplier theorem will be proved for divisible difference lists, i.e., when the underlying group is abelian. As a corollary, we obtain a generalization of a theorem of Ko and Ray-Chaudhuri (1981). We use it to obtain new multipliers of divisible difference sets, which were not obtained from other known multiplier theorems for the divisible case. We do have an analogue of the "-1 multiplier theorem" for divisible difference lists. Using the idea of homomorphic images, we obtain a few results in connection with the prime power conjecture for cyclic affine planes. We obtain some new values n for which there is no cyclic affine plane of that order. We indicate how one can possibly get infinite families of values of n satisfying this property. We point out that some of these are related to classical unsolved problems in number theory. Positive answers to these number theoretic questions would straight away produce infinitely many nonprime powers n for which there cannot be a cyclic affine plane of order n.

AN IMPROVED ALGORITHM FOR THE OPTIMAL SELECTION PROBLEM WITH SHARED FIXED COSTS (133)

S.K.M. Wong and W. Ziarko*, University of Regina

Given a set F of objects (facilities) and a collection S of sets (options) each of which is a subset of F. Each facility in F is associated with a cost and each option in S provides some benefit. The optimal selection problem is to choose a subset S' of S such that the net benefit associated with S' is maximum. This combinatorial problem can be solved by the standard network flow model. It is shown in this paper that the complexity of such a computation can be greatly reduced by an improved algorithm.

GOLOMB'S TRIANGLE AS THE BASIS OF OPTIMAL DIAGNOSABLE SYSTEM DESIGN (134)

Anton T. Dahbura*, and N.F. Maxemchuk, AT&T Bell Laboratories, Murray Hill

Consider a directed graph $G=(V,E)$, a positive integer $t_1<|V|$, and a test outcome $a_{ij}\in\{0,1\}$ for each edge $(i,j)\in E$, where $(i,j)\in E$ if and only if vertex i tests vertex j. If i is faulty then a_{ij} can be either 0 or 1, but if i is fault-free then $a_{ij}=0$ if and only if j is also fault-free. A graph G is said to be t_1/t_1-diagnosable if any set of at most t_1 faulty vertices can be isolated to within a set of at most t_1 vertices. If $E=\{(i,j) : j=i+x_k(\text{mod } n) \text{ and } x_k\in X\}$, where $X=\{x_1,\ldots,x_{t_0}\}$, then G is said to be a $D(n,t_0,X)$ system. In general, for such systems $t_1\le 2t_0-1$. We show that $t_1=2t_0-1$ if and only if the difference between any two elements in X is distinct. Such sequences which also minimize n are considered optimal and are members of *Golomb's Triangle*. Optimal $D(n,t_0,X)$ systems for $t_0\le 23$ are given.

RELIABILITY OF Δ-Y NETWORKS (135)

Ehab S. El-Mallah* and Charles J. Colbourn, University of Waterloo

A graph is Δ-Y reducible if it can be reduced to an edge by recursive application of series, parallel and Δ-Y replacements. Partial 4-trees are subgraphs of 4-trees. Δ-Y graphs have been studied in the context of network reliability. Interest in 4-trees and their partial subgraphs arises since efficient algorithms exist for solving many NP-hard problems that have linear time algorithms for trees. These problems include vertex cover, independent sets, graph k-colorability, network reliability, Hamilton circuits and many others. In this paper we show that all Δ-Y reducible graphs are partial 4-trees. The proof yields a simple algorithm to embed a Δ-Y graph into a 4-tree. Combining this with efficient reliability computations on partial 4-trees generalizes known algorithms for reliability of Δ-Y networks.

LONGEST CYCLES IN 3-CONNECTED PLANAR GRAPHS (142)

Lane Clark, University of New Mexico

We give exponential lower bounds for the circumference of a class of 3-connected graphs which includes 3-connected planar graphs.

A NOTE ON DEGREE CONDITIONS FOR HAMILTONIAN CYCLES IN LINE GRAPHS (143)

Douglas Bauer, Stevens Institute of Technology

We consider the problem of finding the best possible sufficient conditions on the vertex degrees of a graph G to insure the existence of a hamiltonian cycle in its line graph L(G). Specifically, we seek to find the smallest positive integer f(n) such that if G is a graph on n vertices with minimum degree at least f(n) then L(G) is hamiltonian. Some partial results and a conjecture are presented for the class of 2-connected graphs.

STRATIFICATION FOR HAMILTONICITY. (144)

I.J.Dejter, University of Puerto Rico.

Let k be a positive integer. Consider a building with only one entrance, a revolving door inside (resp. outside) of which there are k (resp. k+1) individuals. A displacement of one individual is said to be permissible if it leaves k+1 individuals (resp. k) individuals inside (resp. outside) the building. The configurations formed by the different partitions of the whole population of 2k+1 individuals in the inner and outer subpopulations made adjacent by permissible displacements as above form a bipartite graph Gk. P.Erdos conjectured that for every k, Gk is Hamiltonian. We reduced this to the search of Hamilton paths in certain quotient graph Hk (Graph Th.Conf.,Kalamazoo,1984). Now we got a quotient graph Jk of Hk, almost all of whose vertices, called principal vertices, have the same maximal cardinality when considered as classes of vertices of Hk. For each k<8 we succeeded in reducing the latter search to the one of an induced subgraph formed by a finite number of disjoint cycles of principal vertices and paths of non-principal vertices, from which a Hamilton path as desired is obtained in Hk. The used methods are applicable to every Hk.

Some hamiltonian properties of the powers of a graph. (145)

M. Paoli, University of South Carolina

A graph G is said to be strongly q-edge hamiltonian if, for any set S of q edges of the complete graph built on V(G) such that no three edges are incident with a same vertex, G + S contains a hamiltonian cycle C so that $S \subset E(C)$. A strongly (p,q)-hamiltonian graph is a graph so that after the removal of any k vertices with $0 \le k \le p$, the resulting graph is strongly q-edge hamiltonian. There exist a number of results stating whether the kth power of a graph G is strongly (p,q)-hamiltonian, the values of p and q, and more often of p+q depending on the initial properties of G and of k. We survey some of these results and conjectures and give some new results, in which the property imposed to G is to be either h-connected or h-hamiltonian.

Common Transversals for Families of Partitions of a Finite Set (149)

M. L. Livingston, Southern Illinois University at Edwardsville

Let S be a finite set and suppose, for i=1,2,...,t, $F_i = \{f_{ij} \mid j=1,2,\ldots,s\}$ is a family of s-cell partitions of S which separate the points of S. Define m(s,t) as the least integer m such that if $|S|>m$ then every such partitioning family of S in which each cell f_{ij} is non-empty has a common transversal. We determine m(s,t) for $s > ct$ and also for certain small values of s and t and give bounds for m(s,t) in the remaining cases.

Uniform Hypergraphs with no Block Containing the Symmetric Difference of any Two Other Blocks (150)

D. de Caen, Northeastern University

Let $G_k(n)$ be the maximum possible number of blocks in a k-uniform hypergraph on n points with the property that $A \Delta B \not\subseteq C$ for any three distinct blocks A, B and C. The function $G_k(n)$ is known in the cases k = 2 and 3; these are theorems of Turán and Bollobás, respectively. Here we give a method of estimation in the general case. In particular we show that $G_4(n) \le (1.3)\binom{n}{4}$ (an old conjecture is that $G_k(n)$ equals $\binom{n}{k}$, approximately). This improves upon a recent result of Frankl and Füredi.

A LOWER BOUND FOR THE CARDINALITY OF A MAXIMAL FAMILY OF MUTUALLY INTERSECTING SETS OF EQUAL SIZE (151)

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David A. Drake, University of Florida, Gainesville
Zoltán Füredi, Mathematical Institute, Budapest
Jean A. Larson, University of Florida, Gainesville

A k-clique is a collection of k-subsets (called lines) of a set V (of points) such that any two lines meet in at least one point. A k-clique is maximal if it cannot be extended to another k-clique by adding a new line (and possibly new points). We prove that for k > 3 every maximal k-clique has at least 3k lines, improving a previous lower bound.

A Subset Generation Algorithm with a Very Strong Minimal Change Property (152)

Margaret Carkeet and Peter Eades*
University of Queensland

The algorithm presented in this paper generates all k-subsets of an n-set, where n is even and k is odd. It has a very strong minimal change property: if S and T are successively generated, then either $S = (T-\{x\}) \cup \{x+1\}$ or $S = (T-\{x\}) \cup \{x-1\}$. The algorithm is essentially an elementary implementation of the theoretical results proved by Eades, Hickey and Read in "Some Hamilton Paths and a Minimal Change Algorithm" (JACM, January, 1984).

Extreme points of certain hypergraph classes (153)

G.O.H. Katona, Ohio State University, Columbus, Ohio

Let f_i denote the number of i-element members of a given family $\mathcal{F}$ of subsets of a finite n-element set (that is, of a hypergraph). $(f_0, f_1, \ldots, f_n)$ is called the profile of the hypergraph. Taking a class of hypergraphs satisfying certain conditions, their profiles form a set of points in $\mathbb{R}^{n+1}$. The extreme points of this set are determined for certain classes. Among others, for the class of hypergraphs satifying $F_1, F_2 \in \mathcal{F} \Rightarrow F_1 \cap F_2 \neq \emptyset$.

ON THE HAMILTONIAN PATHS AND CIRCUITS PROBLEM (146)

Sunil R. Das, University of Ottawa

There is no general method of solution to the problem of proving the nonexistence of a hamiltonian path or circuit in a graph. To show that a given graph has a hamiltonian path or circuit, the usual approach is to resort to explicit construction of such a path or circuit. In fact, the hamiltonian path or circuit problem like some of the classical problems of combinatorics such as the traveling salesman problem, the set covering problem, etc. belongs to the class of nondeterministic polynomial-time complete (NP-complete for short) problems, and as such is quite intractable. In this paper systematic procedures are developed for finding all the hamiltonian paths or circuits in undirected graphs, if such paths or circuits do exist. The matrix representation of the graph is used as the mathematical tool in the process. The higher-order forms of such matrix representations are defined, and many important properties concerning the existence or nonexistence of hamiltonian paths or circuits are then established. The results are next extended to the case of directed graphs. Closely related to the notion of a hamiltonian circuit is that of 2-factor of a graph. The possible utilization of the proposed concepts to graph factorization is also investigated. The procedures suggested in the paper are simple, straightforward, and efficient, as is evident from the simulation studies which are carried out to evaluate the orders of time complexity and storage requirements.

FINDING THE SHORTEST HAMILTONIAN CIRCUIT THROUGH N CLUSTERS: A LAGRANGEAN RELAXATION APPROACH (147)

Gilbert LaPorte* and Helene Mercure, Ecole des Hautes Etudes Commerciales de Montreal, and Yves Nobert, Universite du Quebec a Montreal

This paper deals with a generalized version of the Travelling Salesman Problem which consists of finding the shortest Hamiltonian circuit through n clusters, in the case when the distance matrix is asymmetrical. The problem is formulated as an integer linear program. A relaxation of this program possessing a network structure is then considered. The relaxed problem is solved by imbedding a network flow routine into a branch and bound algorithm. Some versions of the algorithm make use of Lagrangean relaxation. Computational results are reported.

FAMILIES OF GRAPHS COMPLETE FOR THE STRONG PERFECT GRAPH CONJECTURE (148)

D. G. Corneil, University of Toronto

The Strong Perfect Graph Conjecture states that a graph is perfect iff neither it nor its complement contains an odd chordless cycle of size greater than or equal to 5. In this paper it is shown that many families of graphs are complete for this conjecture in the sense that the conjecture is true iff it is true on these restricted families. These appear to be the first results of this type.

SOME PROPERTIES OF A GENERALIZED SPERNER LABELING (154)

Lidia Filus, University of Kansas

A generalization of Sperner labeling for simplices is considered. It allows us to give any label to points in the relative interior for each facet and it preserves the Sperner labeling for other points. Some properties of this labeling and its behavior on the facets of the simplex are discussed.