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FIFTEENTH SOUTHEASTERN CONFERENCE ON

COMBINATORICS, GRAPH THEORY AND COMPUTING

MARCH 5 - 8, 1984

SCHEDULE AND ABSTRACTS

Sponsored by:

The Department of Mathematics

in cooperation with

Short Courses and Conferences

Division of Continuing Education

Louisiana State University, Baton Rouge

Time	MONDAY - March 5	TUESDAY - March 6	WEDNESDAY - March 7	THURSDAY - March 8	Time
8:00	REGISTRATION lobby, Pleasant Hall	37 Sb.tcY. / 40 Brigham / 4, Kreher	64 Ellingham / 67 Urrutia / fro Freeman	112 t:.,-t, J.I / 117 Cheston / 122 lane	8:00
8:20		38 Rall / 41 Tufekei / 11;; Wefit-.eimer	65 Lies an / r:.r.Wavl:and / 17} Bilonin1-	-LJ / 118 E..hazv / 12.. Hedet niemi	8:20
8:40	m1"TT1" .. TX\1111'1'S: -ro"m 11.A	39 Chfon / 11.. / 4; Hamin	66 Duke / 6g Hare / 172 Fun..!: h	1111 Edwards / 11Q Be:: :;,, / 70!. r.	8:40
9:00	WEU:OME			.1c Piazza / 120 Sud-borc.:." / 1?; Q+ TAn	9:00
9:20	KANTOR	LAGARIAS	SEYMOUR	116 Jones / 121 Dean / 126 Berman	9:20
9:40				BREAK	9:40
10:00		46 Jamison / 8 Moon / 56 Colbourn	IT, Hamacher / 75 Steffens / 177 Burli{!$;::	127 Stinson / 11<1 Dahbura / 135 White	10:00
10:20	1 Jv'ata / ? Magl¼ " / I.n Car.lin	47 Luod ;;n / 2.'" n-.1,.:-- / 1c:1 T"'!'"'.."	'71, Ovlpy / 7h R".."..."" / 178 T-e1M.. J	128 Bennett / 1,2 Shier / 1'16 Vince	10:20
10:4o	2 Sun,z / 8 Winkler / 14 Hilton	1>0li'AV	'BRRAK	12Q Ro"er / 1<< Das / 1,7 Richter	10:40
11:00	I, Zing / g Cummings / 15 Wells	<:2 Chou- J' ma.. / 1c::t:, ,?---- / 1r:.n"-11	17Q 1-'.avbee / 82 Mcintvre / 85 Pittel	n o Roth / 1,4 Tolman / 138 Abrham	n:oo
11:20	4 Palem / 10 J. Brown / 16 Cheng	S, LeonaM l'i7 Plantholt / 61 Miller	8c Lundgren / 8, Hsu / 86 Neufeld	ERDOS	11:20
11:4o	1'i Swart. / 11 Jim"o / 17 McKee	<:/. Jh, 2 → / 1t:2 Entinao-,.	R7 Dnnmr2.n / Ah.Ri .strcPh / 187 ThAm<>eAn		n:4o
noon	6 Iwata / 12 Wallis / 18 Gimbel	<:<: P.-an ⇒ 0:a.y / 63 a..:=1			noon
12:20	LUNCH	HAPPY MARDI GRAS!	LUNCH	We hope to see you at the Sixteenth Southeastern Conference on Combinatorics, Graph Theory and Computing.	12:20
1:40	KANTOR		SEYMOUR		1:40
2 4o	SLEATOR		88 Kilt;;;. ", ; / Q1 Faudree / 94 Schmidt		2:40
,:oo			So Evans / 92 Anstee / 95 Reischer		-3:00
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5: 1.io	t24 Za.sla ,::" / Bn Pet<>..s / IV 36 Kotzig-				5:4o
6:oo	WINE & CHEESE -- ISU FACULTY CLUB				6:oo
1:00		7:30- DESSERT PAfrr:l at home of 9:30 K.B. Reid, 2626 Dalrymple Dr.	CONFERENCE SEAFOOD BANQUET/ ISU Student Union Plantation Room		7:00
8:oo					8:oo

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'iilli2.;:-: l:. Kantor, U::ivarsi ty o:.'.OretJOn

Every r;:.:l1::2: builcJ.i q is t! corlnec ed bi0arti te ,;raph of Cia-r:ieter n and c:;irth 2;1, for so:·.:e integer n. :-'or r>2, every rank r buildL,g is a connected r-?arti te gra•:)h in ,-:hich the neighbo!"hood of every vertex is a rank r-1 buildin . hese talks will survey ,various ::iro?erties of these highly structured gra::ihs .

Self-Adjusting Binary Trees

Daniel Dominic Sleator[I]

Robert Endre Tarjan

AT&T Bell Laboratories
Murray Hill, New Jersey 07974

This talk is about a new approach to the following old data structure problem. A binary tree is to be used to represent an ordered list of items. This representation should allow items to be accessed, inserted, and deleted, and these operations should be fast. The usual approaches to this problem enforce a "balance condition" on the tree. In a tree satisfing such a structural constraint the depth of the deepest node is O(log n), and the operations are efficient. We have developed data structures that satisfy no balance condition, and no such worst-case per operation bound, but are still efficient for any sequence of operations. We are motivated by the observation that in most applications of data structures one wishes to perform not just a single operation but a sequence of operations. Whenever an item in such a "self-adjusting" tree is accessed, the tree structure is modified in such a way that the accessed item becomes more easily accessible. In addition to being simpler to describe and implement, our trees have .other remarkable properties not shared by any balanced **trees.**

SOLVING SYSI"EMS OF LINEAR EQUAIJTIES AND INEQUAIJTIES EFFICIENTLY

O

Jose M. Mata.

Department o! Electrical Engineering and Computer Science
Princeton University
Princeton, New Jersey 08544

Ab:drad.

In **many applications, like VLSI layout systems.** we **have to solve** a **system of linear constraints on two variables. Usually these systems involve millions of variables and constnunts.** If we **have only equations of the form** '"<- *i;* :.d **(d>O), we can use the top logical sort algorithm, that** is **linear in time Wld space complexity. Allowing also constraints of the form** %(- :, =e (e>0) **gives more power, but affects the efficiency in solving. Algorithms for the single-source shortest path problem or for the linear programming problem can be used, but they are not efficient enough for lhe size of our problem.**

Tie **present here an algorithm to solve system:, of equations of the fonn** *z,* -:, :.ti.{t>O) **aod:,** -:, =e {e>0). with **time complexily** ***O((n,+n,,+v)n.),*** **where** n, =**number of inequalities,** n,, **= number of equalities, and** i, = **number of variables. It** is **specially efl'.icient when the number of equalities** is **small compared to the number of inequA!ities.**

SOME DIFFICULT PROBLEMS WITH EASY PROOFS

H,i-Lin Sung

Department of Computer Science

State Univeraity of New York at Stony Brook

Stony Brook, New York 11794

The problems or unique-colorability and critical-colorability of graphs have been studied for a long time [2]. Recently, the complexity class $DP = \{L_1 \cap L_2 : L1 \in NP, L2 \in coNP\}$ was introduced and these two problems were shown to belong to this class [3]. However, whether they are complete for DP with respect to $\$$ (the Karp reducibility) was lert open, ai:d there exists strong evidence suggesting that it is very difficult to decide ir the unique-colorability or critical-colorability problem is DP-complete [1,3,4].

In this paper, we show these two problems are complete in DP with respect to nondeterministic polynomial time many-one reductions ($\$$). This is a easy consequence or our theorem that coNP-hard sets in Σ_1^P (the class NP relativized to an NP-complete *oracle*, i.e., queries or the form "$x \in A$?" are answered in one step or computation, where A is an NP-complete set), are Σ_1^P-complete, which is a class disjoint from NP if NP $\neq$ coNP. Since the reductions are easily constructed, the intractability or the unique-colorability and criticakolorability problems (and classes or nniqne- and critical- problems) are easily proved, assuming NP $\neq$ coNP. We also use standard proof techniques to show some interesting structural results, e.g., there exist sets in DP - (NP U coNP) which are neither Σ_1^P-complete nor NP-hard.

Σ_1^P is very userul in that one can show (strong evidence of) intractability or a problem by proving it coNP-hard, thus avoiding a stricter, and likely harder to construct $\$$-reduction. We further illustrate this point by showing, via easy proofs, a class or problems (e.g., deciding if two given graphs have the same chromatic number) Σ_1^P-complete and hence intractable, where Σ_1^P is the class P relativized to 3tl NP-complete oracle. The applicability or $\$$-reductions to settle open questions (e.g., the complexity of Graph Isomorphism or Recognition of Perfect Graphs) ;,, discussed.

An Application of Group Testing to Computer Networks with Priority in System

by

I. Z. Chorneyko and L. L. K. Zingl

* I. z. Chorneyko is associate professor, Department of Mathematical Sciences, McMaster University, Hamilton, Ontario, Canada. L.L.K. Zing is assistant professor, Department of Mathematics, State University College, Potsdam, N. Y. 13676.

An Application of k-sample group testing is found to locate the busy user of highest priority in a system of centralized computer network with priority ordering imposed on each terminal user. Let each user has a probability p of being busy (i.e. having a message for transmission) and a probability q of being idle. We propose a sequence of queries analogous to the procedure of finding a single defective using k-sample group testing.

We assume that the status of each user being busy or idle remains the same throughout the process of a query sequence. The expected number of queries is found for certain ranges of n, where n is the total number of terminals in the system. An asymptotic formula is also given when n approaches infinity.

The Complexlty of Schedullna Systema or Non-Identical ProceAOra[1]

Kriahna V. Palem[1] nd Donald S. Fuaul/6

1Department of Elet rical Engineering and

6Department of Computer Scien=

The Univer•ity of Tezaa

Auatin, TX 78711!

Ab.tract

A basic problem rrom deterministic scheduling theory involves scheduling a partially ordered set T or n equal length tasks on a set M or m non-identical processors. Each task bas a local deadline associated with it and the goal is to rind a schedule which minimizes *mazimum latenua.* The set M is partitioned into *k*

equival.-nce classes under the relationship *proceasor type* with m; elements in the ith equivalence class and each task *t,T* can be completed only by a processor rrom a pre-specified class *c(t).* An instance or a scheduling problem drawn from the above domain is said to belong to the class *r* iff the partial order on T is an *in-tree,* the *type graph* satisfies a *unique predecce••or* condition and the ordering of th.- m,s is consistent with that of the type graph in that if a *type* i vertex precedes a *type j* vertex, then *m m*- We prove that any instance or a scheduling problem belonging to the class *r* can be solved algorithmically in polynomial time. The class *r* properly includes the *in-tree•* or Brucker et al. and Hu for which efficient polynomial time algorithms are known. We also show that for a special case or this problem, namely that of minimizing *makeapan,* the algorithm designed for the class *r* can be extended to a much larger class ,,'::),._ In addition, we prove that the *cyclic /oreals* or Goyal and the *out-tree•* or Coffman et al. are contained in ,.· and ir'-,r respectively. Finally, several simple generalizations or these classes are shown to admit NP-complete instances.

Identification of Strategies for Two-person Games

by

Charles Swart, Department of Computer Science, Oregon State University, Corvallis, Oregon, 97330-4602

and

Dana Richards, Department of Computer and Information Science, Indiana University-Purdue University at Indianapolis, Indianapolis, Indiana, 46223

We investigate the use of automata theory to model strategies for nonzero-sum two-person games such as the Prisoner's Dilemma. We are particularly interested in infinite tournaments of such games. In the case of finite-state strategies (such as ALL D or TIT FOR TAT) we use graph travers.al techniques to show the existence of a (non-terminating) procedure for detecting our opponent's strategy and developing an "optimal" counter-strategy. We also investigate counter machine and Turing machine strategies.

PROBLEM REQUIRING k log n DETERMINISTIC SPACE

Shigeki IWATA[1] (Tokai Univ., Japan)
Takumi KASAI (Univ. of Electro-Communications, Japan)

The first "natural" problems are established solvable in deterministic log space for whose recognition log space lower bound can be shown.

A finite state transducer can be ,egarded as a mapping 6:QxZ* QxZ* such that 6(p,x)=(q,y) implies lxl=lyl, where Q is a set of states, and I: is an alphabet. The j-fold function $S^{(j)}$ of 6 is the composite function 86 .. -S (k times).

The k-finite scace transducer problem (FSTPk) is Che problem: given a finite state tranducer S, initial state q0, and two symbols a and b, to determine whether there is an integer j such that $S^{(j)}(q_0,a^k)=(q,b^k)$ for some state q.

In this paper we snow that there exists a .constant c such that $FSTP_k$ can not be solved within (k-c) log n space. The problem FSTPk is solvable within (k+ll log n space.

Some new 6-designs

Earl S. Kramer, David W. Leavitt and Spyres S. Nagliveras1
University of Nebraska, Lincoln, Ne 68588

Abstract

A t-(v,k,A) design is a pair (x,B) where B is a collection of k-subsets (called blocks) from av-set X, such that each t-subset of X occurs in exactly A blocks of *B.* A design is simple if no block of B is repeated. A design is trivial if each k-subset of X occurs precisely m times in *B.* We produce two non-isomorphic simple 6-designs with parameters 6-(20,9,112) and automorphism gcoup PSL(2,19). It has been previously shown that if q < 19, simple 6-designs on v; q + 1 points do not exist with automorphism g oup PSL(2,q). Hence v; 20 is the smallest v; q + 1 where simple 6 designs occur with automorphism group PSL(2,q).

Venn Diagrams: Some Observations and an Open Problem

Peter Winkler, Emory University, Atlanta GA JOJ22

The definition of an "n- Venn dia ram" and some combinatorial deductions concerning Hamiltonian circuits in hypercubes lead to two things: a Venn dia ram extension theorem (easy) and an open problem (maybe not so easy).

II

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SYCHRONIZABLE CODES IN THE de BRUIJN GRAPH

by L. J. Cummings
University of Waterloo

The de Bruijn graph has as vertices the o^n words of length n over an alphabet of a symbols and there is a directed edge between two vertices precisely when they overlap in n-1 symbols. Block codes can be represented as either sets of vertices or sets of edges in the de Bruijn graph. Certain codes with bounded synchronization delay have strikingly simple characterizations when viewed as subgraphs of the de Bruijn graph. In particular, many maximal codes in the class of comma-free codes appear as complete bipartite subgraphs.

1/:io

G

Some Attempts to Construct. orthogonal Lat.in Squares

John Wesley Brown[1] and E.T. Parker
Department of Mathematics, University of Illinois, Urbana

All of our attempted constructions but. one were for 3 order-10 squares.

Earlier, we ran the turn-square for all S cells on the main diagonal. currently we have done 4 cells and 3 cells. For the first of these the initial transversals were reduced by hand; for the second the cells in the first. row were ordered by number of transversal, least first..

we ran 13 involution squares for extension to triple. These have involutions bet.ween the first. row and every other. They were generated by CO!:lpuler by J;mies G. Peterson, Dept.. of computer Science, University of Illinois.

We tried a "self-orthogonal" sum-composition square of A. Hedayat. Also, on square .of 4 orthogonal except for hole due to Professor Brov.wer of th2 Netherlt:1:1d!:!.

Finally, we ran two gxoups t.o exhaust.ion - an order-15 group for 3 order-10; and a 105-group for 5 is-squares. The fox er group used 5-cycles end-around and $(X)(A_1A_2A_3)(B_1B_2B_3)(c_1c_2c_3)$ on points.

None of these tries succeeded.

The second author got some ideas for this note at a conference, swruner, 1983, Simon Fraser University, Burnaby, B.C.

11:lf(?

A recursive construction for 1-rotational doubly resolvable designs. G

Masakazu JIMBO and Sc.ott A.VANSTONE, University of Waterloo

A Steiner 2--design S(2,k,v) is said to be I-rotational if it admits an automorphism whose cycle structure is a (v-1)-cycle and a fixed point. At first, we give a recursive construction of I-rotational Steiner 2-designs.

Secondly, by applying this construction to resolvable I-rotational Steiner 2- designs, we obtain a recursive construction of resolvable I-rotational Steiner 2--designs(1-rotational Kirkman systems).

A I-rotational resolvable Steiner 2--design is said to be cyclically resolvable if the automorphism which gives the I-rotational property also preserves resolution classes. Thirdly, we give a recursive construction of cyclically resolvable I-rotational Kirkman systems.

Suppose a Steiner 2--design has two resolutions $R = \{R_i : i \in I\}$ and $R' = \{R'_i : i \in I\}$, where I is a indexing set. If $R_i \cap R'_j$ 1 for any $i \in I$ and $j \in I$, then the the design is said to be doubly resolvable. Finally, we obtain a recursive construction of doubly resolvable I-rotational Kirkman systems whose one of two resolutions is cyclic.

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Room Cubes of Side Nine

J.H. Dinitz, University of Vermont, and W.D. Wallis University of Newcastle.

Recently we have completed a lengthy computer analysis of one-factorizations on ten points and their orthogonality relations. In this paper we report the results concerning sets of three pairwise orthogonal factorizations - room cubes - from that study.

G

HOMOMORPHISMS AS A GENERALIZATION OF GRAPH-COLORING

Paul A. Catlin, Wayne State University, Detroit, MI 48202

Let $\theta : G \to H$ be a homomorphism of simple graphs, i.e., a mapping such that $\theta(xy) = \theta(x)\theta(y) \in E(H)$ whenever $xy \in E(G)$. Such a mapping we call an <u>H-coloring</u> of G. When determining whether G has an H-coloring, we lose no generality in assuming

that H has no proper subgraph F such that H has an F-coloring.

We present a few generalizations of well-known results on graph-coloring (the special case H = Kn) to the cases where H is an odd cycle on some other graph. Analogues to some known results on chromatically-critical graphs can be obtained.

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The chromatic index of graphs of high degree.

A J W Hilton (with A G Chetwynd) (University of Reading and Open University, U.K.)

Vizing's theorem states that, for a simple graph G, 6(G) x1G) 6(G) + 1. If 6(G) = x'(G) then G is Class 1, and if x'(G) = 6(G) + 1 then G is Class 2.

We pose the following two conjectures:

Conjecture 1: Let G be an (r - 2) - edge-connected graph of order 2n with r vertices of maximum degree. If 1 r n, then G is Class 1.

Conjecture 2: Let G be an (r - 2) - edge-connected graph of order 2n + 1 with r vertices of maximum degree. Let 1 r n + 1. Then G is Class 2 if and only if IE(G) I > n 6(C).

We prove these conjectures if 1 r 4, and if 6(G) n + $\frac{7}{2r}$ - 3.

G

Signatures and Signed Switching Classes

Albert L. Wells, Jr., Department of Mathematics
Louisiana State University, Baton Rouge, Louisiana 70803

This lecture presents joint work of Peter J. Cameron and the speaker. Several properties of two-graphs and switching classes can be established using ideas from homological algebra. This talk reports on an analagous theory of signatures and signed switching classes, which was derived using similar techniques. It differs from the usual approach in that one simple graph G is fixed, and switching is defined as an operation on edge signings of G. Beyond that, the development relies on choosing certain collections X and Y of subgraphs of G. By taking G to be a complete graph, X the set of triangles of G, and Y the complete 4-vertex subgraphs of G, the known results for two-graphs and switching classes can be recouped. Through other judicious choices of G, X, and Y, these results can be mimicked accurately in great generality. Two important examples of this exist for any simple connected graph G:

(1) Choose X to be the set of circuits of G, and Y the set of theta subgraphs of G.
(2) Choose X to be the set of induced circuits of G, and Y the set of induced circuits of certain types described in the lecture.

These examples give rise to characterizations of the sets of balanced circuits (or induced circuits) of a signed graph.

O

/- utomocphisms of Two-Di

Ying Cheng, Department of Mathematics, Louisiana State University

.e,pstract: This is a joint work with A. Wells. A digraph on a finite set X may be defined as an alternating function,: X<X-'> GF(3), i.e., -r(x,x) ==0 and T(x,y) ==-,(y,x) for all x, y EX .. Two digraphs , , p are said to be switching equivalent if there is a function f of X into GF(3) such that 1(x,y) = p(x,y) + f(x) - f(y) for all x, y EX. Eauivale:,sce classes are called two-digraphs. An siutomorphism of a two-digraph **f** is a permutation of X mapping any (or equivalently, some) digraph in **r** to another digraph in T'. In particular, an automorphism of a digraph is an automorphism of its switching class. We give a necessary and sufficient condition that an automorphism of a two-digraph.**f** be an automorphism of a digraph in p.

G

BALAIKE AND DUALITY IN SIGNED GRAOHS

Terry McKee, \-Jright State University

The theory of balance in signed graphs is reconstructed in terms of classical graph-theoretic duality, illustrating a general duality principle for graph theory. The definition of balance (e.vVty eye.le c.on:tahv., an eve.n nwnbeA 06 ne.gative. e.dgu) leads to a new.characterization: e.ve.ny ne.gative. e.dge. .i..J.n an odd numben 06 comple e.-ty-negative. cocyclu,. Related notions such as clusterability and cobalance are also discussed.

n

J. Gimbel (Department of Mathematics, Colby College, Waterville, M 04901),

Necessary Conditions on Switching Sequences.

Given G, a graph, and H V(G) let G switched on H, denoted SH(G),,. b the graph with vertex set V(G} and edge set

E(G-H) U {uv c ECG) lu c Hl .

Given a labeling v_1, ½, ..., v_p of V(G) let Go = G and

$$G_1 = S_{\{vl'v2, \ldots, vi)}(G)$$

for i = 1, 2, ..., p. We examine properties that the sequence G0, G1, ..., GP must have.

G)

Move-up-k Rules Hith Zipfian Probabilities

A.H. Tenenbaum, D.M. Arnow, I and S.A. Grumet

Dept. of Computer and Information Sciences
Brooklyn College of C.U.N.Y.

Empirical data is presented for a set of sequential search algorithms that dynamically reorder the list being searched, with the well-known move-to-front and transposition algorithms at the extrema. Asymptotic cost values and convergence rates **assUlling** Zipfian probabilities are examined, indicating the relationship between these two measures, their rates of change with respect to the various algorithms, and the crossover points between pairs of the algorithms. The behavior of a set of hybrid rules is also examined in light of the above **measures**, and compared to the first set of **algorithms**.

G

Digital Division: A Left to Right Multiple Precision Method

Don Thompson, Pepperdine University

We present an algorithm for performing multiple precision digital division. Like the other three arithmetic operations (and unlike all other division algorithms) we perform the division operation a div b (integer division) on.operands a= (a0, 1 ,.. 7, and b = (b ,b ,.•• ,b), with m > n, by applyii'ig divilhon to ind£vi ual opRrand digit pairs a., b .. The algorithm uses dynamic self-correction, i.e. eorr ction which is accomplished during the process of calculating the quotient and remainder. The method generates quotient digits which are either correct or too large by the amount +1.

G

TRIVALENT POLYGONAL GRAPHS

Manley Perkel, Department of Mathematics and Statistics, Wright State University, Dayton, OH 45435

In the theory of polygonal graphs, a special role seems to be played by the trivalent, strict polygonal graphs (i.e. trivalent, connected graphs of girth m > 3 such that every path of length 2 Ies in a unique m-gon of the graph). In this paper we investigate such graphs as well as a related family of triangular graphs, viz. graphs in which each vertex has an m-gon as neighbourhood. We will outline methods for construction of graphs from both families as well as considering (and answering partially) the question of when the duals of members from one family belong to the other family. Here dual means: m-gons become vertices for polygonal graphs (or triangles become vertices in the case of triangular graphs), two vertices being adjacent if and only if the corresponding m-gons (or triangles) share an edge.

O

The Amida Number of a Graph

Lisa Orton
Richard Ringeisen [1]
Clemson University

Onaga and Chen describe Amida as a traditional lottery game used to determine the priority for selecting items of limited

supply. A completed amida game results in a graph structure with certain distinquished paths. In this paper we generalize the aforementioned structure to amida paths in arbitrary connected graphs. A qraph is amida of type n when there exists distinct vertices s and t, a set S of pairwise nonadjacent edges, and a set X of n distinct paths betweens and t satisfyinq: the edges of each path are alternately in S, any edge in more than one path of X is in S, and no edge of s is in more than two paths of X. After defining the arnida number of a graph to be the maximum n for which it is of type e proceedto find bounds for amida number, a heuristic algorithm for its determination, and its exact value for many well known graphs.

FINITE HAr-lJI PROBLEMS: HOW MANY DISCS CAN BE HANDLED?

Ernst L. Leiss
Department of Computer Science
University of Houston
Houston Texas 77004

The problem of the Towers of Hanoi is generalized in such a way that moves of discs may be made only along edges of a given directed graph G. In previous work we showed that arbitrarily many discs can be moved from the start vertex S to the destination vertex D iff there is a loop of length 3 or more in the transitive closure of G ich can be reached from Sand from which D can be reached. In this paper we consider graphs which do not satisfy this characterization; therefore there is only a finite number of discs which can be handled. These Hanoi problems will be called finite Hanoi problems. The most interesting question for finite Hanoi problems is how many discs can be moved from S to D, for a given graph with m vertices. We give a rather surprising answer to this question, namely we show that there are graphs with m vertices giving rise to a finite Hanoi problem for which the number of discs which can be moved is superpolynomial in m.

Partial support of this research under grant ECS-8303579 from the National Science Foundation is gratefully acknowledged.

Multipartite Analogs of Two-graphs

Thomas Zaslavsky, Ohio State University

A two-graph (G. Higman, O.E. Taylor) is a set of triples from an n-element set, such that (T1) each quadruple contains an even number of the triples. It can be thought of as the set of frustrated triangles in a signed complete graph Kn • This point of view suggests generalizations to other base graphs G , the analog of the two-graph being the frustrated members of a fixed, highly symmetrical spanning set S of polygons of G • We deduce the analog of condition (T1) for G = K(n1,••·,np), the complete p-partite graph, with S = the set of quadrilaterals (if p = 2) or triangles (p > 3), and for G = C(n1,••·,np), the circular p-partitite graph, with S =-the set of p-gons meeting each vertex type once.

A two-graph T is regular if every point pair lies in the same number of T-triples. Equivalently ,ts signed graph has only two eigenvalues. For bipartite two-graph analogs there are three regularities, corresponding to the left, right, and cross pairs. Left-regular cases satisfy a Fisher-type inequality; they correspond to certain intersecting set systems. Both left- and cross-regular cases have only two eigenvalues. Examples come from Hadamard matrices, block designs, and odd-subset systems.

Homomorphisms of 3-chromatic graphs, II

Michael Aloertson and Luana Giboons
Smith College, Northampton, 11A 01063

A homomorphism frow a graph G to a graph His a mapping from the vertex set of G to the vertex set of H which preserves edges. An r-coloring is just a homomorphism to the r-clique. Deciding whether a graph is 3-colorable is one of the classic NP-complete problems. Maurer et al nave snown that deciding whether there exists a nomomorphism to the 5-cycle is also an NP-complete problem, ana they conjecture tnat this result holds for any non-trivial homomorphism minimal target. Vesztergombi has given necessary and sufficient conditions for a graph to map to the 5-cycle. Let L denote the Petersen graph with two adjacent vertices deleted. We show that any triangle free graph which does not map to the -cycle contains a subgraph which maps to L, verify that deciding whether a graph maps to Lis NP-complete, and present related results and open questions.

A Pratitioning Property of k-cycled Refinements of G r a p h G

Jehuda Hartman, Northeastern University, Boston

A graph is k-cycled if all its cycles are integral multiples of an integer k>2. A partitioninq property of refinements of graphs which are k-cycled is established. The partitioninq property is applied to obtain bounds on the minimal number of edges of k-cycled refinements. The same property is also used to examine refinements of graphs which are subgraphs of a hypercube.

AN APPROXIMATE ALGORITHM FOR THE OGE-COLORING PROBLEM ON MULTIGRAPHS8

Mark K. Goldberg
Clarkson College

This paper is concerned with the problem of coloring the edges of a multigraph in such a way that no two adjacent edges receive the same color. A quadratic approximate algorithm with an approximate ratio of 9/8 is described. It is shown that in case the number of colors used by the algorithm is greater than - ¼(dis the maximum vertex degree) the constructed coloring is minimal.

CIIROM.\TlC SUM EQUATIONS FOR ROOTED PLANAR MAPS O

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after June 30, 1984

The theory of chromatic sums for rooted planar triangulations has been founded by Professor W.T. Tutte in a series of papers of his 01,11.

This paper is an attempt to generalize it for rooted planar maps. A general functional equation of chromatic sums for rooted non-separable planar maps is found.

In addition, the sums of the numbers of spanning trees with certain properties over all the maps considered are expressed as a recursion treated as a Pascal-Yang Huei triangle in the 3-dimensional case by determining the derivatives of the chromatic sums at \ = 1.

The Genus of a 3lock Design O

Joan M. Rahn estern Michigan University

The genus of a design (BIBD or FBIBD) is defined to be the genus of its corresponding hypergraph (objects as vertices, blocks as edges); th t is, the genus of the bip2rtite graph associated with the hypergra h in a natural way. ihe euler formula is used to establish a lower bound *i* for the genus of a block design. An imbedding of the design on the surface of genus *i* is then described by a voltage hypergraoh or a voltage graph. Use of the lower bound leads to a characterization of planar BIBDs. A connection between a block design derived from a graph imbedding and the hypergraph imbedding of the design is established. This leads to the determination of genus formulas for several infinite families of designs. The concept of the generalized pseudosurface characteristic of a design is developed along with formulas for infinite families of designs.

CO:-.!NECTED DOMINATION OF COMPLEMENTARY GRAPHS

Renu Laskar and Ken Peters, Clemson University

Abstract. A set D of vertices in a graph G = (V,E) is a connected dominating set if every vertex in V-D is adjacent to at least one vertex in D and the subgraph <D> induced by the vertices in D is connected. The connected domination number $y_c(G)$ is the minimum number of vertices in a connected dominating set in G. Hedetniemi and Laskar showed that if both G and G, the complement of G, are connected, then $y_c(G) + y_c(G) \leq p+1$, where p is the number of vertices in G. Hedetniemi conjectured that $y_c(G) + y_c(G) = p+1$ *if* and only if G = CS. In this paper we prove this conjecture. Thus, if G i CS, $y_c(G) + y_c(G)$ p. We also characterize those graphs for which $y_c(G) + y_c(G) = p$.

GEODETIC GRAPHS AND FINITE GEOMETRIES

Michael F. Bridgland, Lawrence University

A graph G is geodetic if, for each pair of vertices x,y, there is a unique shortest x,y-path in G. A substantial portion of the existing theory of geodetic graphs concerns those graphs which satisfy the stronger condition that there be a unique chordless x,y-path of length no greater than the diameter of G for each pair x,y; we call such graphs ultrageodetic. For grapns of diameter less than three, the two concepts are equivalent. Independently, Stemple and Kantor have classified the (ultra-)geodetic graphs of diameter two. Adapting a construction of geodetic graphs due to Plesn{k, we define, for each finite geometry r, a sequence rk, k > 0, of distinct finite geometries; we then obtain the following result:

Proposition. Let G be an ultrageodetic graph of diameter greater than two. Then exactly one of the following conditions holds:

i) G has a cutvertex, in which case G is a block graph.
ii) G is homeomorphic to Kn for some n > 3.
iii) All maximal cliques of G have the same cardinality, in which case G is isomorphic to the collinearity graph of a Moore geometry.
iv) There exist positive integers d and k, and a generalized (2d+1)-gon r such that G is isomorphic to the collinearity graph of rk.

According to well-known results on the nonexistence of Moore graphs. Moore geometries, and generalized polygons, the graphs of case iii) are the chordless cycles of odd length, and the geometries r of case iv) are the finite projective planes. A generalization of the construction of rk gives rise to a family of geometries whose collinearity graphs include all known geodetic graphs.

AFFINE SUBPLAHES OF FINITE AFFINE PLANES

Lynn t-largaret Batten, University of Hi nni peg

An affine subplane A• of an affine plane A is a subset of the points and lines of A which itself forms an affine plane.

If, in addition, l_1 and h of A' parallel in A' implies that the extensions of l_1 and l_2 to lines of A are parallel in A, then we say that A is an ∞ - preserving extension of A', and write A' A.

l.Ve say A' is maximal in A if the order m of A' is less than the order n of A, and if there is no affine subplane A of A such that ll' A + A.

fie give conditions on n as a function of m which ensure the maximality of A' in A, both in the general case and in the ∞ - preserving case.

In particular, we examine the rather interesting case (n,m) = (4,3).

On Cycle-Star Graph Ramsey Numbers

Lane Clark
California State University, Long Beach

Abstract

Bounds are given fo the cycle-star graph Ramsey number $r(C_m, K1_{,n})$ in the undecided case even m < 2n.

TILING RECTANGLES WITH PAIRS OF PENTOMINOES

Earl S. Kramer, University of Nebraska

A tiling contractor must bid on a job to tile rectangular regions using sufficiently many T5 and c5(=U5) pentominoes as tiles. Recall that a pentomino is a connected region in a chessboard (with unit squares) comprising S squares interconnected by rook paths. Allowing rotations and flips there are 12 pentomino shapes that are identifiable with the letters T, U(=C), V, W, X, Y, Z, F, I, L, P, and N. Should the contractor make a bid on a 13 x 25 c5T5-tiling or throw in the trowel? These and similar tiling (packing) questions will be raised.

This talk is primarily recreational in nature and pictures will tell most of the story.

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RECENT RESULTS AND OPEN PROBLEMS IN GRACEFUL GRAPHS.

A. Kotzig, C.R.M.A., Universite de Montreal, Montreal, Quebec, Canada H3C 3J7.

New results on graceful labellings and their transformations will be presented for the follo ing graphs : 1. Trees, 2. 2-regular, 3. bipartite regular graphs of degree larger than two. For 1, transformation will be described creating any graceful tree on n vertices from one such tree. For 2, the result will concern graphs with odd circuits and graphs with isomorphic components. For 3, a construction of a special graceful labelling of disconnected graphs will be presented if each component has such a labelling.

Domination and Location in Acyclic Graphs

Peter J. Slater, Mathematics Department,
University of Alabama in Huntsville, Huntsville, Alabama 35899

Locating-dominating sets are of interest in safeguards applications of graphical models of facilities. A subset S of the vertex set V of *a* graph G is *a* dominating set if each vertex v €V - S is adjacent to at least one vertex in S. For each v in V - S let S(v) denote the set of vertices in S which are adjacent to v. A dominating set S is defined to be "locating" if for any two vertices v and w in V - S one has S(v) # S(w). Sharp bounds on the cardinality of locating-dominating sets for graphs (respectively, trees) on p vertices are given, and a linear (that is O(p)) algorithm for finding a minimum cardinality locating-dominating set in an acyclic graph is presented.

On Location - Domination Numbers for Certain Glasses of Graphs

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Peter J. Slater, Mathematics Department, University of Alababama in Huntsville, Huntsville, Alabama 35899

We further investigate locating-dominating sets as defined by Slater (see Domination and Location in Acyclic Graphs). Let RD(G) denote the smallest cardinality of a locating-dominating set in the graph G and !!'h(G) the maximum number of vertices in a graph G from class G with RD(G) = h. It is known that !!'h(G) < 3h if G is the class of trees, and if *G* is the class of all graphs then !!'h(G) = h + 2^h - 1. We show \ (P) = 7h - 10 when P is the class of planar graphs, and we present bounds on!!' for other classes of graphs such as those of genus n or those with *a* forbidden subgraph characterization (for example, those graphs with no subgraph homeomorphic to K4 or K3,3).

FACTORING GRAPHS INTO PRIMAL GRAPHS

G

Phyllis Z. Chinn[1] and Paul A. Thoelecke
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* 1983-4 address Math. Dept. University of Central Florida, Orlando, Fl.

Abstract. Primal graphs are analogous to basib vectors in a ·,tctor space. They can be used to write every graph as sum of distinct, non-isomorphic primal graphs. Several infinite classes of primal graphs are determined and conjectures are posed regarding the structure of other primal graphs.

I

On the Clique Graph of the Line Graph

Q

by Robert C. Brigham Phyllis Z. Chinn* and Ronald D. Dutton
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*Visiting from Humboldt State University, Arcata, California 95521

The clique-line graph CL(G) of a graph G is the graph obtained f om G by first creating the line graph L(G) and then forming the clique graph of the result. A characterization of such graphs is given which lends itself to the investigation of relationships between certain graph invariants of C and CL(G) and to some isomorphism questions.

A GREEDY ALGORITHM FOR A PROJECT SCHEDULING PROBLEM

@

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In this paper we show that a project scheduling problem with continuously discounted cash flows has its optimum solution at an extreme point of the convex polyhedron formed by the activity precedence relationships. This result is then used to develop an optimal greedy algorithm to solve the problem in less than n^2 pivots, where n is the numbers of nodes in the project network.

OPTIMAL SOLUTIONS TO CAPACITATED MULTIDEPOT VEHICLE ROUTING PROBLEMS

G

by Gilbert LAPORTE, Danielle ARPIN, Ecole des Hautes Etudes Corraner-ciales de Montreal, 5255 avenue Decelles, Montreal H3T 1V6, Canada
and Yves NOBERT, Departement des sciences administratives, Universite du Quebec a Montreal, 1495, rue St-Denis, Montreal H3C 3P8, Canada

This paper provides an integer line r prograrraning formulation and an exact algorithm for a class of multidepot vehicle routing problems. This formulation contains degree constraints, subtour elimination constraints, chain barring constraints and integrality constraints. Integrality is obtained by branch and bound; subtour elimination and chain barring constraints are first relaxed and only imposed when they are found to be violated. Computational results are reported.

The Incidence Algebra oft-Designs with Automorphisms

G)

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Abstract

At-design (or generalized Steiner system) S(A;t,k,v) is an incidence structure (X, 8) **with** v points X and b blocks 8, such that each block is incident with exactly k points and in which any t points are simultaneously incident with A blocks. It has a group G a s an automorphism group if G i Sym(X) and G preserves 8. In this paper we study the P(X) by P(X) matrices over the rationals Q which are left invariant under the natural action of G on P(X). By doing so we obtain generalizations of the identities which Wilson has found among the higher incidence matrices of an S(A;t,k,v) and derive yet a further extension of Fisher's inequality.

CONSTANT WEIGHT CODES AS EXTENSIONS OF RESOLVABLE BIBD'S

Michael A. Wertheimer
Department of Defense

Abstract

Upon the removal of a distinguished subset of points from the affine plane of order 2' it is possible to construct a resolvable balanced incomplete block design with parameters (v, k,)., r, b) = ($2^{2''}$·1- 2''·1, 2''·1, 1, 2' + 1, 22' - 1). Each such resolvable design can be extended to construct two optimal constant weight codes, A(n, 26, w), with 0 = *w* - 1 These codes solve the "packing problem" which seeks the largest number of w-subsets of an n-set with the property that any pair of objects occurs in at most one w-subset.

IllTERSEC'IIOH PRESERVING EMBEDDINGS OF PARTIAL MENDELSOHN TRIPLE SYSTEMS

Rose C. Hamm, College of Charleston, SC

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A cyclic triple {(a,b),(b,c),(c,a)} is a collection of three ordered pairs of distinct elements in which each of the ordered pairs represents a directed edge in the above directed triangle.

A Mendelsohn triple system (MTS) is a pair (S,T) where S i s a set and T is a collection of cyclic triples with the property that every ordered pair of distinct elements of S belongs to exactly one cyclic triple in T.

First we show that any pair of partial MTSs of order *v* can always be embedded in an intersection preserving pair of triple systems of order u for every admissible u 6v+3. The obvious question to ask now is whether or not an arbitrary collection of MTSs can always be embedded in an intersection preserving set of MTSs. We will show that **such a collection has infinitely many such embeddings.**

Coronation Numbers of Trees

R. E. Jamison[1] and M. W. White, Clemson University

A partially ordered set P has coronation number c iff for any subset S of P, if any c points in Shave an upper bound, then S has an upper bound. Any tree (with subtrees as convex sets) forms a convex geometry (alias antimatroid or APS shelling structure). This paper determines the coronation number of the poset of subatomic extensions of a tree in the lattice of convex geometries. A subatomic extension is one contained in a geometry obtained from the underlying tree by adjoining a single new shelling. order.

Posets Described By Their Bound Sets

John K. Luedeman[1] and F. R. McMorris
Clemson University Bowling Green State University

Let (P,) be a finite poset. The upper bound set %-(P) = {lu(p) I : p £ P} with U(p) = {x £ Pl x > p}. The lower bound set P) is similarly described. The double bound set.@(P) = -- --- {lo(p) I: p £ P} with D(p) = {y £ P I y r y < p }.

Let s1 and s2 be sets of positive integers.- There is a poset (P,) with s1 = (Pl and s2 =Y(P) iff 1 £ s1 *n* s2.

Let μ(S1;s2) be the minimum number of elements in a poset (P,.:l with s1 =%-(P), s2 =Y(P). We determine bounds on μ(S1;S2) for general (P,.:) and determine μ(s1;s2) when (P,2) is self-dual and s1 = s2.

Let μ(S) be the minimum number of elements in a poset (P,<) with S =9'(P). We determine μ(S) for certain sets S.

GAMES ON TREES

A. Meir and J.W. Moon[1]
University of Alberta

'Suppose two players take turns moving a marker away from the root of a finite tree, one edge at a time; when some player cannot move, he loses and the game is over. Let p(n) denote the probability that the first player to

move wins in a game played on a tree Th chosen at random from then-vertex trees in a family F of rooted trees. We determine the limiting value of p(n) for suitable simply generated families F when the players follow certain types of strategies. For example, suppose p is the radius of convergence of the generating function y(x) for trees in F and ' = y(p); if both players move at random then p(n) + {1 + (p,t2,-p))^2}/2 as n + ∞ .

HEIGHTS OF MONOTONICALLY LABELED BINARY TREES

G)

A. M. Odlyzko
AT&T Bell Laboratories
Murray Hill, New Jersey 07974

Monotonically labeled binary trees are binary trees with internal nodes labeled with 1's and 2's in which a node labeled with 1 is never the son of a node labeled with 2. Kirschenhofer and Prodinger have shown that the average height of such trees with n internal nodes is

; as n -,."'

A simple proof of this result and some extensions of it will be provided.

Optimum Communication Spanning Trees in Chordal Graphs

O

Ehab S. El-Mallah
University of Saskatchewan

Charles J. Colbourn [1]
University of Waterloo

The optimum communication spanning tree problem is to locate a spanning tree which minimizes the sum or the lengths or the shortest routes between all pairs or nodes in a graph, weighted by trarric requirements. Although NP-complete in general, this problem has an efficient solution for series-parallel grnphs when all requirements are equal. We extend this efficient solution to chordal graphs with bounded clique size; our strategy exploits the recursive structure or chordal graphs to deviu a recursive method whoee running time is exponential in the size or the largest clique.

Title : An. O(n^2 log n) E::,)cxted Tine Algorithr.i for the All-m Pairs Shortest Path Probleia over a Certain Class of Grarhs.

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iq1stract: Spira ,:;2ve an alyorithm for the all pairs c,,,ortest µath proble (abbreviated as hLLPAIRS) 11ith O(n^2 loy^1 n) e pectea running time. In tilis al<Jorithm, the outgoing ed(JeS fror,1 each verte are sorted, which takes O(n^1 log n) time over n vertices. Then the single source problem is solved in O(nlog^a n) expected time n times by changing the sources, which takes O(n^1 log^{2}n) e:-:;,ected tine.

This algorithra was improved by Bloniarz, and Takaoka and lioffat. In both ir:1proved methods, the presort technique is used. The overall expected times are O(n^2 log nlog*n> and O(n^1 log nloglogn} respectively.

In the present pa:;_Jer, S;:>ira's al')orithr.1 is iE1,?roveC:. by a different approach from the above t\lo. lie: .::accumulate the information obtained by earlier single source problews. This accui'.lulation of information will save the conputing ti:.1e for later single source pro lems. Ne can establish O(n^2 log nl e;::->ectecl computing tine for ALLPAIRS by this r:iethod.

An algorithm for finding the generalized incidence matrices for group actions.

Leo G. Chouinard II. University of Nebraska - Lincoln.

If G acts on the set S' one technique for determining the t-designs on S fixed by G uses the matrices Atk whose rows are indexed by orbits oft-sets under G, and whose columns are indexed by orbits of k-sets, defined as follows: If ,1 is an orbit oft-sets and r is an orbit of k-sets, the (4,r) entry of Atk is the number of B r containing each T Gd. An algorithm which calculates these numbers by generating a tree-like structure is described. The structure embodies information which also allows the calculation of both the orbit representative for any subset of S, and an element of G taking the subset to its orbit representative. Its efficiency is compared with other algorithms in use for this purpose.

Metric, Cometric Association Schemes

Douglas A. Leonard, Auburn University

If the parameters of an association scheme satisfy the triangle inequality then the scheme is called metric and gives rise to a distance regular graph. If its dual parameters do it is called cometric. If it is both, then all its parameters can almost be given in closed form in terms of pijk' i, j, k, < 3.

Existence for holey SOLSSOHs of type 2^n

L. Zhu

Department of Combinatorics & Optimization

University of Waterloo

Abstract

R.C. Mullin and D.R. Stinson have proved that there exist holey SOLSSOHs of type 2^n for all odd n > 3 **with** the possible exceptions n = 15, 33, 39, 75 and 87. They have also conjectured that holey SOLSSOMs of type 2^n exist for all even n 6. In this paper we construct holey SOLSSOMs of type 2^n for odd n E {39, 75} and for all even n **46 with** the possible exception of v E {48, 50, 52, 54, 58, 62, 68, 72, 74, 76, 80, 84, 88, **108, 114}.**

Generalized Quadrangles with t = s + 2

Stanley E. Payne, Miami University, Oxford, Ohio

A catalogue is compiled of all known models of generalized quadrangles of order (s, t) with t = s + 2. Isomorphisms between these models are discussed and some characterization theorems are given.

Closed Neighborhood Containment Graphs

Margaret B. Cozzens and Ding-I Wang, Northeastern University

If G = (V,E) is a graph, then the closed neighborhood containment graph, denoted N(G), has vertex set equal to V, and {x,y} G E#[N(x) N(y) v{y} or N(y) N(x) v{x}]. N(G) is complete if and only if G is a threshold graph. N(G) can be computed in linear time given the adjacency lists for G. We characterize N(G) for various classes of graphs, including vertex transitive graphs, complete multipartite graphs, and various types of trees. Thus N(G) can aid in determining if a graph is in a particular class.

Closed neighborhooa containment graphs are comparability graphs. Therefore, many parameters for these graphs are easily calculable in polynomial time. We show that the domination number of G is bounded by the vertex clique number of N(G), and that this bound is sharp.

The Chromatic Index of Nearla Bipartite Multigraphs

Larry Eggan and Michael Plantholt, Illinois State University

A rule is given for determining the chromatic index of any multigraph which contains a point whose deletion yields a bipartite multigraph. Some related problems on edge-coloring and multigraph arboricity are also discussed.

Hamiltonian Cycles in Random Graphs

A. M. Frieze, Carnegie-Mellon University, Pittsburgh, PA 15213

There has been a good deal of progress recently on determining when a random graph almost surely contains a hamiltonian cycle. We review this progress and point out some open problems.

ESTIMATING THE CHROMATIC NUMBER OF MODERATELY LARGE RANDOM GRAPHS

David W. Matula! Southern Methodist University
Abhai Johri, A. T. and T.

We describe the application of probabilistic, heuristic, and algorithmic techniques to obtain reliable estimates of the chromatic number of random graphs having several hundred to several thousand vertices. Specifically we develop improved probabilistic lower bounds for the chromatic number of a random graph. We introduce an heuristic estimation formula for the chromatic number of a random graph and a greedy paradigm for a family of coloring algorithms. The proposed algorithms are compared with several heuristic algorithms from the literature and shown to significantly reduce the number of colors required for coloring large random graphs. Our results provide substantial evidence that the chromatic number of a random graph should likely fall in a very narrow range in close proximity to our chromatic number estimate. For the benchmark random graph on one-thousand vertices with edge probability one-half our results indicate that the chromatic number will (with high probability) be in the range 85 ± 12.

ON THE PROBLEM OF BANDSIZE

K. Heinrich and P. Helli, Simon Fraser University, Canada

We introduce the notion of bandsize in analogy to bandwidth, as the smallest number of edge-differences over all vertex-numberings. We make sever ervatio ns which allow us to estimate the bandsize of complete binary trees. It follows from our results that the bandsize of the complete binary tree of height n is between c1n and c2n where 0<c1<c2<1. This is in sharp contrast with the bandwidth of these trees which is roughly $2^n/n$.

Bandwidth minimization for refinements of caterpillars

Zevi Miller, Miami University

Let G be a graph and f: V(G) + ZZ a one-one map. The bandwidth B(G) of G is defined by $B(G) = \min_f \max\{|f(v) - f(w)|: vw \in E(G)\}$. This parameter is connected with the efficient storage of matrices and operation of systolic systems. The problem of given G and k whether B(G) < k is NP-complete, even when G is a tree of maximum degree 3. We describe a polynomial time algorithm for finding bandwidth in a certain class of trees, obtained by refinement of caterpillar graphs. We also show that the bandwidth problem for arbitrary refinements of caterpillars having three points of degree greater than two is NP-complete.

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GRAPH-THEORETIC MODELING OF CELLULAR DEVELOPEMENT: A HYPOTHETICAL EXAMPLE EMBODYING DEATH

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and

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ABSTRACT

Sequences of graphs $G_0, G_1, \ldots$, in which G_i generates G_{i+1} according to some simple rules have been used successfully by Lindenmayer and others to model the morphogensis of plants such as ferns. The rules for obtaining growth stage G_{i+1} from G_i generally have been determined solely by the reproductive characteristics of the cells, i.e., the death of cells was not considered. Here we introduce graph sequences $G_0, G_1, \cdots$ governed by the following rules of growth:

i) $G_0 = K1$

ii) each cell (vertex) m of G_i produces one daughter cell d and $N(d) = N(m) \cup \{m\}$

iii) the positive integer x is fixed and a cell with degree x or more dies.

An extension to an algebraic approach to gra h isomorphism

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Abstract

Some invariants for distinguishing non-isomorphic graphs can be computed in polynomial time by considering an undirected graph as a physical object of point masses and elastic edges. These invariants can distinguish several pairs of graphs with an equal number of nodes and edges.

However, the simplest technique produces invariants which are not complete. That is, a computer program found a pair of non-isomorphic graphs which produced the same invariant.

Extensions to the simple technique for computing invariants are numerous. It remains to be seen whether the extensions can still be computed in polynomial time to provide a complete set of invariants for graph isomorphism.

Petersen subdivisions in some regular graphs

M. N. Ellingham
University of Waterloo

Tutte has conjectured that every 2-connected cubic graph not containing a subdivision of the Petersen graph is 3-edge-colourable, a result which would be a strong form of the Four Colour Theorem. This is shown to be true for graphs which have a 2-factor consisting of two chordless cycles. General r-regular graphs with a 2-factor of this kind are also discussed; if they have no Petersen subdivision they are hamiltonian and r-edge-colourable.

New Methods to Construct Minimal Broadcast Networks

Siu-cheung Chau[1] Ir Arthur L. Liestman
Simon Fraser University

Broadcasting is the process of information dissemination in a communication network in which a message, originated by one member, is transmitted to all members of the network. A minimal broadcast network (mbn) is a communication network in which a message can be broadcast in minimu time regardless of originator. A minimum broadcast graph (mbg) is a mbn which has the fewest number of communication links. No technique is known for constructing mbgs of abitrary size. We present new r:iethods for constructing r.ibns which have approximately the minimum number of links possible. The resulting networks often have fewer links than previously described networks of this type.

SUBGRAPHS WITH SHORT CYCLES

Richard Duke Georgia Institute of Technology, Atlanta
Paul Erdos, Hungarian Academy of Sciences, Budapest
Vojta Rodl, Czech Technical University, Prague

At an earlier Southeastern Conference (1982) two of us considered, for a graph with many edges, the possible size of a subgraph in which each pair of edges lie together in a short cycle. It was shown that for $G(n,cn^2)$ with n vertices and cn^2 edges, ca positive constant, there is a positive constant c' and subgraph H with c'n edges such that each pair of edges of Hare in a cycle of Hof length 4 or 6. (We may also require that two edges with a common end point are on a cycle of length 4.)

Here we study a similar question for graphs with cn^2-£ edges. Let fk(n,£) be the largest integer such that each $G(n,cn^2-£)$ has a subgraph H with fk(n,£) edges, each pair lying on a cycle of len th S2k in H. In general fk(n,c) 5 cn^{2-2c}. We show that fk(n,c) cn^{2-3} and that this result is the best possible for k=3, but do not yet know more when k>3. We also consider the case in which each pair of edges with a common end-point are required to lie in a cycle of length 2k-2.

Containment of Elementary Geometric Objects

J. Sack*

Nicola Santoro*

and

Jorge Urrutia*+ I

Given two simple poligons, Oi , Ok , Ok is said to fit into Oi if it can be moved by translations and rotations in such a way that it lies completely inside 01 . Consider the following problem: It is possible to characterize ge001etrical objects Ok by sets of parameters O(i) , l < i < m , in such a way that the containment problem is equivalent-to vector.dominance; i.e. 01 fits into Ok if and only if $P_1(i)$ 2 Pk(i) , l < i 2 m ?

In [l] an interesting resuTt was established: even in the case of rectangles, no such set of parameters can be found regardless of the size of m .

In this paper we continue this investigation by presenting positive results for some cases of elementary geometrical objects. For example, if 0 is a rectangle and Ok a square, the containment problem can be solved by using vector dominance.

We also study some properties of the partial order defined on the plane as induced by the containment relation in rectangles. For example the dimension of this partial order is not finite.

[1] N. Santoro, J.B. Sidney and J. Urrutia. "Geometric Containment is not Reducible to Peano Dominance". SCS-TR-37, Carleton University.

* School of Computer SCience, Carleton University, Canada

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Getting Your Chickens Elected

Keith Wayland
University of Puerto Rico
Mayaguez, P.R.

Brooks Reid has shown that an n-tournament T without a transmitter can be embedded in an m-tournament W such that the kings of W a r e exactly the vertices of T. Moreover, he has shown that m need be at most 2n. In this paper a construction for W i s given such that
m i (log (n+l)} /2 +5(log (n+l)}/2 - 4.

Max and Min Relationships of Disconnected Colorings

W.R. **Haret**
Renu Laskar
Ken Peters
Clemson University

Let G=(V,E) be an ordinary graph. A partition of V into $V_1, V_2, \ldots, V_k$ is called a disconnected coloring of G if for each V_i, the induced subgraph (V_i) is either disconnected or a K . The minimum number k f6r all such partitions, denoted d(G), is defined to be the disconnected coloring number or G. The maximum number k such that each $\langle V_i \rangle$ is disconnected or a K1, while $(V_i \cup V_j)$ is connected for each pair ilj, will be defi ed to b§ ttte maximum disconnected coloring, denoted by X: (G). Some properties of these two parameters and their rglations with other max and min parametersl of graphs will be discussed.

A Group Theoretic Classification of Polynomial Sequences

J.M. Free!"lan

Florida Atlantic University

A substitution is a linear, degree preserving map on polynomials. These form a group S. Basic, Euler, binomial, and Sheffer sequences correspond to substitutions belonging to certain subgroups of S, denoted S_0, E, B, and S_1. These are characterized and interrelated group theoretically by means of the twist and factorial maps. We show that S_1 is the normalizer of E in S, and the semidirect product of E and B, whereas B is the normalizer of E in S_0. This is applied to generating functions.

COLOURING STEINER SYSTEM.$

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A Steiner system (X,B) , denoted S(T,k,v) , is av-set X of vertices and a collection B of k-subsets of X called blocks. A Steiner system may be coloured either by colouring the vertices, with the requirement that no block be monochromatic, or by colouring the blocks so that different blocks of the same colour are disjoint. This article surveys vertex and block colourings of Steiner systems.

Strategies for a Multi erson Contest with Sequentially-Overlapping Stochastically-Independent Events

Walter W. Funkenbusch, Michigan Technological University

In a succession of three tosses of a die, each die outcoae being classified as even or odd, any of the eight possible results is equally likely to hap en. However, if **we** consider sequentially-overlapping **sets** of **three tosses**, then for any of the eight choices, there is at least one **from aaong** the other seven which will likely occur before that designated choice.

This is an already established fact. **(See** M. Gardner, Scientific American Oct. 1974 and W. Penny, Journal of Recreational Mathe■atics Oct. 1969). Gardner and Penny consider only a two person contest each person having only one choice.

This paper asks and answers the query "how ■any choices can I perait to occur before making my choice(s) **from** among those as yet unchosen and still enter last with an advantage?" (each choice requires a unit wager) and also which choice(s) should I raake?

The co binatorics require the comuutiTT of the probability of winning (i.e. occuring firs for each choice when competing within §WY .§.tl of n choices: n: 2, 3,4,5,6,7. This is not a trivial calculation and was done by Mrs. Mary Kerfoot on a Univac 1110 -at Michigan Technological University.

K Best Bases of **Matroids**

Horst W. Hamacher, Department of Industrial and Systems Engineering, University of Florida, Gainesville, Florida 32611

In a matroid M=(E,l)) with element set E and basis setl> the problem of finding a best basis D can easily be solved by the Greedy algorithm. We show how for a give positive integer K the K best bases Dl, ...Dk can be found. By using an algebraic approach we are able to develop a single algorithm for ranking the bases according to different objective functions such as sum objective and bottleneck objective.

Non-binary 3-connected matroids

James G. Oxley
Mathematics Department
Louisiana State University

It is well-known that a matroid is binary if and only if it has no minor isomorphic to u2,4, the 4-point line. Extending this, R.E. Bixby proved that every element in a non-binary connected matroid is in a u2,4-minor. For 3-connected non-binary matroids this was further strengthened by P,D. Seymour who showed that every pair of elements in such a matroid is in a u2,4-minor, This talk presents several results for 3-connected matroids, the main one of which extends Seymour's theorem by showing that if {x,y,z} is contained in a non-binary 3-connected matroid M, then either M has a u2,4-minor using {x,y,z}, or M has a minor isomorphic to the rank 3 whirl that uses {x,y,z} as its rim or as its spokes,

The Initialization of Variables. and Problems of Forward References: a Graph-theoretic Approach

Hans-Jurgen Steffens, IABG Ottobrunn

Abstract

The high level language CHILL allows forward references between variables which are initialized at their declarations. This implies the danger to construct cyclic definitions. In some cases those "cycles" can be detected at compile-time. In other cases it is not possible to decide at compile-time whether there will be "cycles" at run-time or not. In order to avoid a run-time-overhead it is, however, desirable to detect and exclude such cases at compile-time. Therefore appropriate conditions for the use of forward references must be stated and algorithms must be developed which can examine those conditions.

In the paper the conditions will be specified with reference to a directed graph. Also an algorithm will be presented that checks these conditions by transversing the graph,

DISTRIBUTED ALGORITHMS FOR NETWORK RECOGNITION PROBLEMS

(Abstract)

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Distributed Algorithms for Graph problems are motivated both by the necessity to deal with dynamic reconfigurations of networks and, due to their decisive role in obtaining insights into the communicational structures of problems. As in the sequential case, simpler and less expensive distributed algorithms can be designed when the network has a restricted structure. In several applications, it may be worthwhile to preprocess the network to see if it has such a structure, before using a complicated algorithm. In this paper, we present algorithms to determine if a given network is a tree, mesh, ring, star, complete graph, or, bipartite graph. All algorithms are deterministic, none of them use more than o(,E\) messages and the techniques used for these algorithms vary from divide-and-conquer to problem-specific tricks. Most of these algorithms are asynchronous, thus avoiding any overheads such as the phase-termination detection.

Cograpbs: recognition. applications and algorithms

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L. Stewart Burlingham
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Abstract

Cographs are the graphs formed from a single vertex under the closure of the operations of union and complement. Another characterization of cographs is that they are the undirected graphs with no induced P4s. From this characterization it is clear that cographs arise naturally in such application areas as examination scheduling and automatic clustering of index terms. Furthermore, it is known that cographs have a unique tree representation called a cotree. Using the cotree it is possible to construct very fast polynomial time algorithms for problems which are intractable for graphs in general. Such problems include chromatic number, clique determination, clustering, minimum weight domination, isomorphism and Harniltonicity. In this paper we present a linear time algorithm for recognizing cographs and constructing their cotree representation. and a number of algorithm.s which use the cotree are described.

DOMINATION ALGORITHMS ON SERIES-PARALLEL GRAPHS

S. T. Hedetniemi, Department of Computer Science,
Renu Laskarl and John Pfaff, Department of Mathematical Sciences, Clemson University

The class of series-parallel graphs has been widely studied recently. Kikuno, Yoshida and Kakuda have written a linear algorithm which finds a minimum dominating set in a series-parallel graph. A dominating set is a set D of vertices of a graph such that every vertex is either in D or adjacent to some vertex in D. A total dominating set is a set D such that every vertex is adjacent to some vertex in D. An independent dominating set is a dominating set none of whose vertices are adjacent. We modify the algorithm of Kikuno, Yoshida and Kakuda to produce linear algorithms which find minimum total dominating sets and minimum independent dominating sets of a series-parallel graph;

A Class of Maximal L-matrices

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John S. Maybee; University of Colorado; Boulder, CO 80309

Abstract

A class of L-matrices called maximal L-matrices is defined. We show how to associate with these matrices corresponding maximal digraphs and signed digraphs. We then investigate the connection between maximal L-matrices and trees. In particular, we show how to construct a maximal digraph corresponding to each caterpillar and establish the form of the resulting L-macrix. Finally, we show that our construction will not work if the tree T is not a caterpillar.

SIGNED GRAPHS OF NETFORMS

Harvey J. Greenberg• University of Colorado at Denver
J. Richard Lundgren, University of Colorado at Denver
Johns. Maybee, University of Colorado

This paper applies the theory of economic correlation and signed graphs associated with rectangular matrices to the special class of netforms. We show how the signed graph inversion method can provide a basis for incorporating artificial intelligence into computer-assisted modeling. Netforms and transportation matrices are characterized in terms of signed graphs, and an algorithm is given that determines when two signed graphs are compatible as models of the economic correlation in a netform.

TITLE:ON THE MATRIX CONGRUENCE UAV:A (MOD P^n).

AUTHOR: Timothy P.Donovan, Ph.D.
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ABSTRACT: In this paper the solutions to the congruence UAV=A (mod p") are counted for general U,V as well as for invertible U,V. The motivation is the desire to count the incongruent matrices (mod p^n) having a given canonical fonn. This canonical form was developed in an earlier paper published in the 'Annali di Matematica'.

An Efficient Implementation of a Planar Mesh Plotting Algorithm For Modelling Symmetric Gaussian Elimination

David R. McIntyre

Department of Computer Science
Cleveland State University

The use of graph theory has long been useful for both the analysis and comp rative study of various orderings for sparse symmetric Gaussian elimination. We present an efficient implementation of a plotting algorithm for a planar mesh model of symmetric Gaussian elimination which precisely reflects the zero-nonzero structure of the part of the matrix remaining to be factored. The asymptotic time and space complexities of the algorithm are proved. The algorithm is then shown to be asymptotically optimal.

A VLSI SYSTOLIC ARRAY FOR FAST BAND MATRIX TRIANGULATION

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A high speed systolic comp ting array realizable in VLSI (Very Large Scale Integration) circuit technology iS developed for fast triangulation of large scale, band form, linear equation systems. By combining parallel and pipelined concepts, the computing structure can triangulate an augmented matrix {A : E)Nx(N+1) where is in band form or can be permuted a priori to minimum band form, to upper triangular matrix {Q ; .s! } in 9(N) time steps compared to O(N2) in today's main frame computers.

The new algorithm incorporated with advanced semiconductor technology will likely represent the dedicated architecture of next generation computers in this type of large scale linear equation systems.

Measurement of the Effectiveness of Alternative Priority Rules in Network Scheduling

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Engineering Management Program
University of Southwestern Louisiana

Networks are commonly used to represent scheduling relationships which exist in a project. Either nodes or arcs are used to designate the activities which are to be scheduled. Each activity has attributes such as duration and a vector of resource requirements. It is frequently technologically possible to concurrently schedule certain activities, but there are insufficient resources to do so. Such problems can be formulated as discrete mathematical programs. However, considerations such as the availability of computing equipment and software, uncertain data, and the need for rapid response times result in the widespread use of priority rules in network scheduling. For example, "schedule the shortest activity first" is a priority rule.

The effects of different priority rules upon project makespan are statistically measured via a simulation experiment. Details of the experimental design are discussed. Then linear models of the functional relationships between makespan and network attributes are presented, followed by a marginal comparison of effectiveness of alternative priority rules. The adequacy of the modeling procedure is examined, and then conclusions and inferences are provided.

Asymptotical growth of a class of random trees. G

B. G. Pittel, The Ohio State University, olumbus, OH 43210

Abstract. Consider a countable sequence of independent copies of a stationary ergodic process with a finite state space. These processes are interpreted, for example, as nonterminating, say, m-adic expansions of independent random numbers xn E (0, 1] . Three growth rules (algorithms), each of which leads to a related sequence {tn} of nested m-ary trees associated with $\{x_o\}$, are studied. Ln , the length of the path from the root of tn to its node labelled xn is shown to grow, in probability, as lnn/h , where h is the entropy per symbol of the generic process. Also, lim inf L /lnn , lim sup L /lnn are proved to be, almost surely, some constants c1, c2(c1 < h-1 < c2) whose v lues depend on the rule being used. The results shed light on probable behavior of the (digital) search-insertion algorithms.

Maximizing cohesive reliability in series-parallel networks@

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A common measure or network reliability is *probabilistic connectedness*, the probability that every site in a network can communicate with every other site. Studies typically assume that network links fail independently with uniform probability p and that sites never fail. Though #P-complete for general graphs, probabilistic connectedness can be measured in time linear in n, the number or links in the network, for series-parallel networks. It is remarkable that the most reliable series-parallel networks are *generalized Jans*, maximal series-parallel networks with exactly two sites or degree two. For some applications, a network may not be considered operational unless it has edge connectivity (cohesion) or at least k; for example, high cohesion may be necessary to avoid routing problems and queuing delay. We generalize probabilistic connectedness to k-cohesive connectedness and show that for two-cohesive connectedness, generalized rans are again the m06t reliable maximal seri=parallel networks.

An extremal function for contractions of graphs ®

Andrew Thomason

Louisiana State University

The function c(p) is defined for positive integers P 2 by

c(p)=inf(c; e(G)2c\G\ implies **G**) KP) •

where > denotes contraction. Random graph examples show c (p) >0.265p;(log2p) (1 +o (1)). In 1968 Mader showed that c (p) 8 (p- 2)L log2 (p- 2j and more recently Kostochka showed that pJ(log p) is the correct order for C (p) • we present a simple argument showing c (p) 2.68p/(log2p) **(1** +o **(1))**.

LOCAL MOTIONS @

R. B. Killgrove, U. So. Carolina-Aiken, D. I. Kiel, Cal. State Los Angeles, R. W. Sternfeld, u. So. Car.-Aiken

Let A,B,C,D be a quadrangle in any projective plane and E=AB'!CD, F=ACnBD, G=AD BC be its diagonal §Oints, then a local motion fixes A, sends B to E,C to F,D to G. Local motions are linkable in Fano-free planes. In such Desarguesian planes these motions extend to global ones. In ordered planes (Pano-free) let A be interior to BCD. The sequence of diagonal triangles formed by repeated application converge to fixed point A in Pappian ordered planes iff the field is Archimedean. Thus we define Archimedean ordered planes as those with such convergence.

The Fano configuration is contrary to Fano•s axiom. We define Coxeter's configuration to be contrary to Cor. 2.42, Real Proj. Plane, namely, two quadrangles sharing a vertex and diagonal points. Finite Coxeter-free, Fano-free planes have odd order. We are in t,le midst of a computer calculation either obtaining a new order 9 plane (unlikely) or showing such order 9 planes are Desarguesian.

Title: Planes of Prime Order with Translations.

By: A.B.Evans[1] and R.Mcfarland.

Abstract: It has long been con ectured that any affine plane of prime order must be Desarguesian. In this paper, we study affine planes of prime order that admit translations.

We define a graph whose vertices are permutations of [1 ,... ,p-1), two permutations being adjacent if their difference modulo p is also a permutation of [1 ,... ,p-1). Let G be the subgraph induced by the identity permutation and all permutations adiacent to it.

The search for non-Desarguesian lanes of prime order, admitting translations, is reduced to the search for p-1 cliques in G.

We shew that, if p=11, there is only one p-1 clique in G, and deduce that no non-Desarguesian plane of order 11, admitting trans--lations, can exist.

CONSTRUCTIONS OF FINITE PROPER PROJECTIVE PLANES FROM GROUPS

Julia M. Nowlin Brown, York University

Let G be a finite group and let m be a positive integer. We give necessary and sufficient conditions for the existence of a finite projective plane n having G as a collineation group acting with exactly m point orbits. The conditions are given in terms of a pair (Z,H) where Z is an m x m matrix with nonnegative integer entries and H is an m x m matrix whose entries are subsets of the group G. This representation is a generalization of both the usual incidence matrix representation of a finite projective plane and the difference set representation of a plane.

We describe constructions of all known finite proper projective planes in terms of our conditions using the projective general linear groups PGL(3,q) . (A proper projective plane is a projective plane whose automorphism group fixes no point or line.)

CN 'IHE RAMSEY N1MBER FOR 'IHE FIVE SFOKED l'liEEL

Ralph Fatrlree
Menphis State University

It was conjectured by Paul Erdos that the Ramsey mmnl::er r(G) for any graph G sa.,tisfies the strict inequality rCG) > r(Kn), where n = CG). This has been verified for n = 3. 'Tbe conjecture for n = 4 reduces to showing that r<W_6> > 18, where w_6 represents the five spoked wheel. Chvatal and Schwenk showed that 17 r(W6) 20. In this paper it is shown that r<W6) 19, and good colorings of K18 are oonsidered.

Shortest Even Cycles

R.P. Anstee, University of Waterloo/University of British Columbia

An elementary algorithm .is presented to find a shortest cycle of even length in a graph. Breadth first search is used in analogy to the algorithms for finding shortest odd cycles and the girth. Edmonds had previously solved this problem (and a more general one) using an auxilliary graph and the weighted matching algorithm. The algorithm presented runs in 0(/Vl·IE/J.

M.C. HEYDEMANN, D.SOTTEAU[I]

L.R.I. , Bae. 490, Universice Paris-Sud, 91405 ORSAY Cedex FRANCE

ABSTRACT: In this article we determine the maximum number of arcs of a strong d graph of order n,. without cycle of length at least n-k, for n k^2+2k+5 ; thus we partially solve a conjecture of J.C. BERMOND, A. GERMA, M. C. HEYDEMANN and D. SOTTEAU.

We give a conjecture concerning the maximum number of arcs of a strong digraph of order n, of minimum half-degree r, without cycle of length at least n - k, for n large. We prove it for r = 2, k I or k = 2.

The talk will include a brief survey on the subject.

COUNTING RESTRICTED PERMUTATIONS

Frank Schmidt, Mathematics Department
Southern Illinois University, Carbondale, IL 62901

We are concerned with counting permutations which do not contain certain subsequences. The number of even and odd such permutations is found and the involutions among them are counted. Bijections are determined between sets of such permutations and other combinatorial objects. These results complement previous work by Knuth, Lovasz, Rotem, and Stanley.

ITERATIVE ROOTS OF PERMUTATIONS OF FINITE SETS

by

Corina Reischer Universite du Quebec a Trois-Rivieres, Canada
Dan A. Si.movici, University of Massachusetts at Boston, USA

ABSTRACT

Let f, g two transformations of the same finite secs; g is an iteration root of order m of f if $g^m = f$, where the power is considered with respect to function composition.
We reduce the problem of the existence of iteration roots of permutations to the consistency of a diophantic system of equations related to the graphs atcac ed to functions.
The main result of the paper is a characterisation of permutations of finite sets admitting iterative roots. This is an extension of one of our previous results (Proceedings of the Thirteenth International Symposium on Multiple-Valued Logic, May 1983, Kyoto, Japon, 183-187).

Random permutations and Brownian motion.

J.M. Delaurentis[I], Sandia National Laboratories, Albuquerque, New Mexico 87125;
B. G. Pittel, The Ohio State University, Columbus, OH 43210

Abstract. Consider the cycles of the random permutation of length n. Let $X_n(t)$ be the number of cycles with length not exceeding n^t, $t \in [0, 1]$. The random process $Y_n(t) = (X_n(t) - t\ln n)/\ln^{1/2} n$ is shown to converge weakly to the standard Brownian motion $W(t)$, $t \in [0, 1]$. It follows that, as a process, the empirical distribution function of "loglengths" of the cycles weakly converges to the Brownian Bridge process. As another corollary, an alternative proof is given for the Erdos-Turan Theorem: it states that the group-order of random permutation is asymptotically e^{U}, where U. is Gaussian with mean $\ln^2 n/2$ and variance $\ln^3 n/3$.

CARDINALITIES OF EQUIVALENCE RELATIO S

B. D. McKay, Australian National University and
R. W. Robinson[1], Southern Illinois University

Let T(et) = rzl) + ··· + r.J for any partition a with r parts, and let

f(k) = Min {p: k = T(a) for some et p}.

For given m >n, there is an equivalence relation on an n-set having cardinality m (as a set of ordered pairs) if, and only if, (m-n)/2 is integral and f((m-n)/2) ≤ n.

Clearly f(k) = b (k) + f(k- C b £)) where b(k) is the largest part in any partition a f—f(k) with T(a) = k. If r(k) = Max {r: k ( )}then b (k) r(k) for all k 1. It is shown that, in fact, b (k) = r(k) holds with finitely many exceptions. These exceptional cases are enumerated explicitly, thereby giving a very efficient general method for computing f(k).

THE COMBINATORICS OF RECORDS

G

Heinrich Niederhausen, Department of Mathematics, Arizona State University, Tempe, AZ 85287

A record (or a "high") is a term in a sequence which is not less than any previous term. We calculate the number of sequences with r records, or withs strict records, or with h highs and 1 lows. The enumeration depends on the type of sequence. Such types are arising from bijections [n] [n], or from injections [nJ [k), n, or from surjections [nJ ....[d], d n. The enumeration follows from generating !unctions, using the theory of species (Royal). Applications occur in testing **for** independence in a sequence of observations.

Trees with 1-factors

B

Rodica Simion, Southern Illinois University

A matched tree is a tree which admits a 1-factor. Characterizations of matched trees are given and bijectio s are constructed between matched trees and certain oriented trees. The degree distribution of labeled matched trees is described. Generating functions and asymptotics are obtained for the number of rooted, planted, unlabeled and identity matched trees.

The Maximum Number of Edges in an Oriented Graph without Long Directed Paths

8

Madeleine Paoli

and

William T. Trotter, Jr.[I]

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An oriented graph is strongly connected if for every pair (x,y) of vertices, there exists a directed path from x toy. The length of a directed path xO, xl, x2, ..., xt is t, the number of edges in the path. For positive integers n and t with n ≥ t - 1, let H(n,t) denote the maximum number of edges in a strongly connected oriented graph on n vertices which contains no directed path of length t. M. C. Heydemann gave an exact formula for H(n,t) when t is relatively large in comparison with n, i.e., when n < 2t - 2. She also gave a conjecture for H(n,t) when n > 2t - 2. In this paper we settle Heydemann's conjecture in the affirmative. Our results show that H(n,t) always equals the number of edges in a complete multipartite graph although not all parts in this graph have equal size.

On the Recursive Chrimatic Number of Recursive Strongly Oriented Graphs

by Henry A. Kierstead, University of South Carolina; James H. Schmerl, University of Connecticut; Endre Szemeredi, Mathematics Institute, Hungarian Academy of Sciences.

An oriented graph G is strongly oriented provided that it is acyclic and it contains no induced subgraph of the form

 or  .

This is a strengthening of Chvatal's notion of perfectly ordered graph. G is a recursive oriented graph if there is an algorithm which determines whether or not a given vertex or directed edge is in G. We give best possible bounds on the recursive chromatic number of a recursive strongly oriented graph G in terms of the clique size of G. This result generalizes earlier work of Schmerl on the number of recursive antichains required to cover a recursive poset. This proof leads to a new lower bound on the number of recursive chains required to cover a recursive poset.

Message Authentication: A Game on Hypergraphs

Gustavus J. SilTlnOns, Sandia National Laboratories, Albuquerque, NM 87185

In 1978 the present author showed that the problem of verifying that a message was indeed sent by the purported transmitter and has not been subsequently modified could be modeled in complete generality by replacing the classical noisy communications channel with a game-theoretic channel in which an opponent "plays" so as to optimize his chances of deceiving the receiver. In subsequent papers it was shown that the probability that the receiver would be deceived, PA, could be bounded below by any of several expressions involving the entropy of the source, H(S); of the messages H(M); of the encoding rules, H(I); and of the choices, if any, of messages in an encoding H(MiIS). For example, one such expression is;

$$P_A \geq 2^{H(S)-H(M)+H(M|IS)} \qquad (1)$$

An authentication system in which equality held in (1) was defined to be perfect.

In this paper we show that authentication systems can also be modeled as a game played on hypergraphs in which vertices correspond to messages, hyperedges to encoding rules and vertex labels to source states. For any particular labeled hypergraph (authentication system), the transmitter/receiver move first by secretly chosing a hyperedge. The opponent wins if he successfully chooses an "acceptable" vertex in the hyperedge chosen by the transmitter/receiver. A system is said to be Cartesian or not according as to whether or not it can be represented as a Cartesian product of a partition of the vertices with a labeling of the parts. We first show that for any source, S, and message set, M, PA = ISI/IMI. In addition we prove that for any S and M there exists a perfect non-Cartesian system for which PA = ISl!IMI. We also show that every strongly 1-regular k-partite hypergraph on n = kt vertices corresponds to a perfect Cartesian authentication system with ISi = k, IMJ = n for which PA= ISl!IMI = 1/t.

A Few Results in Message Authentication

Ernest F. Brickell, Sandia National Laboratories, Albuqueruqe, NM¥iss

In message authentication, a transmitter is given a source set S and a probability distribution on S, a set of messages M to be used for transmitting the state of the source set, and a set, I, of encoding rules that determine which messages will be accepted by a receiver as authentic. For a probability distribution on I, let PA be the probability that deception can occur. We will use H(X) to denote the entropy of a set X. We will show that H(IjM) - H(I) = H{S) - H(M) + H(MjIS). Combining this with G. J. Simmons' bound that PA.:: 2H(S)-H(M)+H(MiIS), we show that PA.:: 2-lsH(I)_ This bound was proven by E. Gilbert, F. Macwilliams, and N. Sloane under slightly stronger conditions. We also show that if PA= lsl/lMI, then H(MIIS) = 0. We will demonstrate that group divisible designs with Al= 0 and A2 = 1 can be used to construct perfect authentication systems which also satisfy PA= 2-lsH(I)_

COUNTING CERTAIN CLASSES OF EVOLUTIONARY TREES

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R. W. Robinson, University of Southern Illinois

Evolutionary trees of biology are represented by a special class of labelled trees. They are characterized by having disjoint subsets of the labelling set assigned to the vertices of a tree in such a way that no pendant vertex is assigned an empty set of labels. The exact and asymptotic numbers of evolutionary trees are determined under the presence or absence of three conditions on the labelling. The optional conditions studied require the trees to have singleton labels, no vertices of degree two, and that only pendant vertices be labelled. This paper completes previous work by the authors on the enumeration of evolutionary trees under all possible combinations of the conditions, including the restriction of binary trees.

GRAPHS WITH EULERIAN CHAINS

G

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Donald K. Skilton, Simon Fraser University.

ABSTRACT

An euienan *ahain* in a graph is a continuous route which traces every edge exactly once. It may be finite or infinite, and may have 0, 1 or 2 end vertices. For each kind of eulerian chain, there is a characterization of the graphs which admit such a tracing. This paper derives a uniform characterization of graphs with an eulerian chain, regardless of the kind of chain. Relationships between the edge complements of various kinds of finite subgraphs are also investigated, and hence a sharpened version of the eulerian chain characterization is derived.

HOMOGENEOUSLY REPRESENTABLE INTERVAL GRAPHS

9

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A graph is called an interval graph if every vertex of the graph can be represented by an interval of the real line such that two intervals intersect iff the corresponding vertices are joined by an edge. Such a graph is called homogeneously representable if for each vertex v of the graph, there exists a representation of the graph in which the interval representing v is the leftmost (or rightmost) interval in the representation. Theorem. An interval graph G is homogeneously representable iff G1 and G2 shown below are not induced subgraphs of G.

G1 V GZ V

Quantitative aspects of Szemeredi's theorem modulo n

O

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The multiset P = {a1, ... ,ak} is a k-term arithmetic progression modulo n if a1 f a2 (mod n) and a2 - a1 = a3 - a2 ... = a4 - a_{k-1} (mod n). For k odd and k 3, we find explicit constants ck < 1 - 1/k such that for any n f k and for any subset A of [0,n-1], if \A1 > ck n then A contains a k-term arithmetic progression modulo n. (c3 = .5 and c5 is about .77.)

On a Problem Involving Partitions

B

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ABSTRACT

Given an integer n and given s.(n), one of the p(n) partitions of n, we can dJfine L.(n) to be the Least common multiple of the parts n the partition s.(n). Denoting by L(n) the maximum value over all partitions of n of L.Cn), several natural questions arise: What is thJ maximum L(n) as a function of n? In what manner does the maximum value arise? Can we construct for a given n or a sequence of values of n the partition(s) for which the Least common multiple of the parts is the maximum L(n)? In this paper we address these questions, and we present the results of a computation of the partitions which have the maximum Least common multiple, illustrating primarily the steps in the reduction of the computation to one of manageable size.

Combinatorial sequences of divided powers

e

Earl J. Taft, Rutgers University

Let H be a Hopf algebra, a and b group-like elements, do a 1 and d1 an (a,b)-primitive element, i.e., lldl = a ®1 d1 + ®1 {3) h. We have defined a sequence of divider! powers lying over ®1 as a seauence {dn} for n l. 0 with lldh = l dia.i Cilihid;. Examples include sequences {pn(x)} of binomial or a-binomial type where llx = l®x+ l and $X^n X^m = \binom{n}{n}_q x^{n+m}$ for n,m l. n, where $\binom{n+m}{n}_q$ are the G,mssian polynomials (in q), and sequences {pn(x)} of Eulerian or a-Eulerian type. These latter are to be considered as Laurent series with the usual multiplication and llx = x ® x. Here P1(x) is (l,x)-primitive. In these examples, the Hopf algebra is both commutative and cocommutative. We give here two examples of SDP's in Hopf algebras which are neither COfl1!DUtative nor cocommutative. They come up in the attempt to construct Hopf algebras of finite dimension whose antipode has order a orescribed positive even integer.

CONTINUED FRACTIONS FOR CERTAIN ALGEBRAIC POWER SERIES

W. H. Mills[1] and David P. Robbins
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Let F be a finite field and let $K = F((z^{-1}))$ be the field of formal power series in z^{-1} over F. The usual theory of continued fractions carries over to K, with the polynomials in z playing the role of the integers. We study the continued fraction expansions of certain elements of K which are algebraic over $F(x)$, the field of rational functions of z.

We give the first explicit expansions of algebraic elements of degree greater than 2 for which the degrees of the partial quotients are bounded. In particular we study the continued fraction expansion of the solution f in K of the Baum-Sweet cubic $x^3 + f + x = 0$ when $F = GF(2)$. The first few hundred partial quotients of this continued fraction were obtained by computer, a pattern was guessed, and then later we were able to prove that this pattern was correct. The method of proof can be extended to other continued fractions.

Generalized wreath in o:,mbinata::i.cs

Chris Rowley, Open Univez:sit.y

Generali=d wreath p:oducts of groups were introduced into oombinatoci.cs by Bailey et al. to study certain tioclc structures wed frequently by statisticians in designed experiments. The maj:,r statistical interest is in the case where the constituent groups are full symmetric groups but the case of general permutation groups arises from other oombinata:ial structures, such as a generali2ation cf. the wreath and direct iroduct constructi,:,ns fee grnphs studied by Sabidssi...

This talk will survey the uaes of this construction in oombinata:i.c:s.

Biriiography
R. A. Bailey, Cheryl E. Ptaeger, C. A. Rowley and T. P. Speed, 'Generalized Wreath Products of Pern,utation Groups.' Proc. London Math. Soc. (3), 47 <1983), 69-82.
G. Sabidussi.. 'The Compam:ion cf. Gi::aphs.' Duke Math. J., (1959) 93 96.

A GRAPH CONDITION FOR WORD PROBLEM SOLVABILITY

John Pedersen
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Let I be **a** finite set of identities defining **a** variety V of **universal** algebras. A directed graph G(I) is defined, using the representation of the words occurring in I **as** labelled trees. If the identities have a restricted syntactic form, then the absence of cycles in G(I) implies that the word problem for finite V-presentations is always solvable. The proof involves the use of a completion procedure (adding in 'critical pairs'), which is shown to **always** halt successfully **when** G(I) is acyclic.

THE ROOTED TREE COMPLEXITY OF A DATA DEPENDENCY GRAPH AS A SOFTWARE METRIC

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Metrics for the cognitive complexity of software have been of considerable recent interest, and most of these are graph theoretic in their basis, with the cyclomatic complexity of McCabe being the best known. It *is* proposed that the data dependency graph of a program *is* a useful and not sufficiently investigated model of the program's complexity. Furthermore the count of rooted spanning trees of this graph *is* logically justified as a measure of the graph related to the complexity of the original algorithm, and its calculation from the program *is* reasonable.

On Hamiltonian Cycles in Cycle .Permuta-1:ion Graphs

Barry Piazza Clemson University

Let G be a cycle with V(G) = tl,2,... ,n and letc,<ES . The cycle permutation graph P_c(G) consists of two copie of G, G and G, with V(G)=[_x1,x2, . . . ,x jand v(G)=[y1,y2, ... , y_n) along with the edge $(x_1, y_{(1)})$ fo lSi n.Y

In 1972, Klee discussed th':- roblem of determining for which n does there exist a nonhamiltonian cycle permutation graph. He showed computationally that for n 8, only n= S has a nonhamiltonian c. p. Furthermore, he theoretically proved that for any odd n 9, there exists a nonhamiltonian c. p.

In this paper, we extend Klee's result to even n*S*: 14, using an isomorphism result and a generalization of Klee's "obviously hamiltonian" idea. Computational and theoretical techniques of this problem will be discussed.

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Frar.k Harary, University of t'ichigan
Kathryn F. Jones University of Colorado at Denver

The conditional chromatic number of a graph G with respect to a graph theoretic property Pis the minimum number of colors needed to color V(G) so that each color class has property P. We investigate the conditional chromatic number for properties of restricted maximum degree, girth, and independence and examine the problem of characterizing graphs which arc conditionally ipartite of this type.

GenN:l-ting a Spanning Tree With a SpecifieJ Vertex as the Cenl-roid for Biconnected Grap ,

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There is a specialized information accumulation process caJled receiving whereby every vertex in a net.work is expect<"d to get a fll('S$:t.ge to a specified (receiving) l'('rt.t'x in the rainimum time. If the graph represeot:ition of lbc network h:is a spanning tree with the specified ve,tex as a centroid vertex, then the rcci\"ing process can be done in the minimum time by appropriate message passing along th,· edges of the tree. Thus it is important to know when such a span,iing tree exists and to have efficient techniques to construct ouch a opanning tree.

This paper gi\'es :m algorithm that given any vertex in a biconnected graph, construct, in linear time a spanning tree or the graph with the specified vertex as a centroid vertex. The gen-eraI approacL or the al.Prit hll i, to fir,t constn:ct a dire.::teJ acyclic grap!., (dag) that contains all or the edges of lbe o,i„in:,! raph (dirert,·d one way or the other), bas the spccifit•d vertex as the only source (s), and I::"i vertex :idj:1cenL to the source :isthe only sink (t). Now by taking the vertices in topological order from the dag, a ·subtree can be found with root s such that it contains one !ess t-11:10 h:\lf number of vertices. The rem:\ining half or tile vertices can be formed into a subtree rooted :.t t lly u,ing the reverse topological order. By joining the t.vo subtrees together using the edge connecting s and t, the desired spanning tree with c"ntroid s c.,n be obtained. The mo,;t cL:npl•x part or the aJgoritLrn is the construction of the dag with source • and sink t. This isdm,e by mr:ius of two depth-first searches where the 6rst finds all the paths, and the second direct:, :,!the paths so as to avoid cycles and to lead from s tot.

An Algorithm for Generating the Minimum Length Fundamental Cycle Set in a Graph

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The compu ational co piexity of algo itr.!!is to generate a set of fundamental cycles depends on the length of the spanning tree (T). Thus, Tl: or equivalently the shortest-path-length spanning tree (STDLS) £1 result in the fundamental cycle set of minimum Land, consequentiy have the best time performance among all possible spanning trees. Finding T . or STPLS is unfort nately NP-complete. Therefore, heuristic algorrtBms that minimize (L(T) - $L(T_{m!n})$! are being scught.

This paper examines existing algorithms, suggests improvements and resents a new algorithm that approximates T_m . . The proposed algorithm uses a breacthfirst search to generate tHe spanning tree, always selecting the vertex of highest degree as the one to explore from. In case there are two or more vertices wi h the same degree, the combined degree of all the vertices that would be visited from that vertex is computed. This operation is repeated for each vertex in question. If the resultant is a single vertex, we build the spanning tree from that single vertex, rather than any of the other vertices with the same degree. A total of five algorithms were implemented. The values for L obtained by the proposed algoritnm were compared to those obtained by the other algorithms. Finally, a discussion of the proposed algorithm is given.

Shortest paths in undirected graphs with negative edges

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To find shortest simple paths in an undirected grapb G with negative edges but not negative cycles, a dynamic programming aigorithm, which is similar in spirit to the Floyd-Warsball all-pairs shortest-paths algoritlun is given. This algorithm follows from two properties of such paths: (I) a sbortest path will enter a component of G-, the subgraph of G induced by negative edges, at most once; and (2) subpaths of sbortest paths induced by either the positive or the negative edges are themselves shortest paths. Let *H* initially be the subgraph of *G* induced by the positive edges, and find shortest paths in H. One at a time, union a component of G- into H, and consider decreasing the cost of each path in H by going through each portion of tbis negative component. lo this way shortest paths can be found for *H*. The algorithm just sketched takes time $O(n^2m^2)$, where n is the number of vertices in *G*, and m is the number of vertices in G-. One can also detect if G bas a negative cycle in the course of running the algorithm.

The Complexity of Mlnlmlzlng Dllatlon ln Tree EmbeddlG

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C. Simonson. r.:.:i:r.sudborough, Northwestern Unlv. Evanston, Ill.

An embedding of a graph G in a graph H 1s a one to one assoclatlon of the vertices of G wlth the vertices of H. The dllatlon cost of an embedding ls the maximum·dlstance ln H between the images of vertices that are adjacent ln G. The problem of embedding an arbitrary graph G lnto a binary tree ls considered.

A polynomial tlme algorithm ls glven to sblve the problem for any fixed dllatlon bound k. A generallzatlon of thls problem ls considered and shown to be NP-Complete. The generallzatlon allows integer labels on the edges and asks whether the graph can be embedded ln a binary treer- so that no ed e ls stretched further than lts label.

A Tree-Based Invariant for Compact Graphs

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A class of trees, ***string trees,*** is defined for which the number or such trees embedded in a graph is polynomially bounded for all graph classes (due in part to the imposition of limits on the height of the trees included), and the collection of such trees represents the structure of some graphs sufficiently to be the basis for a complete invariant. Computation of the invariant consists or two-major steps: (1) the set of string trees is generated and partitioned into equivalence classes of isomorphic trees; (2) the ***Tree Structure Profile {TSP},*** in which the positions of each vertex With respect to the string trees of each class are enumerated, is generated for each vertex of the graphs in question. The complexity or computation of the Tree Structure Profile is easily shown to be polynomially bounded since both the number of trees and the number of vertices are polynomially bounded.

For the class of ***compact graphs*** (in which both the graph and its complement are required to have diameter of 2) the profiles based on the string trees are seen to represent enough of the graph structure surrounding each pair of vertices for equality of the Tree Structure Profiles to be a sufficient condition to prove isomorphism. This is because the diameter restriction imposed on these graphs forces the differences between two nonisomorphic graphs to be contained within the "Window" characterized by the Tree Structure Profile.

Idiosyncratic Polynomials

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We consider a generalization of the adjacency matrix of a graph. The generalized adjacency matrix A(x) of a graph G has an x in position (u,v) if vertices u and v are adjacent and a l otherwise. Tutte has called the characteristic polynomial of such a matrix an idiosyncratic polynomial. Techniques are developed to aid in the computation of the coefficients and roots of these polynomials. Spectral information for the generalized adjacency matrices of some well-known classes of graphs is given.

Generalized Gallai Theorems

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In 1959 T. Gallai presented the following, now well-known theorem, involving vertex covering (0:o), vertex independence (e o), edge covering (o:1) and maximum matching (t\1):

Theorem (Gallai). For any nontrivial connected graph G with p vertices,

I. Clo + So = P,

11. °'1 + 111 = p.

Since then a variety of generalizations and similar results, which we c2!! '0 :h i +h r-ems', r. v bee f?:.:nd f-::,r- v :—!e y of s:—:? !—::c ! ·invariants. A typical Gallai theorem has the form: o: • 11 = p, where o: and 11 are numerical, maximum or minimum functions of some type defined on the class of connected graphs.

This paper represents an attempt ot unify results of this type. In particular we present two general theorems; one, of Gallai type I above, is based on hereditary properties of set systems; the other, of Gallai type 11, is based on vertex partitions into subgraphs having tree Like properties. These generalizations encompass most of the existing Gallai theorems. We also present several new results, one of which is, curiously, not encompassed by either of the above mentioned **nP.neralizations.**

On Edge-Graceful Lobelings of Graphs

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ABSTRACT

An edge-labeling of a graph G is an one-to-one function mopping ECG) to the set {1,2,,,,,IECG)I}, The weight of a vertex u in V<G> with respect to on edge-labeling g, denoted wg(u), is the sum of all g(e) withe incident to u modulo IVCG)I, A connected graph G is said to be edge-graceful if there is an edge-labeling of G such that the weights of all vertices of Gore distinct, Such an edge-labeling of G is called an edge-graceful labeling of G, In this paper, we derive so e properties of edge-graceful graphs and prove a necessary condition for a graph to be edge-graceful, We also.identify some classes of edge-graceful graphs,

An Identity Arising from the Counting of Independent Sets

Glenn Hopkins and William Staton! University of Mississippi

An independent set I in a graph G is a set of vertices no two of which are adjacent. For each nonnegative integer k, Fk(G) is the number of independent sets of cardinality k in G, and F(G) is the number of independent sets in G. Summing over k, one obtains }/k(G) = F(G). In the case where G is the 2xn lattice, we determine F(G) and Fk(G), thereby obtaining a combinatorial identity.

Boolean gene|ating functions and parity results on graphs

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Abstract A Boolean generating function $f(x_1,x_2,\cdots,x_n)$ is a polynomial over Z_2 in the indeterminates $x_1,x_2,\cdots,x_n$. We show how Boolean generating functions can be employed to obtain parity results on graphs such as Thomason's theorem on the parity of the number of Hamiltonian circuits and Little's theorem on the parity of the number of 1-factors.

On scheduling perfect competitions

D. R. Stinson
University of Manitoba

The following problem was posed by M. S. Brandly. Let v 4 be an integer. A perfect competition is the set of all possible games, where in each game two of v players play against two other players. Thus, there are $\frac{1}{2}\binom{v}{2}\binom{v-2}{2}$; $3\binom{v}{4}$ games. We desire to schedule these games into rounds, so that any player plays in at most one game in each round. Denote by R(v) the minimum number of rounds required to schedule a perfect competition, subject to this constraint. We prove that, for any integer v 4, R(v); 3(:)!LfJ.

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CONJUGATE ORTHOGONAL LATIN SQUARES AND MENDELSOHN DESIGNS

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Abstract. We shall refer to a Latin square which is orthogonal to its (3,2,1) conjugate as a (3,2,1) - conjugate orthogonal Latin square. A (v, K, 1) - Mendelsohn design is a pair (X,B), where X is av-set (of ints) and B is a collection of cyclically ordered subsets of X (called blocks) with sizes in K such that every ordered pair of points of X are consecutive in exactly one block of B. The existence of idempotent (3,2,1) - conjugate orthogonal Latin squares of order v is known to be equivalent to the existence of a certain class of (v, K, 1) - Nendelsohn designs. This paper investigates the spectrum of idempotent (3,2,1) - conjugate orthogonal Latin squares and it is found that it contains all positive integers v except 2, 3, 6 and possibly 12, 14, 18, 26 and 30.

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Embedding Partial Perpendicular Arrays

D. G. Hoffman and C. A. Rodger[1]
Auburn University

Abstract

A(oartial) perpendicular array of strength k and order n is an ( ) x k array on n symbols such that in any pair of columns of the array each unordered pair of symbols occurs in the same row exactly (at most) once. It is shown that a partial perpendicular array of strength 3 and order n can be embedded in a perpendicular array of strength 3 and order en^2.

he Existence Question for a 2-(46,6,1) Design: A Coding Theoretic View

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Robert Roth, Emory University

The smallest v for which the existence of a 2-(v,6,1) design is unknown is 46. It is known that the automorphism group of such a design has cardinality at most 3; this leaves coding theory as the most likely technique for answering the existence question. The binary code of such a design will be discussed: in particular, partial results on the weight enumerators of this code and its dual will be stated and the correspondences between certain code words and certain subconfigurations of the design will te given.

An Effldent Minimum Verla O»•er Algorithm for a Broad Casa of Undirected Graphs

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Let *G* be an arbitrary undin:ctcd graph with vertex set *V* and edge set *E*. It is well-known that if M•c;;E is a maximum matching in G and if K C V is a minimum vertex cover set of *G* then j,Wls!il- Furthermore, the problem of computing a minimum vertex cover set of *G* is in the class of NP-c:ompletc problems, although O(n2.5) algorithms arc known for computing a maximum matching of *G*.

In this paper, it is shown that if *G* is in the special class of undin:ctcd graphs for which 1M•l= lii then a minimum vertex cover set of *G* can be computed by means of an O(n3) algorithm. In addition, this algorithm can be u s ¢ to determine whether an arbitrary undirected graph is a member of the class of graphs for which 1M•l= lii- Thus, if the above equality holds then the algorithm yields a minimum vertex cover set d. *G*; otherwise, an easily detectable inconsistency is produced.

ALGEBRAIC ASPECTS OF RELIABILITY PROBLEMS

Douglas R. Shier, Mathematical Sciences, Clemson University

Network reliability problems typically require evaluating the probability that a particular pair of vertices can communicate, given individual link failure probabilities and various assumptions about the dependence or independence of link failures. This type of problem is formulated algebraically (in terms of operations on polynomials) and in this form resembles the usual shortest path problem. Iterative techniques for approximating network reliability are derived in a natural way from this association.

ON THE PERFORMANCE OF A CLIQUE DETECTION ALGORITHM IN UNDIRECTED GRAPHS

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ABSTRACT

The clique problem of undirected graphs, in its many different aspects, has been intensively discussed in the literature; in particular, a number of authors have developed a variety of fascinating approaches to the solution of the clique detection problem in undirected graphs. In a recent paper an efficient, new algorithm for generating all cliques of an undirected graph has been proposed by Das, Sheng and Chen. This algorithm is based on a refinement of the technique of successive splitting described by Paull and Unger in the determination of maximal compatibles of states in the context of minimization of incomplete seq'uential machines. One novelty of the proposed algorithm is that in its application the generation of duplicate subgraphs or duplicate cliques is completely suppressed, while the generation of non-clique subgraphs is greatly reduced. For the special case of Moon-Moser graphs, this algorithm generates all cliques without generating any non-clique subgraphs. In the present paper investigations are carried out concerning computational complexity and storage requirements of the aforesaid algorithm, and its performance characteristics are evaluated by implementing the algorithm using FORTRAN IV in a Cyber System 170/720 with memory access time of 460 nsec, 60 bits word length, and 192K words total memory. Two sets of experiments are conducted: the first on small graphs with up to 60 vertices, and the second on large graphs with up to 1000 vertices. The implementation results demonstrate that the present algorithm performs significantly better than most of the existing algorithms for both random graphs and Moon-Xoser graphs. A complete listing of all the progra:as is also provided.

ALTERNATIVE METHODS FOR COMPUTING CHROMATIC POLYNOMIALS

L KIRK TOLMAN[1] AND JACK W LAMOREAUX BRIGHAM YOUNG UNIV

The most common method of computing the chromatic polynomial of a graph G is by use of trefonnula:

$$PG(x) = p_{G\bullet}(x) + p_{G''}(x)$$

1 here G' is obtained by placing an edge between two non-adjacent vertices in G and G'' is obtained by fusing the same two vertices. Alternatives are presented hich, together with the above and other known methods may be played off one with another to simplify the computation of the chromatic polynomial.

Symmetrical Maps

Arthur T. White, estern Michican niversity

A in,e ;,:is a confi uratio1 consisting of a connected graph G and a 2-cell imbedding of in a closed orientable 2-mar..ifolc (a surface). An automorphism of M is an automorp ism of G which also preserves oriented re ior.. boundaries ior the imbedding; the automorphism group for iv;is denoted by Aut t,i, rt is well-known that IAut M1 2 IE(G)I : if this bound is attained, then G is edee-transitive and is said t0 be symmetrical (variously, symmetric, regular). In this paper we discuss three classes of problems concerning s:,rmmetrical maps: (1) For P given edge-transitive graph G, what syr.imetrical maps, if any, does it have? (2) For a given group r, what symmetrical maps, if any, for Cayley t,raphs G(f,) are there? ()) For a given surface S(k), what symmetrical maps exist on S(k)? we contribute toward the study of question {1), by showing, The regular complete tripartite graph K(n,n,n) has a symmetrical map, for all n. Moreover, the imbeddings can be taken as genus imbeddings. For each value of n, three block designs are determined.

Regular Maps on Surfaces

Andrew Vince
University of Florida

A map is a eneralization of a polyhedron; the latter can be regarded as a map on a sphere. The regular maps, those maps with the greatest symmetry, generalizes the five regular polyhedra. Although a classification of the regular maps seems intractible, an algorithm is presented for generating a large class of regular maps. The technique is combinatorial and makes use of graph covering.

LEFT-RIGHT PATHS OFF THE PLANE

8. Richter[1] & H. Shank
Utah State University Univ. of Massachusetts

For graph G embedded in surface S, we consider the cycle space, the cocycle space, the dual cycle space, the boundary cycle space and the left-right path space - all vector spaces over GF[2]. Relations between these spaces and their intersections are given. These generalize several well-known facts about graphs embedded in the plane.

(-VALUATIONS OF CUBIC GRAPHS.

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A. Kotzig, C.R.M.A., Universite de Montreal

The concept of a (-valuation for cubic graphs was introduced by the authors in 1981 (Congressus Numerantium Vol. 32, pp. 89-101); it was shown that (-valuations can be used to construct orthogonal pairs of additive permutations but only two cubic graphs with (-valuations were presented. The existence problem is studied in the present paper; it is proved that for every n > 3, there exists a bipartite cubic graph on 2n vertices with a k-parameter family of (-valuations where k > 2. It is also shown that every bipartite cubic graph which has a (-valuation is edge 3-connected; this statement is not true for cubic graphs which are not bipartite. Certain transformations of graphs are considered which can be used to construct new cubic graphs with (-valuations.