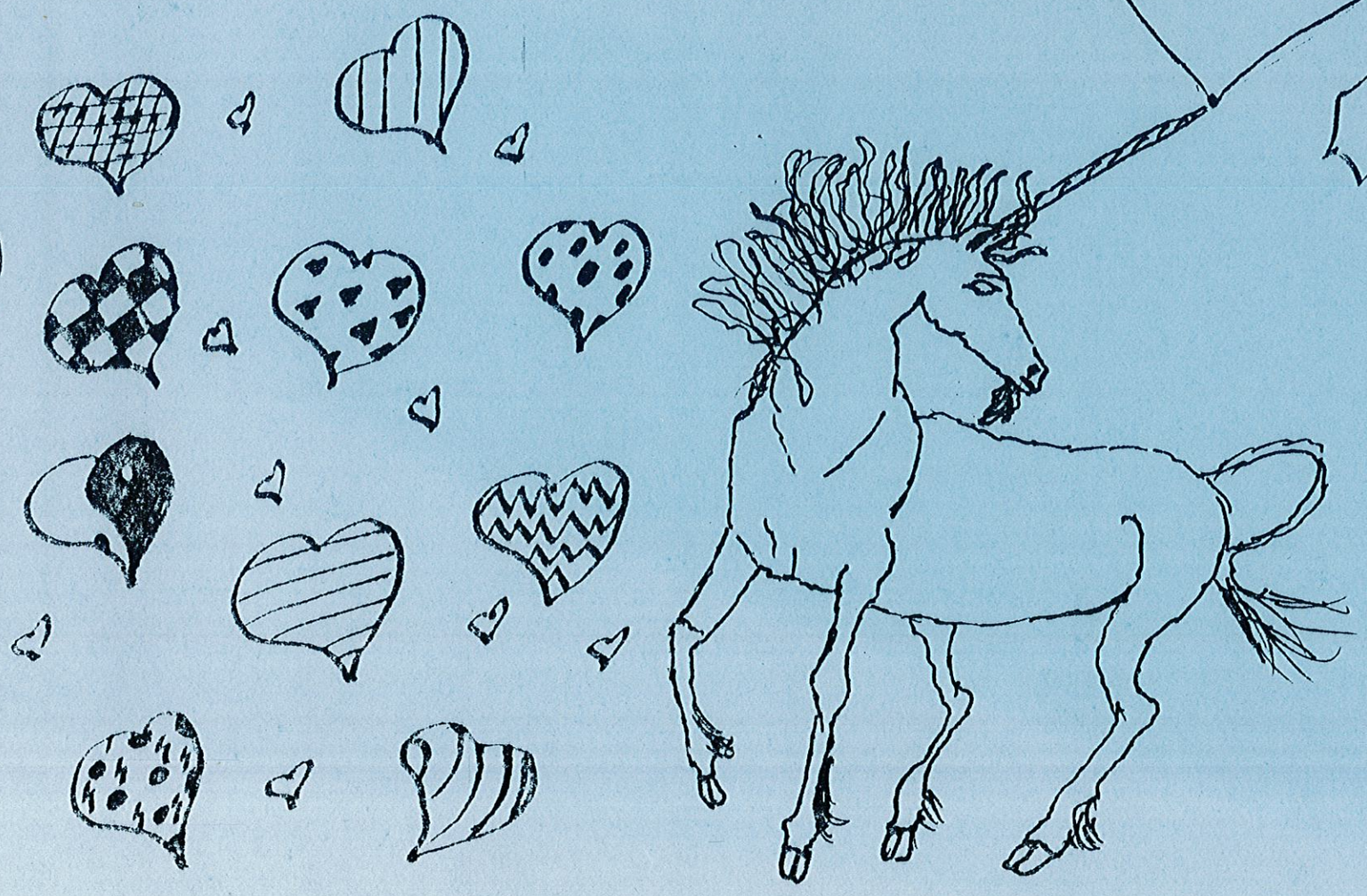
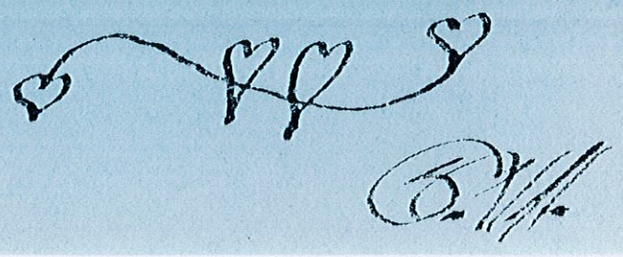
FEBRUARY 14TH and it's the



1983 South-eastern Conference on: combinatorics, graph theory. & computing



Our Conference Proceedings are published by Utilitas Mathematica Publishing Co., P.O. Box 7, University Centre, Winnipeg, Manitoba, Canada, R3T 2N2 in its series Congressus Numerantium. Back issues and additional copies may be ordered from them. The publisher has asked us to announce that the Proceedings of the 13th Conference will be mailed soon. If you participated in last year's conference and either forgot to pay your postage or changed your address you may take advantage of this delay to arrange to obtain your copy.

**Miscellaneous Information**

The phone number at the Conference desk is 393-3719
There is a coin-operated xerox machine in the second floor lounge of the University Center.
The copy center (behind the police station, which is near the south end of the covered walkway) can xerox onto transparencies. Blank transparencies are available in the bookstore in the University Center and at the copy center.
GCN and GCS are the two halves of the Gold Coast Room of the University Center.
Room 207 is entered from the second floor lounge of the U.C.

COFFEE will be available in Room 232 after Monday Morning, as will the book exhibits and the display of menus from many area restaurants. We plan to have demonstrations of the IBM personal computer in Room 219.

Transportation: We shall provide van transportation from Day's Inn and University Inn to the University Center at 8:00AM Monday through Thursday, making two trips if necessary. There will be van transportation to the motels at 5:45PM Monday through Wednesday from the University Center. There will also be van transportation from the reception at the Board of Regents Room on Monday evening. Transportation will be provided at 5:45PM and 6:15PM from the University Center and at 6:30PM from the motels to the Tuesday night beer party. The van will go back to the University Center and to the motels. A van will leave the University Center for the banquet at 6:20PM Wednesday, and vans will leave the motels at 6:30PM. Transportation will be provided back to the motels at the close of the banquet. The vans will be available for transportation back to the motels at the close of the Conference on Thursday. Although we aim to provide all needed transportation, we encourage car-pooling where possible.

## MONDAY

| | GCN | | GCS | | 207 | |
|---|---|---|---|---|---|---|
| 8:40 | Registration (from 8AM) | | | | | |
| 9:00 | Opening Remarks - E.W. Baker, Dean, College of Science | | | | | |
| 9:30 | VAN LINT | | | | | |
| 10:30 | COFFEE | | | | | |
| 10:45 | Freeman | 1 | Smith | 5 | Archdeacon | 9 |
| 11:05 | Allaway | 2 | Magliveras | 6 | D. Berman | 10 |
| 11:25 | Niederhausen | 3 | Kramer | 7 | Kierstead | 11 |
| 11:45 | Robinson | 4 | Taylor | 8 | Bernhart | 12 |
| 12:05 | | | | | | |
| 12:25 | | | | | | |
| 12:45 | | | | | | |
| 1:35 | VAN LINT | | | | | |
| 2:40 | | | | | | |
| 3:00 | Dillon | 13 | Klingsberg | 21 | Bridgland | 29 |
| 3:20 | Vanstone | 14 | Walsh | 22 | Faudree | 30 |
| 3:40 | Mullin | 15 | Saxton | 23 | Laskar | 31 |
| 4:00 | Moreno | 16 | Paul | 24 | Schmidt | 32 |
| 4:20 | Phelps | 17 | Holladay | 25 | McKay | 33 |
| 4:40 | Takefuji | 18 | Culik | 26 | Barkauskas | 34 |
| 5:00 | Wilson | 19 | Trenary | 27 | K. Berman | 35 |
| 5:20 | | 20 | | 28 | Lesniak | 36 |

## TUESDAY

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| | Kotin | 37 | Hsu | 43 | Dewdney | 49 |
| | Kotzig | 38 | Danhof | 44 | Stahl | 50 |
| | KLEE | | | | | |
| | COFFEE (ROOM 232) | | | | | |
| | Chung | 39 | Henminger | 45 | Matula | 51 |
| | Winkler | 40 | Friedlander | 46 | Hamacher | 52 |
| | Ringeisen | 41 | Klerlein | 47 | Bailey | 53 |
| | Ross | 42 | Tannenbaum | 48 | Christopher | 54 |
| | Conference Photo: Outdoor Stage (12:05) | | | | | |
| | LUNCH | | | | | |
| | KLEE | | | | | |
| | Miller | 55 | Holliday | 63 | Cornell | 71 |
| | Proskurowski | 56 | Anderson | 64 | Hell | 72 |
| | S.T. Hedetniemi | 57 | Stinson | 65 | Rosenfeld | 73 |
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| | Cockayne | 80 | White | 86 | Burstein | 92 |
| | POMERANCE | | | | | |
| | COFFEE (Room 232) | | | | | |
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| | Dawes | 82 | Livingston | 88 | Richmond | 94 |
| | Wagstaff | 83 | Silverman | 89 | Opatrny | 95 |
| | Brickell | 84 | Gibson | 90 | Entringer | 96 |
| | LUNCH | | | | | |
| | POMERANCE | | | | | |
| | Wachs | 97 | Grace | 105 | Hobbs | 113 |
| | Georges | 98 | Turgeon | 106 | Fisk | 114 |
| | Santoro | 99 | Chen | 107 | Hoskins | 115 |
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| | Chinn | 101 | Beasley | 109 | Bennett | 117 |
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| | Cozzens | 103 | Lee | 111 | Fuji-Hara | 119 |
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## THURSDAY

| | GCN | | GCS | | 207 | |
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| | ERDOS | | | | | |
| | COFFEE (Room 232) | | | | | |
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Monday, February 14, at 9:30 A.M. and 1:35 P.M., Professor Jacobus H. van Lint of the Technical University of Eindhoven speak on, "Preparata and Kerdock Codes."

Tuesday, February 15, at 9:30 A.M., Professor Victor Klee of the University of Washington will speak on, "Graph-Theoretic Methods in the Qualitative Analysis of Linear Systems."

Wednesday, February 16, at 9:30 A.M. and 1:35 P.M., Professor Carl Pomerance of the University of Georgia will speak on, "Primality Testing."

Thursday, February 17, at 9:30 A.M., Professor Paul Erdos of the Hungarian Academy of Sciences will speak on, "Problems in Set Theory and Number Theory."

At 2:30 P.M., Thursday, February 17, Carl Pomerance will give a public lecture in Room 141 of the Business Building, sponsored by the FAU Sigma Xi Club. Conference participants are cordially invited to attend.

Conference participants are invited to the Cocktail Party Reception in the Board of Regents Room of the Administration Building from 6:00PM to 7:30PM on Monday, February 14, 1983. (The drink tickets are for accounting purposes -- BROWN TICKETS for a mixed drink, GREY TICKETS for beer or wine and no tickets required for soda. Additional tickets may be requested.) Transportation will be available back to the motels after the Reception. The vans going to the motels at 5:45 will return to the University, leaving the motels at about 6:15.

Conference participants are also invited to a Beer Party, Tuesday February 15 from 6:00PM to 8:00PM at the home of Dr. and Mrs. John M. Freeman, 741 Azalea Street (but park on Aurelia!). Vans will leave the University Center at 5:45 and 6:15PM for the party, and will leave the motels at 6:30PM. Transportation will be provided back to the motels and the University Center after the party. It is about a one-mile walk from the University to the Freeman's home.

The Conference Banquet will be held at the Boca Teeca Country Club, Wednesday February 16. The cash bar opens at 6:30PM. Seating will be at 7:45 and service at 8:00PM. A van will leave for the banquet from the University Center at 6:20PM and vans will leave the motels at 6:30PM. It is more than a two-mile walk from the University Center to Boca Teeca.

There will be a Survivor's Party 6:00PM to 8:00PM Thursday evening February 17 at the home of Dr. and Mrs. Frederick Hoffman, 4307 N.W. 5th Avenue. Conference participants who are still in the area are most welcome. Please let us know if you can come, and if you need transportation. It is a walk of less than two miles from the University to the Hoffman's home.

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## ① BAXTER'S FUNCTIONAL EQUATION AND POLYNOMIAL CHAINS

J. M. Freeman, Florida Atlantic University

Atkinson and Rota have pointed out a connection between Baxter's functional equation

$$f(t) = 1 - tB(\alpha f(t))$$

and certain Euler identities involving the polynomial chain $p_n(x) = (x-1)(x-q)\cdots(x-q^{n-1})$. In essence, this connection reduces to the fact that $q^{\hat{n}}p(x) = p(qx)$ is a homomorphism on polynomials, making $B = q^{\hat{n}}(1-q^{\hat{n}})^{-1}$ a Baxter operator. Surprisingly, there is a quite different connection which generalizes to arbitrary polynomial chains $p_n(x) = (x-a_0)(x-a_1)\cdots(x-a_{n-1})$, and which reduces instead to the fact that the twist maps $\phi$ and $\phi_-$ are homomorphisms on the umbral algebra. Namely, the dual generating function of a polynomial chain $f(t)$ satisfies the above Baxter equation where $B = \phi_-(1-\phi_-)^{-1}$ and $\alpha$ is an umbral element simply related to the zero sequence of the chain. This gives several identities including,

$$f(t) = \prod_{k=1}^{\infty}(1-t\phi_-^k(\alpha)) = 1 - tB(\alpha) + t^2B(\alpha B\alpha) - \cdots,$$

which are the dual forms of the Euler identities for arbitrary polynomial chains.

## The Umbral Calculus and Integral Valued Polynomials ②

Wm. R. Allaway, and M. W. Islam, Lakehead University

It is well known that the polynomials, which together with their derivatives have integral values at a fixed point $x = a$ have the basis $(x-a)^{\hat{n}}/n!$, $n = 0, 1, 2 \ldots$, over the integers. Let $\gamma$ be the fundamental sequence $(q_n)$ where $q_0 = 0$, $q_1 = 1$ and $q_n \neq 0$ for $n \geq 2$, and define the linear operator $Q$ on the set of all polynomials by $Qx^n = q_n x^{n-1}$. $Q$ is known as an Eulerian differential operator. Let its basic Eulerian polynomial sequence be denoted by $q_n(x)$, $n = 0, 1, 2 \ldots$. If $\phi$ is the set of all polynomials $p(x)$ such that the evaluation of $Q^\lambda p(x)$ at $x = a$ is an integer for $\lambda = 0, 1, 2 \ldots$, then by using the umbral calculus we find in this paper a number of bases of $\phi$ over the integers. One of these bases is $a^n q_n(x/a)$, $n = 0, 1, 2 \ldots$. Also we characterize those fundamental invariant operator relative to $\gamma$ that map $\phi$ into $\phi$.

## ③ STIRLING NUMBER REPRESENTATION PROBLEMS

Heinrich Niederhausen
Department of Statistics, University of Toronto, Toronto, Canada M5S1A1

Nielsen's Stirling polynomials $\psi_n(x)$ are essentially Sheffer polynomials. These polynomials have Stirling numbers as *values*, not as *coefficients*, unlike the exponential polynomials. Hence, the binomial theorem for Sheffer sequences and its generalizations lead to (new?) representations of Stirling numbers.

## ⑤ NEARLY KIRKMAN SYSTEMS OF BLOCK SIZE GREATER THAN THREE

H.P. Smith, University of Victoria

If $\mathcal{V}$ is a set of points with $|\mathcal{V}| = v$, a nearly Kirkman system (NKS[k,v]) is a collection of blocks (k-subsets) of $\mathcal{V}$, $k \geq 3$, which can be partitioned into subcollections (resolution classes) such that each resolution class covers $\mathcal{V}$, each pair of points appears together in not more than one block, and each point appears in $\lfloor (v-1)(k-1) \rfloor$ blocks, where $(k-1) \nmid (v-1)$. We extend results of Kotzig and Rosa to demonstrate the existence of infinite classes of nearly Kirkman systems for all $k \geq 3$ whenever $k$ is a prime or a prime power.

## SIMPLE 6-(33,8,36) DESIGNS FROM $P\Gamma L_2(32)$ ⑥

Spyros S. Magliveras & David W. Leavitt, Univ. of Nebraska

A $t$-$(v,k,\lambda)$ design is a pair $(X,\mathcal{B})$ where $\mathcal{B}$ is a collection of $k$-subsets (called blocks) from a $v$-set $X$, such that each $t$-subset of $X$ occurs in exactly $\lambda$ blocks of $\mathcal{B}$. A design is simple if no block of $\mathcal{B}$ is repeated. A design is trivial if each $k$-subset of $X$ occurs precisely $m$ times in $\mathcal{B}$. Using the 4-homogeneous group $P\Gamma L(32)$ we construct over 500,000 new simple, non-trivial 5-designs and six pairwise non-isomorphic, simple, non-trivial 6-(33,8,36) designs.

## THE EXACT COVERING NUMBERS g(1,3,14), g(1,4,15), and g(1,5,16) ⑥

E.S. Kramer*, University of Nebraska, Lincoln, NE 68588-0323
R.C. Mullin, University of Waterloo, Ontario, Canada

Let $X$ be a $v$-set and let $\mathcal{B}$ be a family of proper subsets (called blocks) of $X$ with the property that any given $t$-subset $T$ of $X$ is contained in precisely $\lambda$ members of $\mathcal{B}$. Then $\mathcal{B}$ is said to be an exact $(\lambda,t)$ cover of $X$. Let $g(\lambda,t,v)$ denote the least number of blocks which form an exact $(\lambda,t)$ cover of $X$. The values of $g(1,3,v)$ are known for $v \leq 26$ with the exception of $v \in \{11,13,14,23\}$. In this paper, it is shown that $g(1,3,14) = 63$, and that up to isomorphism there are precisely four ways of achieving this bound. All such configurations contain blocks of cardinality 4 and 6 only. We also show that $g(1,4,15) = 183$ and that $g(1,5,16) = 478$.

## ⑨ Coupled Colorings of Planar Maps

Dan Archdeacon
University of Vermont

A coupled coloring of a planar map is a simultaneous coloring of the vertices and faces such that two elements which are either adjacent or incident are colored differently. It is an unsolved problem whether 6 or 7 colors are necessary and sufficient. We present a new proof that 7 colors are sufficient, and prove that 6 colors suffice if the map is either 4-connected or of girth exceeding 4. We also investigate conditions under which 5 colors are sufficient.

## Cliques, Colorings, and Locally Perfect Graphs ⑩

Michael O. Albertson, Smith College
*David M. Berman, University of New Orleans

Suppose a graph $G$ is properly colored with colors $(1, 2, 3, \ldots)$. We say that a vertex $v$ is <u>tightly colored</u> if the color given to $v$ is equal or less than the size of the largest clique containing $v$.

If $G$ has a proper coloring in which every vertex is tightly colored then we call $G$ <u>locally perfect</u>. We show that every perfect graph is locally perfect, but not conversely. There is no counterpart to the perfect graph theorem, as it is possible for a graph to be locally perfect but its complement not.

We consider the question of coloring a planar graph so that as many vertices as possible are tightly colored. We show that this must be at least 5/8 of the vertices, and give an example in which this is only 7/10 of the vertices.

## On the Chromatic Index of Multigraphs without Large Triangles ⑪

Henry A. Kierstead, University of South Carolina

Vizing proved that for any multigraph $M$, $\chi' \leq \Delta + \mu$, where $\chi'$, $\Delta$, and $\mu$ are respectively chromatic index, maximum degree, and maximum multiplicity. We shall give a new proof of Vizing's Theorem that provides the additional information that if $\chi' \geq \Delta + s$, where $s > \frac{\mu+1}{2}$ then $M$ contains a $2s$-sided triangle, i.e. a triangle whose edges have multiplicities that sum to $2s$. In particular: (*) If $M$ is a multigraph of multiplicity 2 that has no 4-sided triangle then $\chi' \leq \Delta + 1$. Previously J. Schmerl and the author had proved that (*) implies: (**) If $G$ is a graph that does not induce $K_{1,3}$ or $K_5 - e$ then $\chi \leq \omega(G) + 1$ where $\chi$ and $\omega$ are respectively chromatic number and maximum clique size.

(4) ASYMPTOTIC NUMBER OF SELF-CONVERSE DIGRAPHS

R.W. Robinson, Southern Illinois University, Carbondale, IL

The exact number $D_n$ of nonisomorphic self-converse digraphs on n vertices has long been known, as it is straightforward from Burnside's lemma. However there is no single term in the resulting sum formula for $D_n$ which dominates the others asymptotically. It is shown that $D_n$ is asymptotic to

$$\frac{e^{\sqrt{n/2}}}{e^{1/8\sqrt{2}}} \cdot \left(\frac{2n}{e}\right)^{n/2} \cdot \frac{2^{n(n-1)/2}}{n!}$$

as $n \to \infty$. A similar result is obtained for the number $R_n$ of self-converse rooted digraphs on n vertices. Finally, it is shown that almost all self-converse digraphs are weakly connected.

SDP DESIGNS (13)

J.F. Dillon, Dept. of Defense

We characterize designs with the Symmetric Difference Property as arising from the 1st order Reed-Muller code via extension by a difference set. We thus generalize some pioneering work of W.M. Kantor and shed some light on some more recent work of p.J. Bush and Kantor.

Some results on the discrete logarithm problem (or how to rob a bank) Parts 1 and 2. (15) (14)

I.F. Blake, R. Fuji-hara, R.C. Mullin & S.A. Vanstone.
University of Waterloo.

Let $\alpha$ denote a primitive element of $F = GF(q^n)$. For any non-zero element $\beta$ of F, the exponent k (where k is a residue mod $q^n-1$) with $\alpha^k = \beta$ is called the discrete logarithm of $\beta$ (with respect to the base $\alpha$). The determination of such a k in general appears to be a difficult problem and thus exponentiation is used as part of various crypto-systems. For example, Hewlett Packard has developed a VLSI chip (PKDS) based on $GF(2^{127})$. Other institutions, including certain major banks, also employ exponentiation in $GF(2^{127})$ as part of their security systems, hence the sub title of this pair of talks. The authors have implemented an algorithm which finds discrete logarithms in $GF(2^{61})$ efficiently. This algorithm and its implementation and statistics will be discussed in Part 1. In Part 2 it will be shown that it is possible to implement this algorithm in $GF(2^{127})$. Heuristic methods, as yet not fully analyzed, which should be explored prior to such an implementation, will also be discussed.

(5) NON-ATTACKING ROOKS WITH DISTINCT DIFFERENCES

HERBERT TAYLOR COMMUNICATION SCIENCES INSTITUTE
UNIVERSITY OF SOUTHERN CALIFORNIA

A COSTAS ARRAY IS A CONFIGURATION OF N NON-ATTACKING ROOKS ON AN NXN BOARD WITH THE "DIFFERENCES" (VECTORS) BETWEEN PAIRS OF ROOKS ALL DISTINCT. FOR N>3 EVERY COSTAS ARRAY HAS DIAMETER>N IN THE LEE METRIC (TAXI METRIC). ANY TWO NXN COSTA ARRAYS HAVE AT LEAST ONE DIFFERENCE IN COMMON IF $N \geq 4$. DOUBLY PERIODIC COSTAS ARRAYS DO NOT EXIST FOR N>2. S.W. GOLOMB'S QUESTION WHETHER ANY COSTAS ARRAYS EXIST WHICH ARE ALSO CONFIGURATIONS OF NON-ATTACKING QUEENS IS STILL OPEN. NONE EXIST FOR $N \leq 10$.

*Isomorphism of Distributive Lattices in Polynomial Time*

*László Babai, Eötvös U., Budapest
*Paul Klingsberg, St. Joseph's U., Phila.
Eugene M. Luks, Bucknell U., Lewisburg (21)

*Via the duality with posets, isomorphism testing of distributive lattices of size n is reducible to isomorphism testing of vertex-colored graphs with color size less than or equal to log(n). Previously, Babai used this reduction to give an O(n**log(n)) Las Vegas algorithm, and a deterministic version of Babai's "tower of groups" method [Furst, Hopcroft, Luks] preserves this time bound.*

*We offer several improvements through which isomorphism of distributive lattices is tested-- in fact canonical forms are found-- in $O(n^6 \log(n))$ time.*

(22) A CASE FOR ITERATION

T. R. Walsh, Dept. of Computer Science, Univ. of Western Ontario

Certain problems originally solved by recursive algorithms have been shown by the author and others to be susceptible to iterative solutions. Among these are

a) the Towers of Hanoi and two modifications thereof,
b) Gray code generation,
c) Merge-sort.

Of particular interest is a new iterative merge-sort algorithm which, unlike the well-known one, uses no more comparisons than the recursive version.

These iterative algorithms can be made to execute faster than the recursive versions, since they can be programmed in FORTRAN, which does not support recursion but which compiles into more efficient code than any high-level recursion-supporting language. They also require less memory than the stacks needed to implement recursion - a great convenience when one is working by hand.

As an illustration, the fastest of the Gray code generation algorithms was used to make a table of the number of Cribbage hands with score 0,1,...,29, using 20 seconds of CPU time on a VAX 11.

It would be useful to find a scheme for obtaining iterative versions of a class of recursive algorithms large enough to include such asymptotically fast algorithms as Strassen's matrix-multiplication algorithm.

(12) THE LINEAR PROGRAM OF A FOUR COLOR-RING

Frank R. Bernhart, Rochester Inst. of Tech., Rochester, N.Y.

There is an unusual characteristic linear program associated with the model of reduction of an n-ring configuration in a planar cubic map, or dually a planar triangulation. The feasible region of the LP has not been previously investigated, although its structure for rings of order $n \leq 5$ can be inferred from standard work on the four color problem. A very crude version of the inequalities may be said to carry the same information used to check configurations for the property of four-color reducibility -- this is the phase of the Appel-Haken proof of the four color conjecture which required extensive computer aid, to check configuration of ring order up to 14. Also, the LP contains many equations, which are closely related to the special equations coming out of the theory of chromatic polynomials.

This article is devoted to the description of the system $L_6$ for the six-ring, and the search for the extreme or 'prime' solutions. The search for extreme points of a finite and highly degenerate polytope is known to be a hard task. On the other hand, elementary graphs and the six special six-ring solutions of A.F. Bernhart provide a major helpful subset. It is hoped that resolution of these problems for orders six and seven will lead to helpful insights.

CONVEXITY IN GRAPHS: A Survey of Extremal Problems (29)

Michael F. Bridgland, Louisiana State University

In this expository progress report, we discuss work of the author and others concerning a collection of problems of this form: in which graphs are the convex sets of vertices as "small" ("large") as possible? Such problems have been investigated (from the point of view of convexity) primarily for one type of convexity: a subset S of the vertices of a graph is said to be metrically convex if all of the vertices lying on each shortest path joining vertices in S also lie in S.

Following the lead of R. Jamison [1], we broaden our scope to include various other alignments (families of convex sets) on graphs. With that generality, the extremal problems mentioned above encompass several problems that originally were introduced in contexts other than convexity: friendship graphs, Moore graphs, Moore geometries, geodetic graphs, and ultrageodetic graphs.

1. Jamison-Waldner, R., A perspective on abstract convexity: classifying alignments by varieties, Convexity and Related Combinatorial Geometry, D. Kay and M. Breen, ed., Marcel Dekker, 1982, 113-150.

(30) RAMSEY NUMBERS INVOLVING SPARSE GRAPHS

R. J. Faudree, C. C. Rousseau, R. H. Schelp

Let G be a fixed graph and H a connected sparse graph on n vertices. (A graph is sparse if the number of edges is not to much larger than the number of vertices of the graph.) Results and problems on the generalized Ramsey number r(G,H) will be considered. For example, r(G,H) will be determined precisely when G is a graph of small order and H is an arbitrary tree. Also, it will be shown that when H is sufficiently sparse and n is sufficiently large, r(G,H) can be expressed precisely in terms of n and parameters connected with the chromatic number of G.

## FURTHER RESULTS ON QUASIPERFECT CODES RELATED TO THE GOPPA CODES

Oscar Moreno, University of Puerto Rico, Rio Piedras

Gorenstein, Peterson and Zierler in 1960 proved that double error correcting primitive B.C.H. codes are cyclic. In 1979 Helleseth proved that only those primitive B.C.H. codes are quasiperfect. In 1980 O. Moreno found a new class of double error correcting codes related to the Goppa Codes. This class includes the Zetternberg Codes which are B.C.H. Codes. In this paper further double error correcting quasiperfect codes, related to the Goppa codes are found. This includes the Melas Codes which are cyclic. A preliminary examination of the class of cyclic extended four error correcting Goppa Codes for possible quasi-perfection. A subclass of this codes is found not to be quasiperfect.

## A COMBINATORIAL CONSTRUCTION FOR ERROR CORRECTING CODES

Kevin T. Phelps, Georgia Institute of Technology

A purely combinatorial construction for a class of distance 6 codes is presented. This approach is then applied to give another construction for the Nordstrom-Robinson code (16,256,6). The fact that these codes are "product" codes leads to a relativoly efficient decoding approach.

## A NEW CONVERSION SCHEME OF MULTIRESIDUE CODES IN GF(2) POLYNOMIAL RING ; TAK.

Yoshiyasu Takefuji, Takakazu Kurokawa, Hideo Aiso
Keio University

Abstract. We have developed a new conversion scheme of multiresidue codes in GF(2) polynomial ring. In number ring, there exist two conversion schemes; the one is the chinese Remainder theorem, the other is the Mixed Radix Conversion scheme. However, no one have discussed on a conversion scheme of multiresidue codes in polynomial ring. Our scheme can be realized by solving a regular matrix equation of

$$A \cdot X = b$$

where A is a regular matrix of $n \times n$, X is a vector of

$$X = (x_1\ x_2\ x_3\ \cdots\cdots\ x_n)$$

and b is also a vector of

$$b = (b_1\ b_2\ b_3\ \cdots\cdots\ b_n).$$

In the matrix equation all the elements are 0s or 1s. The matrix equation can be easily solved in a paralled and pipelining way which is suitable for VLSI iterative implementation with a flexible expandability toward the increase of matrix dimension. The multiresidue codes in GF(2) polynomial ring have various application fields such as error-correction, detection, cryptography and so on.

## On Minimum Parallel Computing Times for Gaussian Elimination

Michael R. Leuze, Vanderbilt University
Lawrence V. Saxton*, University of Regina

Consider the problem of solving a symmetric system of linear equations by Gaussian elimination on a parallel machine. A graph-theoretic model of this problem based on an existing parallel machine is developed; to this model, a function representing parallel execution time is applied. A graph ordering which minimizes the parallel timing function is studied. Graph orderings which produce certain desirable structures in the corresponding coefficient matrices, such as envelope and low-fill orderings, are well-known for the sequential case. It is demonstrated that the optimum parallel ordering has no relationship with either the envelope ordering or the low-fill ordering. An efficient algorithm is also presented to produce the optimum parallel ordering for trees.

## Coding Strings by Pairs of Strings

F.Chung, R.Tarjan, Bell Labs
W.Paul*, R.Reischuk, IBM Research, San Jose

Let $X,Y \subseteq (0,1)^*$. We say Y codes X if every $x \in X$ can be obtained by applying a short program to some $y \in Y$. We are interested in sets Y that code X robustly in the sense that even if we delete an arbitrary subset $Y' \subseteq Y$ of size k, say, the remaining set of strings Y-Y' still codes X. In general, this can only be achieved by making in some sense more than k copies of each $x \in X$ and distributing these copies on different strings in Y. Thus if the strings in X and Y have the same length, then $\#Y \geq (k+1)\#X$.

If we allow coding of X by Y in a way that every $x \in X$ is obtained from two strings $y,z \in Y$ by application of a short program, then we can do better. Let $Y = (\oplus_{x \in S} x : S \subseteq X)$ where + denotes bitwise sum mod 2. Then $\#Y = 2^{\#X}$, yet Y codes X robustly for $k = 2^{\#X-1} - 1$. This paper explores the limitations of coding schemes of this nature.

## SOME FAMILIES OF CONTEXT-FREE 2-IRREDUCIBLES

Kenneth Holladay, University of New Orleans

A simply connected shape composed of an even number of tiles is 2-irreducible iff it can not be partitioned into two simply connected subsets each with an even number of tiles. The 2-irreducibles on a given tiling can be generated by a context-free array grammar. This grammar can use blanks as context. For some tilings, this grammar does not need to use blanks as context. We give numerous illustrations of the possibilities using tilings whose symmetry group has one or two transitivity classes of tiles.

## ON MAXIMUM CHORDAL SUBGRAPH

Paul Erdős, Math Inst. of the Hungarian Academy of Sciences
Renu Laskar*, Clemson University

A graph G(n) with n vertices is chordal if every cycle of length greater than 3 has a chord. Let f(n) denote the smallest integer such that every graph G(n) can be made chordal by deleting $\leq f(n)$ edges of G(n).

Theorem. $f(n) = \frac{n^2}{2} - (1+\sigma(1))\sqrt{2}n^{3/2}$.

## Nearly Reconstructing Infinite Rayless Graphs

Rüdiger Schmidt, Department of Mathematics, University of Michigan

A graph is called rayless if it contains no infinite path. Two graphs are called hypomorphic, if they have (up to isomorphism) the same set of vertex-deleted subgraphs. A graph G is called reconstructible if G hypomorphic to H always implies that $G \cong H$.

It is well-known that infinite rayless graphs in general are not reconstructible. In fact, there are n-connected rayless graphs which are not reconstructible, for every $n \in \mathbb{N}$. We show the following theorem:

Theorem: For every infinite rayless graph G the number of pairwise non-isomorphic graphs which are hypomorphic to G is at most 2.

As is well-known this is not at all true for arbitrary graphs.

The proof makes use of a structure theory for rayless graphs which was developed by the author.

## APPLICATIONS OF A TECHNIQUE FOR LABELLED ENUMERATION

Brendan D. McKay
Vanderbilt University, Nashville, Tennessee 37235

A method involving summation over roots of unity was successfully used by Liskovec to enumerate labelled regular tournaments. In this paper we apply similar techniques to count many other classes of directed and undirected labelled graphs. An appendix lists the numbers of regular tournaments to 21 points, regular graphs to 21 points (by degree), regular bipartite graphs to 40 points (by degree), eulerian digraphs to 16 points, and eulerian oriented graphs to 15 points. Mention is also made of related work (jointly with R. W. Robinson) on enumerating eulerian circuits in complete graphs.

(19)

## A Method of Bounding the Minimum Distance of Cyclic Codes

Richard M. Wilson, California Institute of Technology

We describe a method of obtaining a lower bound for the weight (i.e. the number of nonzero terms) of a polynomial in terms of its set of roots and apply it to several examples of cyclic codes where it gives better results than the BCH-bound and the Hartmann-Tzeng bound on the minimum distance.

Let $S$ be a subset of a field $E$. We inductively define a family of subsets of $E$ (to be called *independent with respect to* $S$) as follows:

(o) $\emptyset$ is independent w.r.t. $S$.

(i) If $A$ is independent w.r.t. $S$, $A \subseteq S$, $b \notin S$, then $A \cup \{b\}$ is independent w.r.t. $S$.

(ii) If $A$ is independent w.r.t. $S$ and $c \in E$, $c \neq 0$, then $cA$ is independent w.r.t. $S$.

Theorem. Let $S$ be the set of roots $\{a \in E : f(a) = 0\}$ of a polynomial $f(x)$ over $E$. ($f(x)$ need not split in $E$.) Then the weight $wt(f)$ of $f(x)$ satisfies

$$wt(f) \geq |A|$$

for every subset $A$ of $E$ which is independent w.r.t. $S$.

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Constructivistic graph theory and computer programming

(26) (27)

K. Culik and R.G. Trenary, Comp. Sci. Dept. of Wayne State Univ., Detroit, Mi. 48202

PART I-II

1. If one compares (1) a constructive proof of an existential graph theoretical theorem in which a finite object is constructed with (2) a computer program by which the same object is computed, one may observe that the main difference between (1) and (2) consists of the difference between the sets and their computer representation by corresponding data structures (arrays, records, linear lists), respectively. The first are (3) unordered (unsets), and there are no names of their elements, while the second are (4) totally ordered sets (tosets), and therefore each element can be identified by its position (positive integer). Natural operations and relations on finite tosets are introduced.

Metatheorem If one takes a proof (1) and replaces all unsets (3) by arbitrary corresponding tosets (4), then having access to all elements one can make the proof so detailed and specific that it becomes a program (2) (where tosets correspond to arrays available in each programming language).

Examples of constructive proving (1), and then of programming, which is passing from (1) to (2), are presented.

2. Logical types concern unsets only, but can be extended to tosets, as it is required in data types. Syntactical and semantical definitions are presented and studied.

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Graphs With Unique Subgraphs (34)

David W. Bange and Anthony E. Barkauskas, University of Wisconsin-La Crosse and Peter J. Slater, University of Alabama, Huntsville.

Let $E_k$ denote the class of graphs all of whose subgraphs obtained by removal of $k$ or fewer edges are pairwise non-isomorphic. Let $V_k$ denote the class of graphs all of whose induced subgraphs obtained by deleting $k$ or fewer vertices are pairwise non-isomorphic. It is shown that $V_{2k}$ is a subclass of $E_k$ and by construction that this result is best possible.

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Zero-sum weightings, spanning trees and perfect matchings (35)

K. A. Berman and M. Mawrylycz, Wesleyan University, Middletown, CT

Abstract. Let $G$ be a connected multi-graph with vertex $V$ and let $F$ be a field of characteristic $p$. A zero-sum weighting over $F$ is a weighting of the edges of $G$ with elements of $F$ such that for each vertex $v \in V$ the sum of the weights over all the edges incident with $v$ equals zero. Now consider an orientation $O$ of the edges of $G$. For $v \in V$, let $\tau^+(v)$ ($\tau^-(v)$) denote the number of spanning trees $T$ of $G$ such that an even (odd) number of edges of $T$ "point toward $v$". In this paper we show that there exists a zero-sum weighting over $F$ that is a coboundary with respect to orientation $O$ iff for all $v \in V$, $\tau^+(v) = \tau^-(v)$ in the case $p = 0$ and $\tau^+(v) \equiv \tau^-(v) \pmod p$ in the case $p \neq 0$. We also show that if $G$ is an Eulerian graph and $O$ a balanced orientation of $G$ then for any two vertices $v, w \in V$, $\tau^+(v) - \tau^-(v) = \pi^+(w) - \pi^-(w)$ where $\pi^+(w)$ ($\pi^-(w)$) denotes the number of "even" ("odd") pairs of perfect matchings in the graph $G - v$.

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NEAR 1-FACTORS IN GRAPHS (36)

G. Chartrand, S.F. Kapoor, L. Lesniak, Western Michigan University, and S. Schuster, Carleton College.

A near 1-factor of a graph of order $2n(\geq 4)$ is a factor of the type $(n-2)K_2 \cup P_3 \cup K_1$. The Chartrand - Nebeský result that every $r$-regular, $(r-2)$-edge-connected graph $(r \geq 3)$ of even order containing less than $r$ distinct edge cut sets of cardinality $r-2$ has a 1-factor is shown to be best possible in the sense that for each $r \geq 3$, there exists an $r$-regular, $(r-2)$-edge-connected graph $G$ of even order containing exactly $r$ distinct edge cut sets of cardinality $r-2$ that has no 1-factor, but can be (isomorphically) factored into near 1-factors. The Chartrand - Nebeský result is extended in such a way to produce the following as a corollary: If $G$ is an $r$-regular, $(r-2)$-edge-connected graph $(r \geq 3)$ of even order containing less than $2r$ distinct edge cut sets of cardinality $r-2$, then $G$ has a near 1-factor.

CONNECTIVITY AND THE DESIGN OF OPTIMALLY SURVIVABLE SYSTEMS
Leon Kotin, Dept. of the Army (37)

Abstract. We represent a distributed system by a graph $G = G(p,q)$ with $p$ points, to which we assign weights, and $q$ arcs, which are vulnerable to being severed. We define "survivability" of $G$ after severance of some arcs, and then determine graphs, for prescribed $p$ and $q$, which optimize survivability.

(38) REGULARLY k-PATH CONNECTED GRAPHS
Anton Kotzig, University of Montreal

A graph $G$ is called $r$-regularly $k$-path connected ($P_r(k)$-graph) if, to every pair of vertices of $G$, there exists exactly $r$ paths of length $k$ connecting these two vertices. $P_1(1)$-graphs are complete graphs. $P_1(2)$-graphs are friendship graphs and known results will be reviewed. The present state of research for $k > 2$ and $r > 1$ will be discussed.

DIAMETER BOUNDS FOR ALTERED GRAPHS (39)
F.R.K. Chung* & M.R. Garey, Bell Labs, Murray Hill, N.J.

The main question addressed in this paper is the following: If $t$ edges are removed from a $t$-connected graph $G$ having diameter $D$, how large can the diameter of the resulting graph be? (The diameter of a graph is the maximum, overall pairs of vertices, of the length of the shortest path joining those vertices.) We provide bounds on this value that imply that the maximum possible diameter of the resulting graph, for large $D$ and fixed $t$, is essentially $(t+1)\cdot D$. The bulk of the proof consists of showing that, if $t$ edges are added to an $n$-vertex path $P_n$, then the diameter of the resulting graph is at least $(n/(t+1)) - 1$. Using a similar proof, we also show that if $t$ edges are added to an $n$-vertex cycle $C_n$, then the least possible diameter of the resulting graph is (for large $n$) essentially $n/(t+2)$ when $t$ is even and $n/(t+1)$ when $t$ is odd. Examples are given in all these cases to show that there exist graphs for which the bounds are achieved.

We also give results for the corresponding vertex deletion problem for general graphs.

Such results are of interest, for example, when studying the potential effects of node or link failures on the performance of a communication network, especially for networks in which the maximum time-delay or signal degradation is directly related to the diameter of the network.

D. Frank Hsu

On generalized complete mappings (43)
Fordham University

Generalized complete mappings, recently defined and studied by Hsu and Keedwell, have shed new light on a number of new connections between sequencings of groups, R-sequencings of groups, complete mappings, graceful directed graphs, and the construction of neofields and block designs. We briefly survey these results and their connections.

Generalized Set Covering Problems

K.J. Danhof and Y.L. Varol (44)
Department of Computer Science
Southern Illinois University, Carbondale, Illinois 62901

Let $S=(x_1 \cdots x_n)$ be some universe and $\underline{F}=(F_1,\ldots,F_t)$, where for $i \le t$, $F_i \subseteq P(S)$. In addition, a cost, constant over each $F_i$, is associated with the subsets of $S$ thus determined. We are interested in the problem of selecting subsets of $S$, at most one from each $F_i$, in such a way that the extent of $S$ thus covered is maximal while the sum of the costs of the subsets selected is minimal. It is not difficult to show that this problem is NP-hard and subsumes a variety of other such problems including the usual set covering problems and the maximum satisfiability problem.

A greedy heuristic is proposed for this generalized set covering problem and the heuristic is analyzed with respect to its worst case behavior. It is shown that the heuristic will result in a solution which is within a ratio of $\log \gamma$ ($\gamma$ denotes the maximum size of the subsets) or 2 of the optimal solution according as the maximum size of the families $F_i$ is 1 or larger than 1 respectively.

More on the Chessmaster Problem (45)
R.L. Hemminger* and B.D. McKay, Vanderbilt University

At the Silver Jubilee Conference at The University of Waterloo in June 1982 the following sequence problem was presented along with some claims and conjectures.

Generalized Chessmaster Problem (GCMP): Let $k,n,b$ and $c$ be positive integers with $b \le n,c$ and let $A=(a_i)_{i=0}^{\infty}$ be a sequence of integers. Consider the following conditions on $A$:

(1) $0 = a_0 < a_1 < a_2 < \ldots < a_n$,

(2) $a_{i+b} \le a_i + c$, $\forall\, i (0 \le i \le n - b)$ and

(3) $\exists i,j$ $(0 \le i < j \le n)$ such that $a_j = a_i + k$.

Call $A$ acceptable if $A$ satisfies (1) and (2) and define $N(b,c,k)$ by
$N(b,c,k) = \min\{n \mid (3) \text{ holds for all acceptable } n\text{-sequences}\}$.
The problem is to find $N(b,c,k)$.

We will present further results and conjectures concerning this problem. Many of our conjectures are based on computer generated information.

AVERAGE-TIME TESTING OF SATISFIABILITY ALGORITHMS
A.K. Dowdnoy*, T.R.S. Walsh, D.L. Wohlau
The University of Western Ontario (47)

An enumeration of instances of the satisfiability problem reveals a sharp concentration, over all instances of length $\ell$, in the neighborhood of $n = .72\ell$, where $n$ is the number of variables. As $\ell \to \infty$, this peak appears to become sharper. Using duality, one obtains a scheme for generating "typical" instances of satisfiability for submission to various satisfiability algorithms undergoing testing.

The Davis-Putnam procedure and some new algorithms for the satisfiability problem are tested under this scheme and the results compared. All algorithms tested appear to run in $O(\ell)$ average time.

THE AVERAGE GENUS OF GRAPHS
Saul Stahl, University of Kansas (50)

Topological graph theorists first computed minimum genera of some classes of graphs. In the 70's, a considerable amount of attention was given to the maximum genus of a graph. The author now proposes to look at the average genus of graphs. The use of rotation systems translates this into questions about the symmetric group. The resolution of these questions calls for some standard counting techniques, as well as some more sophisticated ones that involve the characters of the symmetric group. Some partial results will be presented and an explanation will be given for the fact that in the few cases where the average genus has been computed, it seems to be much closer to the maximum genus than to the minimum one.

THE MAXIMUM CONCURRENT FLOW/MINIMUM RELATIVE CAPACITY DUALITY THEOREM - A LABELING ALGORITHM PROOF
David W. Matula, Southern Methodist University (51)

$P(i,j)$ denotes the set of all paths of the graph $G$ between vertices $i$ and $j$ for $i,j \in V(G)$, and $P(e)$ denotes the set of all paths containing the edge $e \in E(G)$. A concurrent flow function $f: \bigcup_{i,j} P(i,j) \to R$ associates a flow value $f(p) \ge 0$ with every path $p \in \bigcup_{i,j} P(i,j)$, which is feasible relative to the capacity constraints if $\sum_{p \in P(e)} f(p) \le 1$ for all $e \in E(G)$, and realizes the concurrent flow level $z$ if $\sum_{p \in P(i,j)} f(p) = z$ for every pair $i,j \in V(G)$. We describe a duality theorem between the maximum concurrent flow level and a concept termed the minimum relative capacity, providing a labeling algorithm proof of the result. Applications in telecommunication networks, transportation networks, and cluster analysis will be described.

## THE DIGRAPH DISTANCE LABELLING PROBLEM (40)

Peter Winkler, Emory University

The "distance" from vertex u to vertex v in a strongly connected digraph is the length of the shortest directed path from u to v. The distance labelling problem, first formulated in the graph case by Graham and Pollak, entails addressing the vertices in such a way that the distance from one vertex to another may be found by comparing their addresses. The problem occurs in packet-type communications networks, and has some interesting connections with linear algebra.

The results and open questions to be presented are due to F. Chung, E. Dent, R.L. Graham and R. Roth as well as the writer.

## More on Mission Networks (41)

K. E. Buell and R. D. Ringeisen*, Clemson University

L. Lesniak-Foster, Western Michigan University

Mission networks were first introduced by R. Ringeisen and L. Lesniak-Foster in the paper, "Decision Points in Mission Networks". This follow-up paper discusses three types of mission networks, namely uniform, fundamental, and multi-goal mission networks. A mission network N is said to be uniform if every decision point has the same importance. Fundamental mission networks are ones for which the number of decision points (complexity) reaches its upper bound. We show that any mission network may be embedded as an induced subnetwork of a uniform mission network. Moreover, every mission network can be expressed as the edge sum of fundamental mission subnetworks, where exactly one such subnetwork is spanning as well as importance and decision point preserving.

## Superfluous Paths in Strong Digraphs (42)

Jeffrey A. Ross*, University of South Carolina, USA
Claudio L. Lucchesi, IMECC-UNICAMP, Brasil

A branch of a strong digraph is a path whose endpoints have degree at least three and whose internal vertices have degree two. We obtain results concerning branches whose removal yields a strong digraph. Among other things we obtain upper and lower bounds on the number of such branches and characterize the cases of equality.

## Permutations of Integers
Richard Friedlander, University of Missouri-St. Louis (46)

It has been conjectured that, given any finite subset $D = (D_1,\ldots,D_n)$ of the set N of positive integers, there exists a permutation $(a_k)$ of N such that $(|a_{k+1}-a_k| \; K \in N) = D$ if and only if $(D_1,\ldots,D_n) = 1$. P. J. Slater and W. Y. Vélez have shown that this is a necessary condition, and have proved that it is sufficient if, corresponding to each $D_i \in D$, there is a $D_j \in D$ such that $(D_i,D_j) = 1$. B. Gordon and myself have recently shown the condition to be sufficient in the more general setting stated in the conjecture.

## ON THE REVERSE OF A PERMUTATION (47)
Joseph B. Klerlein* & A. Gregory Starling, Western Carolina U.

The reverse of the permutation $(v_1,v_2,\ldots,v_n)$, in functional notation, is the permutation $(v_n,v_{n-1},\ldots,v_1)$. We consider several algorithms that generate the n! permutations and for each consider the problem of selecting a complete reverse-free collection.

## MINIMAL COST PERMUTATION GENERATING ALGORITHMS (48)
Peter Tannenbaum, University of Arizona

Let $S_n$ denote the symmetric group on n symbols and let $w:S_n \to R$ be a given cost function in $S_n$ (where w(P) denotes the cost of multiplying a permutation by P.) A minimal cost permutation generating algorithm for w is an algorithm which generates a sequencing $P_1,P_2,\ldots,P_{n!}$ of all permutations in $S_n$ in such a way that the cost $w(C_1) + w(C_2) + \ldots + w(C_{n!-1})$ (where $C_i = P_{i+1}\cdot P_i^{-1}$ is the change needed to pass from $P_i$ to $P_{i+1}$ in the sequence) is minimal. In general, the cost function w is defined by the specific nature of the problem at hand. We will give several examples of minimal cost permutation generating algorithms and some of their applications in combinatorics, statistics and theoretical computer science.

## THE MIN COST DYNAMIC FLOW PROBLEM (52)
Horst W. Hamacher, University of Florida

In a network G = (N,E) we have given three functions defined on the arc set E: the capacity function u(e), the transversal time function τ(e) and the cost function a(e). Then we consider the Min Cost Dynamic Flow Problem (MCDF). Find a maximal dynamic flow the linear cost of which is minimal. Problems of this type occur often in practical applications and we present two algorithms for solving this problem.

In the first algorithm we reduce MCDF to a classical min cost flow problem in an extended network. This extended network contains T + 1 many copies of each node of the original network, where T is the number of time periods considered. The second algorithm avoids this disadvantage of dealing with this large network. MCDF is in this algorithm solved by finding a lexicographical min cost flow in the original network. An additional decomposition step will then generate an optimal solution of MCDF. Whereas the time and space complexity of the first algorithm is dependent on the number T of time periods the second one has the same complexity as an algorithm for solving the classical min cost flow problem with linear cost.

## FUNDAMENTAL ORBITS 53
Craig K. Bailey, U. S. Naval Academy

The orbit size of a point in a tree is the cardinality of the orbit of the automorphism group of the tree in which the point lies. The fundamental orbit size of a point is a similar concept but so defined such that the cardinality of the automorphism group of the tree is the product of the fundamental orbit sizes of its points.

This is accomplished by defining two points to be in the same fundamental orbit if and only if they are in the same orbit and have a common neighbor. Points in fundamental orbits are counted using generating functions and asymptotic techniques are applied to determine average sizes of these orbits.

Thus, the average size of the automorphism groups for trees of maximum degree 3 and maximum degree 4 is determined. These trees are used as models of organic molecules.

## REPEATED GRAPHS WITH DIRECT PRODUCT AUTOMORPHISM GROUPS (54)
Peter R. Christopher, Worcester Polytechnic Institute

A graph X whose automorphism group is isomorphic to a group G is referred to as a G-graph. Let G be of the form $S_m \times S_k$, a direct product of symmetric groups $S_m$ and $S_k$, with $m \geq k \geq 2$. Necessary and sufficient conditions are found for $S_m \times S_k$ to be isomorphic to a complete monomial group. One implication for graphs is the following:

An $S_m \times S_k$-graph X has repeated non-asymmetric components if and only if m=4, k=2, and X or its complement is the disjoint union of three isomorphic $S_2$-components and possibly an asymmetric graph.

## GRID LAYOUTS OF GRAPHS

Zevi Miller*, Miami University, Oxford, Ohio
J. Orlin, Sloan School of Management, M.I.T.

(55)

Let G and H be graphs, $|H| \geq |G|$ and H connected, and $f: V(G) \to V(H)$ an injection. Let $C(G,H)$ be $\min(\max d_H(f(x), f(y)))$, where the max is taken over all edges $xy \in E(G)$ with f fixed, the min is over all f, and $d_H$ denotes distance in H. Let $H_1$ and $H_2$ be the graphs of the one and two dimensional integer lattices. Determining $C(G,H_1)$, known as the bandwidth of G, is known to NP-complete, even when G is restricted to being a tree. We show that determining $C(G,H_2)$ is NP-complete.

## EXTREMAL GRAPHS WITH NO DISCONNECTING MATCHING

by
Arthur M. Farley and Andrzej Proskurowski*
Computer and Information Science Department
University of Oregon

(56)

We describe classes of graphs with the minimum number of edges for a given number of vertices that have no disconnecting, independent set of edges. An independent edge set (i.e., a matching) has no pair of edges incident to the same vertex. A disconnecting set is one which, by its removal, transforms a given, connected graph into one having at least two connected components. We note a relationship to previously defined extremal graphs having related connectivity and forbidden subgraph properites.

## A Max-Min Relationship between Matchings and Domination

Stephen T. Hedetniemi
Department of Computer Science, Clemson University

(57)

In 1959 Gallai established two theorems, one relating vertex coverings ($\alpha 0$) to independent sets ($\beta 0$), the other relating edge coverings ($\alpha 1$) to matchings ($\beta 1$), of the form:

$$\alpha 0 + \beta 0 = p, \text{ and} \quad (1)$$
$$\alpha 1 + \beta 1 = p, \quad (2)$$

where p is the number of vertices in a graph. In 1974 Nieminen established a result of the same form, relating end-eges in spanning forests ($\varepsilon$) to domination ($\gamma$), i.e.

$$\varepsilon + \gamma = p. \quad (3)$$

In this paper we define a new set of parameters which can be used (i) to obtain a result which is dual to (1) above, and (ii) to show that (2) and (3) are, in fact, dual to each other. Thus, we obtain a somewhat surprising 'max-min' relationship between matchings ($\beta 1$) and domination ($\gamma$).

## A Simple, Automated Construction Technique for the Family of Singer Designs

Robert L. Holliday, Lake Forest College

(63)

Let q be the power of a prime and let $P_q$ be the $(q^2+q+1, q+1, 1)$ projective plane of order q. Using only a complete set of mutually orthogonal latin squares and simple counting arguments, a direct construction technique is given for the family of Singer designs with parameters $(q^n+q^{n-1}+\ldots+q+1, q^{n-1}+\ldots+1, q^{n-2}+\ldots+1)$. The technique is illustrated with an example and an implementation of the algorithm in Pascal is presented. The algorithm is of linear complexity (in terms of the number of points of the design), makes use of the set feature of Pascal and for an input of a particular Singer design, returns the next design in the family.

## AN IMPROVED PRODUCT THEOREM FOR BALANCED HOWELL ROTATIONS

B. A. Anderson Arizona State University

(64)

Recently Du and Hwang have given a product theorem for balanced Howell rotations. Their argument requires that one of the factors have order one more than the order of a cyclic Hadamard difference set and that certain other conditions be satisfied. In this paper it is shown that these rather severe conditions can be relaxed a little. At the same time it is possible to simplify the Du-Hwang arguments by using the well known ideas of (1) the Kronecker product of Hadamard matrices and (2) 1-factorizations associated with the composition of complete graphs.

## Perfect Pair-coverings with Blocksizes Two, Three and Four

R.C. Stanton and D.R. Stinson*, University of Manitoba

(65)

We investigate the minimum number of blocks in a perfect pair-covering (pairwise balanced design) which contains only blocks of size 2,3, and 4. We denote this covering number by $g^{(4)}(v)$, where v is the number of points. We determine $g^{(4)}(v)$ for all v, with three exceptions: v = 17, 18, and 19.

## ON NEW FAMILIES OF RECURSIVELY DEFINED PERFECT GRAPHS

D.G. Corneil* University of Toronto
D.G. Kirkpatrick, University of British Columbia

(71)

A cograph is either an isolated vertex or a graph formed recursively by applying the operations of complementation or union to cographs. This construction allows a cograph to be represented uniquely by a tree thereby resulting in linear time algorithms for many problems which are NP-complete for general graphs. We extend this recursive construction to include other operations which preserve perfectness. In some cases the new class of graphs have a unique tree representation; in others the tree is not unique. We study the properties of these graphs and compare them with known families of perfect graphs.

## CONCERNING THE ACHROMATIC NUMBER OF TREES

M. Farber (U. of Waterloo), G. Hahn (U. Paris Sud),
P. Hell* (Simon Fraser U.) and D.J. Miller (U. Victoria)

(72)

It is NP-complete to determine, given an integer k and a graph G, if the achromatic number of G is at least k. However, for each fixed k, there are polynomial algorithms which for any graph G determine if achr $G \geq k$. We concentrate on the case of trees. When a tree T has at least (roughly) $\frac{4}{5}\binom{n}{2}$ edges and maximum degree $\Delta < n$, then achr $T \geq n$. For trees T with exactly $\binom{n}{2}$ edges and $\Delta < n$ we obtain some necessary and some sufficient conditions for achr T=n. In particular, we solve the achromatic number problem for the class of trees with a bounded number of endpoints, and for a class of trees in which the number of endpoints grows only slightly slower than the total number of vertices.

## HAMILTONIAN DECOMPOSITION OF PRISMS OVER SIMPLE 3-POLYTOPES

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CANADA

(73)

In search for Hamiltonian cycles in simple 4-polytopes, we observed that certain families of such polytopes not only have a Hamiltonian cycle, but actually have a Hamiltonian decomposition. It was conjectured that the prisms over simple 3-polytopes have a Hamiltonian decomposition. Partial results and related ideas will be presented.

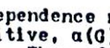
Reducing space and time requirements for processing trees

Sandra Hedetniemi * and Stephen Hedetniemi
Clemson University (58)

During the past few years several dozen combinatorial algorithms on trees have appeared. The large majority of these algorithms have been designed using the parent-array (or recursive representation) of a tree. Using this representation it is natural to think in terms of an algorithm which passes one or more times over the array and stops, in O(N) time, with the desired result, where N is the number of vertices in the tree. In this paper we introduce several of many possible ways of compressing the recursive representation of a tree into a smaller, implicit data structure. In particular we introduce the concept of the family representation of a tree and we study the corresponding space and time requirements for computing a variety of parameters of tree such as: (i) the independence number, (ii) a b-matching, (iii) the domination number, (iv) the diameter, and (v) a shortest path.

Maximal Independent Sets in Vertex-Transitive Graphs (59)

Mark Ramras, Northeastern University

Let $\alpha(G)$ denote the independence number of the finite graph G. For G vertex-transitive, $\alpha(G)/|G| \leq \alpha(H)/|H|$ for every induced subgraph H. Thus, for example, $\alpha(G)/|G| \leq k/2k+1$ if G contains a $2k+1$ -cycle, and $\alpha(G)/|G| \leq 1/k$ if G contains a k-clique. (The trivial example G =  for which $\alpha(G)/|G| = 2/5$ shows that some hypothesis is needed.) For G = the truncation of a 3-regular graph G' (every vertex of G' is blown up to a triangle), or more generally, if each vertex of G has degree 2 or 3 and V(G) is the union of disjoint triangles, an algorithm is proved which constructs an independent set achieving the cardinality $(1/3)|G|$.

ON MATROIDS OF THE GREATEST WHITNEY CONNECTIVITY (60)

Li Weixuan, University of Waterloo

Abstract. The Whitney connectivity of a matroid M is defined by T. Inukai and L. Weinberg as the smallest integer k for which there exists a subset S of the ground set E of M such that $\rho(S) \geq k$, $\rho(E-S) \geq k$ and

$$\rho(S) + \rho(E-S) - \rho M + 1 = k,$$

where $\rho$ is the rank function of M. M is called a Whitney matroid (W-matroid) if there exists no such an integer. In this case, we defined the Whitney connectivity of M to be the rank of M.

In this paper, several properties of W-matroids are discussed. A characterization of the W-matroids whose dual are also W-matroids is given. And, the binary W-matroids are investigated in some detail.

W. H. Mills
Institute for Defense Analyses (66)

There are only four values of $v \leq 136$ for which the existence of a balanced incomplete block design with $k = 6$ and $\lambda = 1$ is an open question. These values of $v$ are 46, 51, 61, and 81. Let $G$ be the automorphism group of a BIBD with one of these four values of $v$, and $k = 6$, $\lambda = 1$. If $p$ is a prime greater than 5, then it can be shown that $p$ does not divide the order of $G$. In two cases this result depends on computer runs. The remaining primes are 2, 3, and 5. For each of these three primes and each of the four values of $v$, we give a realistic upper bound for the maximal power of the prime that can divide the order of $G$. Some of these upper bounds depend on computer runs.

EMBEDDING MAXIMAL PACKINGS OF TRIPLES INTO MAXIMAL PACKINGS OF TRIPLES (67)

Eric Mendelsohn, U. of Toronto & Alexander Rosa*, McMaster U.

The Doyen-Wilson theorem states that a Steiner triple system (STS) of order u can be (properly) embedded in an STS of order v if and only if $v \geq 2u + 1$. Since an STS is simply a maximal packing of triples which happens to be exact, the following more general question arises naturally: for given $s,t \in \{0,1,\ldots,5\}$ and $u, u \equiv s \pmod 6$, what are the values of $v$, $v \equiv t \pmod 6$, such that a maximal packing of triples on u elements can be (properly) embedded in a maximal packing of triples on v elements? We present results concerning the smallest such values of v.

An Efficient Algorithm for Directing Triple Systems (68)

Janelle J. Harms and Charles J. Colbourn*,
Department of Computational Science,
University of Saskatchewan,
Saskatoon, Saskatchewan, Canada.

A triple system $B[3,\lambda;v]$ is an edge-decomposition of the complete multigraph $\lambda K_v$ into triangles, and a directed triple system $DB[3,\lambda;v]$ is an arc-decomposition of the complete directed multigraph $\lambda K_v^*$ into transitive tournaments of order three. Every triple system $B[3,2\lambda;v]$ is the underlying system of some $DB[3,\lambda;v]$. We give a new proof of this result, which provides the basis for a remarkably efficient algorithm for directing triple systems.

The algorithm proceeds in two steps. First, the b blocks are sorted into lexicographic order in O(b log b) time. Secondly, the set of blocks is partitioned into segments. The segment for an element x is the collection of all blocks having x as their smallest element. These segments are then processed in reverse order. The undirected triples are replaced by transitive tournaments in such a way that no directed pair appears in more than $\lambda$ of the directed blocks. This latter constraint is ensured by using a modified Euler circuit algorithm which operates in linear time.

COMPUTING THE INDEX OF A GRAPH (74)

Kenneth Lane, Colby College, Waterville, Maine

The index, or spectral radius of a graph is defined to be the eigenvalue of the graph of maximal absolute value. A computational technique for finding the index is presented. An analysis of the method provides for a shortcut for the computation which enables us to write down a polynomial which divides the characteristic polynomial. This short cut can be used on a wide class of graphs, including all trees and other graph exhibiting sufficient symmetry.

GRAPHS WITH CYCLIC DEFECT (75)

S. Fajtlowicz, University of Houston

Let G be a graph of maximum degree d and diameter 2. A graph H is said to be the defect of G if its adjacency matrix B satisfies the equation

$$(*) \qquad A^2 + A - (d-1)I = J + B$$

where J is a matrix whose every entry is 1.

If B is the zero matrix then (*) is the equation of the Moore graph (of diameter 2). With the exception of the case d = 57, all graphs satisfying this equation were found by A.J. Hoffman and R. Singleton. The case of B being the adjacency matrix of a matching was solved by Erdős, Hoffman and myself.

We shall show that if G is the adjacency matrix of a cycle then G is the Möbious Ladder $M_4$.

A CHARACTERIZATION OF UPPER BOUND GRAPHS (76)

J. Richard Lundgren & John S. Maybee*, Univ. of Colorado

The upper bound graph and strict upper bound graph of a poset are characterized in terms of clique covers. These characterizations allow one to find all posets that have the same upper bound graph or strict upper bound graph.

Generalization of the Cycle Matroid of a Graph (61)

Andrew Vince University of Florida

The cycles of a connected graph G are the circuits of a matroid M(G), called the cycle matroid of G. The bases of M(G) are exactly the spanning trees of G. A set S of subgraphs of G is called a cover if $\cup S = G$. We introduce a generalization $M_S(G)$ of the cycle matroid of G with respect to a cover S. Fundamental questions about $M_S(G)$ concern its bases. These questions are motivated by problems in PL-topology.

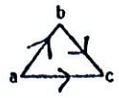
"The ✱-Sperner Property" (62)

Prof. Jerrold R. Griggs, University of South Carolina

A ranked poset P is said to have the Sperner property if no antichain in P is larger in size than the largest ranks. If the largest ranks are the only maximum-sized antichains, P is said to have the ✱-Sperner property For instance, Sperner's theorem implies that $B_n = 2^n$, the Boolean algebra of subsets of {1,...,n}, order by inclusion, is ✱-Sperner. Several other nice posets can be shown to be ✱-Sperner. If time remains, some other interesting recent results in Sperner theory will be presented.

EMBEDDING PARTIAL TRANSITIVE TRIPLE SYSTEMS (69)

Rose C. Hamm, College of Charleston, S.C.




A transitive triple ((a,b),(b,c),(a,c)) is a collection of three ordered pairs of distinct elements in which each of the ordered pairs represents a directed edge in the above directed triangle.

A Transitive Triple System (TTS) is a pair (S,T) where S is a set and T is a collection of transitive triples with the property that every ordered pair of distinct elements of S belongs to exactly one transitive triple in T.

From Wilson's Theorem we know that a partial TTS of order v can be embedded in a TTS of order n for n sufficiently large and $n \equiv 0$ or $1 \pmod 3$. Here we will embed a partial TTS of size v in a TTS of size n for some $n \leq 6v^2-15$.

PERFECT SYSTEMS OF DIFFERENCE SETS WITH A SMALL NUMBER OF COMPONENTS (70)

J. Abrham*, Dept. of Industrial Engineering, University of Toronto
A. Kotzig, C.R.M.A., Université de Montréal
P.J. Laufer, College Militaire Royal, St. Jean, P.Q.

Let c, m, $p_1, \ldots, p_m$ be positive integers. Let

$S_i = (x_{oi} < x_{1i} < \ldots < x_{p_i,i})$, i=1,...,m be sequences of integers, and

let $D_i = (x_{ji} - x_{ki},\ 1 \leq k < j \leq p_i)$, i=1,...,m be their difference sets.

Then we say that the system $(D_1,\ldots,D_m)$ is a perfect system of difference sets (PSDS) starting with c if

$\bigcup_{i=1}^{m} D_i = (c, c+1, \ldots, c - 1 + 1/2 \sum_{i=1}^{m} p_i(p_i + 1))$. Each $D_i$ is called a component of the PSDS $(D_1,\ldots, D_m)$; the size of $D_i$ is $p_i$. All PSDS with at most three components are described. There exist two PSDS with one component, one PSDS with two components, three PSDS with three components starting with 2 and twenty-five PSDS with three components starting with 1.

Measures of Hashing Schemes and Information Theory. (77)

by Alan J. Coppola, Dept. of Mathematics and Computer Science, Oregon State University, Corvallis, OR, 97331.

A relationship between probabilistic measures of complexity for external hashing schemes [Lipton, Rosenberg, Yao; JACM, Jan. 1980, pp.81-95] and information theory will be given.

Complexity of Graphs for Software Measurement (78)

W.R. Edwards and J.M. Bieman
CMPS Dept., U.S.L., Lafayette

The increasing importance given to software engineering and management has motivated intense search for appropriate numerical measures for the complexity of software systems, programs, or program modules. The metric of McCabe, based on the cyclomatic number of the control flow graph, is well known and is currently being used to set limits on complexity of modules and direct program testing. Other graph-based metrics and current research is discussed and necessary characteristics of complexity measures are analyzed.

## MULTICRITERIA SPATIAL PRICE, EQUILIBRIUM NETWORK DESIGN: THEORY AND COMPUTATIONAL RESULTS (79)

Terry L. Friesz and Patrick T. Harker, Univ. of Pennsylvania

In this paper we consider the problem of determining the optimal design of a transportation network using a vector valued criterion function when the users of this network are assumed to behave according to the principles of spatial price equilibrium. This formulation allows for a broader analysis of the effects of investments in transportation networks on not only the users of this network, but also of those agents indirectly affected by transportation costs, time delays, etc. We describe and test two alternative solution techniques which use in the network equilibrium subproblems a nonlinear complementarity pivoting algorithm.

## MFST82; A PROGRAM FOR MINIMUM FULL STEINER TREES IN THE PLANE (80)

E.J. Cockayne* & D.B. Peters, University of Victoria

An essential step in all existing algorithms for minimum total length networks connecting a given set A of plane points is the construction of minimum length full Steiner trees (MFSTs) on subsets of A. We present a new program for MFSTs called MFST82 which is based on earlier programs of the first author and W.M. Boyce. Geometry of the given points is utilized to eliminate steps in the earlier algorithms, thereby substantially reducing machine times.

## A Generalization of Tarjan's Depth First Search Algorithm for the Biconnectivity Problem (81)

Yung H. Tsin
Department of Computing Science
University of Alberta

A sequential algorithm for finding all biconnected components and all seperation vertices of an undirected graph is presented. The algorithm requires $O(n+|E|)$ time and space, where $n$ is the number of vertices and $|E|$ is the number of edges of the graph being examined, and is therefore optimal. This algorithm does not rely on the well-known depth-first search spanning tree but uses *any* spanning tree of the graph. It is also shown that the algorithm is a generalization of Tarjan's depth-first search algorithm for the biconnectivity problem.

## Linear constraints related to Turán's problem (85)

D. de Caen, Univ. of Waterloo

The Turán number $(Tn,\ell,k)$ is the smallest possible number of edges in a k-graph (= k-uniform hypergraph) on n vertices such that every $\ell$-set of vertices contains an edge. Given a k-graph $H = (V,E)$, we let $\lambda_i$, $0 \le i \le \binom{\ell}{k}$, equal the number of $\ell$-sets contained in V that contain i edges of H. Turán's problem is equivalent to estimating the linear form $\sum_i \lambda_i$, given that $\lambda_0 = 0$. This paper presents a few linear relations that hold among the $\lambda_i$.

## The Bracket of 2-Extensors (86)

Neil L. White, University of Florida

A 2-extensor ab is the vector of Plucker coordinates of the line determined by two points a and b in an n-dimensional Euclidean space. This vector has length $\binom{n+1}{2}$, and a bracket of 2-extensors is the determinant of $\binom{n+1}{2}$ of these 2-extensors. Theorem 1: Two infinitesimally rigid bodies in n-space which are joined by $\binom{n+1}{2}$ rigid bars attached at flexible joints yield a single rigid body if and only if the bracket of the 2-extensors determined by the bars is non-zero. Theorem 2: The bracket of 2-extensors is an invariant of the general linear group, and hence may be expressed in terms of ordinary brackets (i.e. (n+1)x(n+1) determinants ) of the points determining the 2-extensors. The problem of determining explicitly such an expression in dimension 3 is discussed.

## UNIT DISTANCES (87)

Joszef Beck and Joel Spencer* SUNY (Stony Brook)

In 1946 Paul Erdős asked for the maximal number f(n) of unit distances among n points in the plane. He showed $n^{1+O(1)} < f(n) < n^{3/2}$. We improve the upperbound to $f(n) < n^{3/2-\epsilon}$ where $\epsilon > 0$ is an absolute constant.

## Some Results on the Ochromatic Number of a Graph* (91)

Gustavus J. Simmons, Sandia National Laboratories, Albuquerque, NM 87185

Given a graph G and an ordering $\varphi$ of its vertices, denote by $\chi_\varphi(G)$ the least number of colors that suffice to properly color the vertices of G in the order $\varphi$; subject to the rule that a new color is introduced only when a vertex cannot be properly colored in its order with any of the colors already used. The ochromatic number, $\chi^o(G)$, of G is defined to be the maximum of $\chi_\varphi(G)$ over all possible orderings, $\varphi$, of the vertices of G.

It is true, and nearly obvious, that;

$$\chi(G) \le \chi^o(G) \le \Delta(G)$$

Brook's Theorem characterizes the graphs for which equality holds between the outer terms of the inequality. Several results will be given defining when inner equalities hold.

It is true of almost all graph theoretic parameters, P, that if $H \subset G$, then $P(H) \le P(G)$. The ochromatic number however can violate this rule; i.e., $H \subset G$ and $\chi^o(H) > \chi^o(G)$ can occur! Examples are exhibited.

* This work performed at Sandia National Laboratories supported by the U. S. Department of Energy under contract number DE-AC04-76DP00789.

## UPPER BOUND FOR CHROMATIC NUMBER OF A HYPERGRAPH (92)

Michael Burstein

IBM Thomas J. Watson Research Center, Yorktown Heights, New York

A *hypergraph* is a pair $G = (V, E)$ where $V$ is a set of vertices and $E$ is a set of subsets of $V$, called edges. A *coloring* of the hypergraph $G = (V, E)$ is a mapping $f: V \to P$ from the set of vertices $V$ to a set of colors $P$, which is not constant on every edge, i.e. for any edge $e \in E$ there exist two distinct vertices $x, y \in e$ such that $f(x) \neq f(y)$ (we assume that every edge has cardinality of at least 2). The *chromatic number* $\chi(G)$ of a hypergraph is the minimal cardinality of the set of colors for which a coloring exists: $\chi(G) = \min\{|P|\}$.

Notations: for any subset $A \subseteq V$ of vertices $E(A)$ denotes the set of edges belonging to $A$: $E(A) = \{e \in E : e \subseteq A\}$; $\Delta(G) = \max\{|E(A)| - |A| : A \subseteq V\}$.

**Theorem.**

$$\chi(G) \le \left[\frac{3+\sqrt{9+8\Delta(G)}}{2}\right].$$

In the case when $|E(A)| < |A|$ for every $A \subseteq V$, i.e. when $\Delta(G) < 0$, we have $\chi(G) < 2$. This fact was conjectured by Erdős and proved by Lovász in 1967. Since then it has been and is being independently rediscovered by numerous authors with a frequency of once-in-two-years.

In the case when $G$ is a connected graph, $\Delta(G) = q - p$, where $q$ and $p$ are numbers of edges and vertices respectively. In this case the above inequality results in an exact upper bound for the chromatic number of connected graphs originally obtained by Ershov and Kozhukhin in 1962.

## The Group Reduced Chromatic Polynomial of a Graph (93)

Allen J. Schwenk, U.S. Naval Academy

The standard chromatic polynomial, counting the number of proper coloring of the graph G using $\lambda$ colors, in effect counts the patterns for a labeled graph. We introduce the concept of allowing a group of symmetries to reduce the number of distinctive colorings. Both automorphisms of the graph and permutations of the colors can be allowed. Using the full automorphism group of G produces the unlabled chromatic polynomial. Allowing the symmetric group to permute the colors give the number to patterns with "color indifference."

The unlabeled chromatic polynomial distinguishes among trees. It is conjectored that every tree has an unlabeled polynomial that can be expressed as a linear combination of falling factorials with nonnegative unimodal coefficients. Another problem is to determine the structure of graphs which are uniquely colorable in the unlabeled sense.

## CONSTRUCTIONS OF MINIMALLY 2-CONNECTED AND MINIMALLY 3-CONNECTED GRAPHS

Robin W. Dawes, University of Toronto (82)

Two known constructions of minimally 2-connected graphs are examined and compared, and problems with each are considered. A modification is suggested which allows a construction of precisely the minimally 2-connected graphs. The relationship between this construction and a construction for precisely the minimally 3-connected graphs is shown.

## HOW TO CRACK AN RSA CODE

Samuel S. Wagstaff, Jr., University of Georgia (83)

We describe the algorithms and devices which may be used to factor large integers. The algorithms include the continued fraction method, the quadratic sieve method, and enhancements of these techniques such as the early abort strategy. The devices include serial computers, parallel processors, and special hardware designed especially for the above algorithms.

## Are Most Low Density Knapsacks Solvable in Polynomial Time?*

Ernest F. Brickell, Sandia National Laboratories, Albuquerque, NM 87185 (84)

A knapsack (or subset sum) problem consists of a set of n+1 integers $a_1,\ldots,a_n$, and s. Let $A = \max\{a_1,\ldots,a_n\}$. The density of the knapsack is defined to be $n/\log_2 A$.

The algorithm we will describe is a modification of the Shamir-Adleman algorithm for breaking the Graham-Shamir cryptosystem. The algorithm is designed for knapsacks of density $\leq 1$. In particular the algorithm should break multiple-iterated Graham-Shamir or Merkle-Hellman cryptosystems. Tests of the algorithm have been completed for $n \leq 20$ and density $\leq \frac{1}{2}$, but a theoretical proof is not yet available.

* This work performed at Sandia National Laboratories supported by the U. S. Department of Energy under contract number DE-AC04-76DP00789.

## COMMON TRANSVERSALS FOR FAMILIES OF PARTITIONS

M. Livingston, S. Illinois Univ. at Edwardsville (88)

Let S be a set and suppose $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_t$ is a family of partitions of S where $\mathcal{F}_i = \{F_{ij}: j=1,2,\ldots s\}$, $F_{ij} \neq \emptyset$, $\bigcup_j F_{ij} = S$, and $F_{ij} \cap F_{ik} = \emptyset$ for $j \neq k$. Let $m = \min_{i,j} |F_{ij}|$ and $k = \lceil \log_2 m \rceil$. For certain values of s and t we determine the least positive integer $N = N(s,t,k)$ such that for $|S| \geq N$, every such family of partitions of S has a common transversal. Moreover, when $|S| = N$, we determine the structure of those families which fail to have a common transversal.

## INTERSECTION PROPERTIES OF FAMILIES CONTAINING SETS OF NEARLY THE SAME SIZE

P. Erdös, Hungarian Academy of Sciences; R. Silverman*, Fairfield University; A. Stein, University of Connecticut (89)

A family F of sets has property B(s) if there exists a set S whose intersection with each set in F is non-empty but contains fewer than s elements.

P. Erdös has asked whether there exists an absolute constant c such that every projective plane has property B(c).

In this paper, the authors, as a partial answer to this question, obtain the result that for n sufficiently large, every projective plane of order n has property B(c log n). The result is a corollary of a theorem applicable to somewhat more general families of finite sets.

## DISJOINT CYCLES IN TOURNAMENTS

Peter M. Gibson, University of Alabama in Huntsville (90)

Let $T(R_n)$ be the set of all tournaments T with score vector $R_n = (r_1,\cdots,r_n)$. Let c(T) denote the maximal number of disjoint cycles in T, $c^*(R_n) = \max\{c(T): T \in T(R_n)\}$, and $S_n = \{c^*(R_n): R_n$ is a strong score vector$\}$. It is shown that $\min S_n = [\frac{n-1}{2}]$ and $\max S_n = [\frac{n}{3}[\frac{n-1}{2}]]$. Let $G(R_n)$ be the graph whose vertices are the tournaments in $T(R_n)$ with $T,T' \in T(R_n)$ joined by an edge when one can be obtained from the other by reversing a 3-cycle. It is shown that $G(R_n)$ contains an $[\frac{n-1}{2}]$-cube graph for each strong score vector $R_n$, and that $G(R_n)$ contains an $([\frac{n}{3}[\frac{n-1}{2}]] - 1)$-cube graph for some strong score vector $R_n$.

## A DIFFERENTIAL EQUATION ARISING IN CHROMATIC SUMS

A.M. Odlyzko, Bell Labs., Murray Hill, N.J.
L.B. Richmond*, University of Waterloo (94)

Let $P(T,\lambda)$ denote the chromatic polynomial of the graph of the rooted loopless triangulation, T, of the sphere. Let $h_n$ denote the sum of $P(T,\lambda)$ over all rooted triangulations of the sphere with 2n faces. The asymptotic behaviour of $h_n$ is determined using Tutte's differential equation for a function which is very nearly the generating function for the $h_n$. The asymptotic results are used to show that the values of $P(T,\tau^2)$, where $\tau$ is the golden ratio, over all T with 2n faces have a variance very large compared to the mean. It is also shown that $P(T,4) > (1,4)^n$ for $(9.45)^n$ of the T with 2n faces (there are roughly $(13.5)^n$ such T).

## The Bandwidth of $\Theta$ graphs

J. Chvatalova, Vanier College and J. Opatrny, Concordia U. (95)

The bandwidth of a graph G, denoted by B(G), is $\min \max |f(u) - f(v)|$ where the maximum is taken over all adjacent vertices and the minimum is taken over all one-to-one mappings of vertices to integers. Graph $\Theta_{k,n}$ for $k \geq 1$, $n \geq 2$ consists of two vertices which are joined by n vertex-disjoint (except for the end-vertices) paths of length $k+1$. In this paper we study the bandwidth of $\Theta_{k,n}$ for $n \geq 3$. It is shown that

$$B(\Theta_{k,n}) = n \quad \text{for } k \geq \max\{3,(n-2)/2\},$$
$$= \lceil (3n+2)/4 \rceil \quad \text{for } k = 2.$$

For $3 \leq k \leq (n-3)/2$ we show that

$$2\lfloor (kn+1)/(2k+1) \rfloor \leq B(\Theta_{k,n}) \leq 2\lceil (kn+1)/(2k+1) \rceil.$$

## DEGREE SUMS OF PATHS IN EXTREMAL TREES

Roger C. Entringer*, University of New Mexico
Douglas E. Jackson, Eastern New Mexico University (96)

A problem in the design of an efficient communication network has the following graph-theoretic model. Given a path $P$ in a tree $T$ denote by $D(P)$ the sum of the degrees, in $T$, of the vertices of $P$ and think of $D(P)$ as the delay in transmission of information between the end vertices of $P$. We let $D(T) = \max(D(P))$ where the maximum is taken over all paths of $T$ and we wish to construct trees $T$ on $n$ vertices for which $D(T)$ is minimum. We show such extremal trees are regular except near end vertices or at a special vertex and we determine $D(T)$.

## Spreads and Packings for a Class of $((2^r+1)(2^{r-1}-1)+1, 2^{r-1}, 1)$-Designs

R. D. Baker and G. L. Ebert*, University of Delaware (145)

If $\phi(x,y) = ax^2 + hxy + by^2$ is an irreducible quadratic form over $GF(2^r)$ and $H$ is an additive subgroup of $GF(2^r)$ of order $2^i$, Denniston (1969) showed that $\Omega = \{(x,y): \phi(x,y) \in H\}$ is a collection of $(2^r+1)(2^i-1)+1$ points in $AG(2,2^r)$ such that every (affine) line of $AG(2,2^r)$ meets $\Omega$ in 0 or $2^i$ points. In this paper we construct various spreads and packings of the induced BIBD, concentrating our efforts on the case when r is even and $i = r-1$. The motivating force was an attempt to construct new semiplanes by using one spread to designate groups and thus finding sufficient packings to extend the associated divisible design to an elliptic semiplane.

## Shellable Order Ideals in Modularly Complemented Lattices

Dennis Stanton, University of Minnesota
Michelle Wachs*, University of Miami (97)

Modularly complemented lattices are upper-semimodular lattices in which every element has a modular complement. They include fundamental classes of lattices such as the subset lattices, the subspace lattices, the partition lattices and the Dowling lattices. Although all upper-semimodular lattices are shellable, this is not necessarily true for the order ideals of these lattices. We consider those lower order ideals which are generated by the complements of modular elements. It is shown that such order ideals are shellable partially ordered sets.

## On the Decomposability of Cubic Graphs (98)

John P. Georges and Agnes H. Chan, Northeastern University

A graph is said to be decomposable if and only if it has a disconnecting set of independent edges. In this paper we show that every cubic graph is decomposable, except $K_4$ and $K_{33}$. The zonality of a decomposable graph is the cardinality of a minimum disconnecting set of independent edges. We classify cubic graphs according to zonality and discuss the interrelationships among zonality, edge connectivity, and girth in cubic graphs. Finally, we obtain a lower bound for $n(\gamma)$, the smallest order of a cubic graph with girth $\gamma$.

## MORE GRACEFUL TREES

Thom Grace, Illinois Institute of Technology (105)

A graceful labelling of a tree T with e edges is an assignment of the labels 0, 1, ..., e to the vertices of T which induces a label for every edge of T by taking the positive difference of the endpoint labels; furthermore, these edge labels are distinct. In 1973, Stanton and Zarnke presented a construction whereby n copies of a gracefully labelled tree are joined to another graceful n node tree to build a larger graceful tree. Gabow gave a related construction in 1975 joining n copies of a graceful tree to the vertices of degree 1 in an n-star. In the present work, we show similar constructions in which the n trees used are not necessarily isomorphic. Under certain conditions, these techniques can be shown to produce a more restrictive labelling called a "balanced" labelling. Verification of these methods is based on the matrix representation of graceful trees, first proposed by G. Bloom in 1979.

## GRACEFUL GRAPHS GENERATED BY PERFECT SYSTEMS OF DIFFERENCE SETS

Jean M. TURGEON, Université de Montréal. (106)

Let G be a simple, non-oriented graph with n edges. We consider certain numberings of the vertices and of the edges of G in which the number associated with an edge is always the absolute value of the difference of the numbers associated with its endpoints. Such a numbering is called graceful if the vertex values belong to the set $\{0, 1, \ldots, n\}$ and the edge values are all the numbers from 1 to n. This notion has been generalized to c-graceful numberings, where the vertex values belong to $\{0, 1, \ldots, n+c-1\}$ and the edge values range from c to n+c-1. A graph is graceful [c-graceful] if it admits a graceful [c-graceful] numbering. We consider the graceful graphs that can be obtained from perfect systems of difference sets and the graphs that are known not to be graceful from the lack of a corresponding perfect system.

## HOMOGENEOUS LATIN SQUARES CONSTRUCTED FROM GROUP TABLES (113)

by
Arthur H. Hobbs, Mathematics Department, Texas A&M University, College Station, Texas
and Anton Kotzig, C.R.M.A., Universite de Montreal, Montreal, Quebec, Canada

A latin square L is h-homogeneous if every cell of the square is in exactly h order 2 latin subsquares of L. Given order n latin squares L having cells $c_{ij}$ and L' having cells $d_{ij}$, L and L' are k-opposed if for each cell $c_{ij}$ in L, there are exactly k other cells $c_{rs}$ in L which contain the same symbol as $c_{ij}$ such that the cells $d_{is}$ and $d_{rj}$ have equal symbols in L', and for each cell $d_{ij}$ in L' there are exactly k other cells $d_{rs}$ in L' which contain the same symbol as $d_{ij}$ such that the cells $c_{is}$ and $c_{rj}$ have equal symbols in L. We have shown: Theorem 1: If there is an h-homogeneous latin square L of order n and if there exist h'-homogeneous, pairwise k-opposed, order m latin squares $M_1, \ldots, M_n$, then there is an (h'+h+hk)-homogeneous latin square of order mn (constructed from the named squares). Theorem 2: Let $C_n$ be the Cayley table of the additive cyclic group of order n. Then there is a latin square L which is k-opposed to $C_n$ and which is obtained by permuting the rows of $C_n$ if and only if k+1 divides n and $k \neq 0$ when n is even. Theorem 3: if $D_n$ is a Cayley table of the dihedral group of order 2n, and if $n \equiv e \pmod 2$, with $e \in [0, 1]$, then there are n-1 other latin squares which are (1-e)-opposed to $D_n$ and pairwise (1-e)-opposed to each other. The proof of Theorem 2 includes an interesting application of graph theory to latin squares.

## Latin Squares of Automorphisms (114)

Steve Fisk, Bowdoin

Suppose G is a subgroup of the symmetric group $S_n$ of permutations on $1,2,\ldots,n$. A Latin Square is called a G-Latin Square if all the rows and columns of the Latin Square are permutations which lie in G. In this talk I show the existence of G-Latin Squares for the alternating group, and PSL(n,q), the group of all projectivities of PG(n-1,q). We also determine all the Latin Squares for the group of all semi-linear automorphisms of a field to itself.

## A SHOUT-ECHO ALGORITHM FOR FINDING THE MEDIAN OF A DISTRIBUTED SET (99)

D. Rotem, University of Waterloo
N. Santoro*, Carleton University
J.B. Sidney, University of Ottawa

A shout-echo system of size $d$ is a connected network of $d$ processors $p_1, \ldots, p_d$ where each processor can only broadcast a message to all other processors or reply to a received message. The process of broadcasting a message to and receiving a reply from all other processors constitutes a (basic) communication activity. Given a $nd$-dimensional totally ordered set $D$, a distribution of $D$ on a shout-echo system of size $d$ is a $d$-tuple $\langle D_1, \ldots, D_d \rangle$ where $D_i = D$, $|D_i| = n$, and $i \neq j \rightarrow D_i \cap D_j = \emptyset$. The problem is to determine the (lower) median of $D$, using communication activities, when each processor $p_i$ is only aware of the set $D_i$, and messages are constrained to contain an identifier, an element of $D$ and a value in the range $(1,nd)$, or a subset of the above. We present a median finding shout-echo algorithm and show that it requires $O(\log d \log n)$ communication activities, thus improving the previous $O(d \log n)$ upper-bound for the problem.

## SHORTEST PATHS THROUGH k SPECIFIED NODES

by Gilbert Laporte*, Ecole des Hautes Etudes Commerciales de Montréal
5255 avenue Decelles, Montréal H3T 1V6, CANADA (100)
and Yves Nobert, Université du Québec à Montréal
Département des Sciences administratives, Montréal H3C 3P8, CANADA

The problem of finding the shortest path from a source to a sink and passing through k specified nodes is considered. Two particular cases of this problem are the standard shortest path problem and the travelling salesman problem. Only the symmetrical case of the problem is considered in this paper. The problem is formulated as an integer linear program containing degree constraints, subtour elimination constraints and integrality constraints. We propose an algorithm in which the last two types of constraints are first relaxed and introduced only when they are found to be violated.

## Uses of Graph Theory Measures for Predicting and Assessing Diversity of Wildlife Habitat (101)

Bruce G. Marcot, Dept. of Wildlife Management, Humboldt State University
*Phyllis Z. Chinn, Dept. of Mathematics, Humboldt State University

In wildlife studies, different habitat types are called "patches" and various assessments can be made of wildlife diversity on the basis of the physical proximity and juxtaposition of various types of patches. As part of a program to respond to recent Federal legislation to maintain and monitor habitat diversity on some Federal lands, the authors have used various graph theoretical measures to model wildlife requirements. Three types of measures are explored: using various graph invariants to measure the general diversity of a given habitat map; using measures of graph centrality to assess the influence and arrangement of key habitat patches; using measures of graph connectedness to assess traversability of habitat networks.

## NEW PROPERTIES FOR THE VALUES OF AN (m,p) URN (107)

Robert Chen*, University of Miami
and
Frank K. Hwang, Bell Laboratories

Shepp [3] considered the following problem: An (m,p) urn contains m balls of value -1 and p balls of value 1 and one is allowed to draw balls randomly, without replacement, until one wants to stop. Let E(m,p) denote the expected value of drawing (at least one draw) from an (m,p) urn assuming an optimal drawing strategy is followed after the first draw. Let V(m,p) = max(0,E(m,p)) denote the value of an (m,p) urn. Shepp was interested in finding for what m and p there is a drawing strategy for which V(m,p) is positive. Boyce [1] showed that this model has applications to financial and marketing problems.

Since the recursive equations defining V(m,p) can be solved only numerically, it is helpful to obtain properties of V(m,p) which give additional insight into the structure of the optimal drawing strategy. Many such properties have been given by Boyce [1] and Shepp [3] (a good review was given by Johnson and Kotz, p. 305-310 of [2]). In this paper we prove three convex properties and a quadrangle property for V(m,p). We also give asymptotic results for V(λp,p) by using Shepp's result on V(p,p).

## SELECTING MOST PROBABLE EVENTS (108)

*F. K. Hwang*
Bell Laboratories
Murray Hill, New Jersey 07974

We consider the problem of selecting $m$ events which must include at least $c$ out of the $h$ most probable events. The selection is based on a sample of fixed size. We allow more generality to the problem by working with an urn model which includes sampling from the multinomial distribution and the hypergeometric distribution as special cases. We prove that if we select the $m$ events with the highest counts in the sample, then the probability of a correct selection is monotone nondecreasing in the sample size. We then use this result to obtain the least favorable configuration in selecting the most probable event, i.e., $c = h = 1$, while the underlying probability distribution is multinomial.

## BOOLEAN RANK PRESERVING OPERATORS (109)

by LeRoy B. Beasley*, Utah State University and
Norman J. Pullman, Queen's University.

We study the extent to which certain theorems on linear operators on field-valued matrices carry over to linear operators on Boolean matrices. We obtain analogues and near analogues of several such theorems dealing with linear operators which preserve the rank of every matrix as well as those which preserve the rank of a specified subset of matrices.

## DEVELOPMENT OF AN ALGORITHM FOR MAPPING BINARY INTERLACEMENT ARRAYS INTO STRUCTURAL CROSS-SECTIONS (115)

J.A. Hoskins, Dept. of Clothing & Textiles, U. of Manitoba

It is often useful for woven textile structures, which are conventionally represented by binary interlacement arrays, to be displayed in terms of their structural cross-sections. An algorithm for mapping such binary interlacement arrays onto an optimal set of graphic primitves to produce the required sections is discussed for up to four fabric layers. Illustrative examples are included.

## BALANCED TWILLS WITH BOUNDED FLOAT LENGTH (116)

Janet A. Hoskins, Department of Computer Science, University of Manitoba, Winnipeg, Canada.
Cheryl E. Praeger, Department of Mathematics, University of Western Australia, Nedlands, Western Australia 6009, Australia
Anne Penfold Street*, Department of Mathematics, University of Queensland, St. Lucia, Queensland 4067, Australia.

The balanced simple twills on n harnesses can be classified according to their maximum float length. The number of balanced n-harness twills with specified maximum float length is determined both by Burnside enumeration and, for $n \leq 20$, by an adaptation of a sieve algorithm for twills.

## ON A CLASS OF $n^2$x4 ORTHOGONAL ARRAYS AND ASSOCIATED QUASIGROUPS

F.E. Bennett, Mount Saint Vincent University (117)

Abstract An $n^2$x4 orthogonal array is a pair $(P,B)$ where $P = (1,2,\ldots,n)$ and $B$ is a collection of ordered 4-tuples of elements of $P$ (called rows) such that if $1 \leq i < j \leq (1,2,3,4)$ and $x$ and $y$ are any two elements of $P$ (not necessarily distinct) there is exactly one row of $B$ whose $i^{th}$ coordinate is $x$ and whose $j^{th}$ coordinate is $y$. An $n^2$x4 orthogonal array invariant under cyclic permutation of its first three columns is shown to be equivalent to a quasigroup satisfying the identity $((xy)y)y = x$. Thus the spectrum of quasigroups satisfying the identity $((xy)y)y = x$ is precisely the set of all $n \equiv 0$ or $1 \pmod 3$, $n \neq 6$. We present constructions which give the maximum number of idempotents in such quasigroups for every order $n \equiv 0$ or $1 \pmod 3$, $n \neq 6$.

## A RESOURCE LOCATION PROBLEM ON GRAPHS (102)

Ioannis G. Tollis* and Brendan D. McKay, Vanderbilt University

Given a graph G with non-negative edge lengths and a pair of vertices a and d, the resource location problem considered here is that of locating a minimum number of gas-stations on the edges of G so that a car of range R may drive from a to d along any simple path without running out of gas.

Amongst other things, we present a linear-time algorithm to solve the problem in the case that G is a series-parallel graph.

## The General Channel Assignment Problem (103)

Margaret B. Cozzens and Ding-I Wang, Northeastern University

In this paper we study the general form of the radio frequency assignment problem. Let $V = \{x_1, x_2, \ldots, x_n\}$ be a set of transmitters. Given k levels of interference between transmitters, for each $i = 0,1,\ldots,k-1$, let $G_i$ be a graph with vertex set V and an edge between x and y if and only if x and y interfere at level i. For the $i^{th}$ level of interference we are given a finite set T(i) of nonnegative integers which represents the separations between channels which are not allowed if the two channels interfere at the $i^{th}$ level. A T-coloring of $\{G_i\}$ is a function f which assigns to each $x \in V$, a positive integer or channel such that for $x \neq y$, and $i = 0,1,\ldots,k-1$:

$$(x,y) \in E(G_i) \Rightarrow |f(x)-f(y)| \notin T(i).$$

T-colorings were introduced by Hale in 1980 and the special case of only one level of interference was studied by Cozzens and Roberts in 1982. We present results on minimum order (least number of distinct channels used) and minimum span (least distance between largest and smallest channels used) T-colorings of nested graphs $G_0 \supseteq G_1 \supseteq \cdots \supseteq G_{k-1}$.

## A HEURISTIC PROCEDURE FOR LARGE-SCALE EXAMINATION SCHEDULING PROBLEMS (104)

J.G. Fisher, D.R. Shier*, Clemson University

A type of examination scheduling problem is formulated in which examinations are to be assigned to time periods in order to minimize the occurrence of consecutive examinations for any student. It is shown that this optimization problem is in fact NP-hard, suggesting the appropriateness of good heuristic procedures for the problem. One such procedure is presented, which relies on ways of optimally arranging terms in a sequence. Empirical evidence suggests that this procedure, while not guaranteed to be optimal, does tend to provide solutions within a few percent of optimality.

## Finite Manuals and their Event Structures - A Reconstruction Problem in Empirical Logic (110)

CAROL G. CRAWFORD, U.S. Naval Academy, Annapolis, Maryland

This presentation concerns a reconstruction problem of a mathematical theory relating to experiments in an abstract setting. This field generalizes the conventional notion of sample spaces by allowing for the simultaneous representation of the outcomes of a set of random experiments.

A manual $(X, \mathcal{L}, \mathcal{O})$, consisting of the operations involved in a collection of experiments, can be represented by a connected graph consisting of complete graphs intersecting on one or more vertices. From the graph representing $(X, \mathcal{L}, \mathcal{O})$ one can construct the event structure $(\mathcal{D}, |)$, a disconnected graph whose components are made up of stars and paths, where each line segment represents an operation and the endpoints represent events which form a partition of the outcomes making up the operation.

An algorithm to reconstruct the possible manuals which have $(\mathcal{D}, |)$ as their event structure will be discussed. Further graph theoretic relationships between manuals and their event structures will also be included.

## INVERSES OF BOOLEAN MATRICES (111)

Jin Bai Kim, W. Virginia U. & Yong M. Lee*, Trenton State Col.

Let X be a set with more than two elements and $\Sigma$ be the collection of all subsets of X. $M_n(F)$ denotes the $n \times n$ Boolean matrices with entries from $\Sigma$. $I_n \in M_n(\Sigma)$ denotes a diagonal matrix with all diagonal entries are equal to X, and we call $I_n$ the identity matrix in $M_n(\Sigma)$. A in $M_n(\Sigma)$ is invertible if there exists B such that $AB = BA = I_n$. We characterize the invertible Boolean matrices.

## TOP-DOWN: FROM COMPUTATIONAL MATHEMATICS TO TERNARY RINGS (112)

Killgrove*, USC Aiken; M. Cates, R. Patenaud, Cal. State LA

Computational mathematics means using the computer to do mathematics, ie prove theorems. There are two approaches: directing the computer in the proof, and letting the computer search for a proof. Proponents of the first include Lehmer's theorems in number theory, the 4 color problem, our own work on finite planes; the second include Wang's reproving Mathematica Principia, Golenter's Geometry theorem prover, the program which solves integration problems using by parts, partial fractions, and trig substitutions. At Denver, members of the second group reported on a program checker. Members of the other group sometimes show additional evidence as to the correctness of their results. In the future it would be nice to have a program directed to prove something be checked by the program checker; in this way the two groups could combine efforts. This talk will be mostly at the top while our write-up will be mostly at the bottom. We intend to include in the final write-up such things as: comparison of the number of transversals for successful and unsuccessful Latin Squares of order nine, the polynomial representation of these transversals, of the Latin Squares themselves, and of the Ternary Rings, albeit not all as there are many for the known planes of order nine.

## On Nonisomorphic Room Squares (118)

J. H. Dinitz* and D. R. Stinson
University of Vermont and University of Manitoba

Let NR(s) denote the number of non-isomorphic Room squares of side s. We prove that for s sufficiently large, $NR(s) \geq \exp(cs^2)$ for some absolute constant c. More precisely, $NR(s) \geq .19 \exp(.01s^2)$ for $s \geq 153$ odd; and $NR(s) \geq .19 \exp(.09s^2)$ for $s \geq 1001$ odd.

## Skew Resolutions of Affine Geometries and Their Applications (119)

R. Fuji-Hara* and S.A. Vanstone, University of Waterloo

A skew class of the affine geometry AG[n,q] is a set S of lines of AG[n,q] such that each point of AG[n,q] appears in exactly one line of S and no two lines of S are parallel. Let $S_1, S_2, \ldots, S_r$, $r = \frac{q^n-1}{q-1}$ be skew classes of AG[n,q]. If each line of AG[n,q] is contained in exactly one of these classes, the set of classes is called a skew resolution. We discuss constructions of skew resolutions a special type of skew resolution (strong skew resolution), and their applications to balanced Kirkman systems, blank balanced Kirkman systems, packings of PG[n,q], etc.

## Some Non-Existence Results on Twisted Planes Related to Minimum Covers. (120)

E.R. Lamken*, R.C. Mullin and S.A. Vanstone; Univ. of Waterloo

Let V be a finite set of v elements. A covering of the pairs of V by k-sets is a family F of k-subsets of V, called blocks, such that each pair of V occurs in at least one member of F. We are interested in coverings of the pairs of an $(n^2+n)$-set with $n^2+n$ blocks of size $n+1$, where n is a positive integer. We give a necessary condition for such coverings to exist. We apply the Hasse-Minkowski Theorem to get two non-existence results.

## A Catalogue of Sporadic Slope-Critical Configurations (121)

Dale Hill and Robert E. Jamison-Waldner*
Clemson University, South Carolina

In 1970 P. R. Scott conjectured that n noncollinear points in the plane always determine at least n-1 slopes. This was finally shown in 1981 by P. Ungar using the allowable sequences of Goodman and Pollack. A set of n points determining the minimum n-1 slopes will be called slope-critical. Scott gave one infinite class of such configurations and two others have since been found. Presently computer search has revealed about 80 "sporadic" examples. The properties of these will be discussed, the efforts to classify them described, and some reasons why they really seem "sporadic" will be given. A catalogue of the sporadic configurations should be available.

## All Triangles are "Nearly" Euclidean Ramsey in the Plane (122)

Leslie E. Shader
University of Wyoming

If the plane is colored with two colors and T is a triangle, T is said to be Euclidean Ramsey if and only if there is some congruent monochromatic triangle. Numerous families of Ramsey triangles as known, also known that the equilateral triangle is not Ramsey. In this paper it is shown that every triangle T is "nearly" Euclidean Ramsey in the sense that for every two coloring of the plane there is a monochromatic triangle "nearly" congruent to T.

## DISTANCE IN FINITE GEOMETRIES II (123)

Anthony Evans, Wright State University

Let A be an affine plane with point set P. d is called a distance function on A if the following hold.

i) (P,d) is a metric space.

ii) Opposite sides of a parallelogram have the same "length".

In I we gave several examples of "non-trivial" distance functions on translation planes. In this paper we will study the possible distance functions on the non-translation planes of order 9. Our main tool will be the concept of a "distance graph".

## ON THE NUMBER OF LINES IN MAXIMAL PARTIAL PROJECTIVE PLANES (124)

Stephen Dow, University of Alabama in Huntsville

A partial projective plane (ppp) of order n is an incidence structure on $n^2 + n + 1$ points such that every line has size n + 1 and any two lines meet in a unique point. A finite projective plane is then a ppp having $n^2 + n + 1$ lines. A ppp is said to be maximal if it cannot be extended to a ppp of the same order having an additional line. We investigate the set B(n) of values of b for which there exists a maximal ppp of order n having exactly b lines. We report that B(2) = {7}, B(3) = {5, 13}, B(4) = {9, 13, 21}, and give some information for $5 \leq n \leq 10$. We show that min B(n) $\geq$ n + 2 with equality if and only if n is odd; we give constructions yielding upper bounds for min B(n), n even. As a counterpoint to known results stating that a ppp having nearly $n^2 + n + 1$ lines can be embedded in a projective plane, we show that $n^2 - 3n + 9 \in B(n)$ whenever n > 3 is the order of a projective plane.

## A TWO-COMMODITY NETWORK FLOW APPROACH TO THE TRAVELING SALESMAN PROBLEM (129)

G. Finke*, E. Gunn, Technical University of Nova Scotia
A. Claus, Concordia University

The traveling salesman problem (TSP) is the problem of finding a minimum cost tour or Hamiltonian circuit in a weighted graph. We will present a new integer formulation for the TSP which has the form of a two-commodity network flow problem. It is shown that the linear programming relaxation yields a better lower bound than the assignment problem. This improvement can be attributed directly to the existence of the two distinct commodities in the network. In fact, both lower bounds coincide if the two commodities are interchangeable. The partial graph generated by the feasible solutions of the relaxed problem will be strongly connected and, above all, a solution of the assignment problem may be attached to the two-commodity flow which satisfies a weak form of the subtour elimination constraints.

## RANDOMLY BITRACEABLE GRAPHS (130)

JOHN FREDERICK FINK, Department of Mathematics
University of Louisville, Louisville, Kentucky 40292

If a graph G is expressed as the edge sum of two factors R and B of G, then G = R ⊕ B is called a bifactorization of G. We call the edges of R red edges, and those of B blue edges. A path in such a bifactorization is an alternating path if for every pair of adjacent edges in the path, one is red and the other is blue. A bifactorization G = R ⊕ B is randomly bitraceable if for each vertex v of G, every nonhamiltonian alternating path beginning at v can be extended to a longer alternating path (and thus to a hamiltonian alternating path) with initial vertex v. A graph G for which a randomly bitraceable bifactorization exists is also said to be randomly bitraceable. We characterize the randomly bitraceable graphs that are also randomly traceable, and show that each of these graphs has (up to automorphism) precisely one randomly bitraceable bifactorization. Other results are also presented.

## A Localized Backtracking Algorithm for Finding Hamilton Circuits (131)

Michael Gilpin, Michigan Technological University

Rubin (JACM, Oct 74) extended the work of Hakimi (IEEE Region Six Conference, 1966) in giving criteria for determining when to terminate partial paths in the search for Hamilton paths on a connected graph. The tests given by Hakimi and Rubin are global in nature in that they are applied to all nodes. We present an algorithm which is local in nature in that the criteria for determining when to terminate partial paths are only applied to nodes located one edge away from the current node. An example is given which indicates the effectiveness of this localized procedure.

## ON WINDY POSTMAN PROBLEM (132)

Guan Meigu, University of Waterloo

The so-called "windy postman problem" was proposed by E. Minieka in 1979. The problem is: "Given an undirected graph G = (V,E) such that for each edge e = (x,y), the distance from x to y can be different to the distance from y to x. How to choose a shortest postman route for graph G?"

The main results of this paper are:

(1) to prove that the windy postman problem is NP-complete.

(2) to prove that when the distance function satifies a condition (we call it condition Q), the windy postman problem can be solved by a polynomial algorithm.

(3) to give a simple method for checking whether condition Q is satisfied or not.

(4) to give an approximation algorithm for the case when condition Q is "nearly" satisfied.

## Extensions of the notion of conformality in hypergraphs (137)

R.P. Anstee, University of Waterloo

We consider a generalization of conformality as follows. From a hypergraph H we form its k-section $(H)_k$, which is a hypergraph consisting of all possible edges of size k contained in edges of H. Attempting to reverse this process, we form a hypergraph K consisting of the maximal edges (of size ≥ k) such that any edge of size k contained in any one is in $(H)_k$. Results of Gilmore and Berge are generalized to determine when $H = {}^k K$ in which case we say H is k-conformal. A new result of Quinn describes the class which has the hereditary k-conformal property. Other examples where information akin to $(H)_k$ is enough to determine H are considered.

## Enumeration of connected invariant graphs. (138)

E. A. Bender, University of California, San Diego; and
E. R. Canfield*, University of Georgia (presenter).

Let h be a finite group acting on unlabeled graphs which does not change connectivity. Examples include edge reversal in directed graphs and permutations of colors in edge and/or vertex colored graphs. We relate the generating functions of h-invariant (directed) graphs and h-invarinat (weakly) connected (directed) graphs. This leads to a recursive formula for calculating the number of connected graphs when the total number of graphs is known. We apply this to self-dual signed graphs, self-converse digraphs, and color cyclic graphs. We also obtain asymptotic expansions. As expected, almost all of the above h-invariant graphs are connected and the asymptotic number of disconnected graphs has a simple interpretation.

## Bounds on Nordhaus-Gaddum Type Bounds for Clique Cover Numbers (139)

D. Taylor, Harris Corporation; R. Dutton and R. Brigham*,
University of Central Florida

Nordhaus-Gaddum type bounds are presented for edge and total clique cover numbers. For the most part these bounds are not sharp and probably differ significantly from sharp ones. However, families of graphs are given which in some sense show that the stated bounds are not too bad. Finally sharp bounds are demonstrated for the class composed of graphs G where either G or its complement is bipartite.

## ON MIXED RAMSEY NUMBERS: TOTAL CHROMATIC NUMBER VERSUS GRAPHS (140)

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The total chromatic number of a graph G, $\chi_2(G)$, is the least number of colors which can be assigned to the edges and vertices of G so that no adjacent or incident elements are assigned the same color. For an integer m and a graph G, the mixed ramsey number, $\chi_2(m,G)$, is the smallest integer p such that for every graph H on p vertices, either $\chi_2(H) \geq m$ or $G \subseteq \bar{H}$. The known upper bound for $\chi_2(m,K_{1,n})$ is shown to be exact for small values of m, where $K_{1,n}$ is the star graph. Upper bounds for $\chi_2(m,T_n)$ and $\chi_2(m,P_n)$ are determined where $P_n$ is the path on n vertices and $T_n$ is any tree on n vertices. Other results are included.

## Automorphisms of the 45-point elliptic semiplane

R. D. Baker, University of Delaware (125)

The author found the first elliptic semiplane which is not a projective plane minus a Baer subset (An elliptic semiplane, J. Combinatorial Theory A-25(1978), 193-195.). Recently this semiplane was rediscovered in investigations of the tactical decomposition of the projective plane of order eight induced by a subplane of order two. Here we exploit this connection to further investigate the automorphisms of this elliptic semiplane. In particular the actions of $PGL_3(2)$ and $PGL_4(2)$ on the semiplane will be discussed.

## GRAPH - THEORETICAL ASPECTS OF PROPERTIES OF TRANSFORMATIONS OF FINITE SETS (126)

Corina Reischer*, Universite du Quebec a Mont Simovici, University of Massachusetts

We obtain results related to iteration, decomposition and the existence of n-ary roots of transformations of finite sets using the graphs attached to these transformations. We retrieve, as special cases, certain iterative properties of switching functions.

## Directing Graph Designs

Charles J. Colbourn and Marlene J. Colbourn, (127)
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For G a (directed) connected graph, a $(v,k,\lambda)$ G-design is a partition of the edges of the complete (directed) multigraph $\lambda K_v$ ($\lambda K_v^*$) into k-vertex subgraphs isomorphic to G. When $\vec{G}$ is an oriented graph with underlying graph G, a $(v,k,\lambda)$ $\vec{G}$-design yields a $(v,k,2\lambda)$ G-design. In this paper, we study the converse question: for which G and $\vec{G}$ does a $(v,k,2\lambda)$ G-design always underly a $(v,k,\lambda)$ $\vec{G}$-design? An affirmative answer was given by C. Colbourn and J. Harms when G is $K_3$ and $\vec{G}$ is the transitive tournament of order three.

We examine all remaining oriented graphs on three and four vertices, and establish a negative result in every case but one. When G is $K_4$, and $\vec{G}$ is the transitive tournament of order four, the question remains open.

## 1-TRACEABLE GRAPHS (133)

C. N. Pareek, Kuwait University

Abstract. A graph G is called 1-traceable if G and G-v respectively have Hamiltonian paths for each vertex v of G. In this note, we study properties of 1-traceable graphs and show that the minimum order of a 1-traceable non-homogeneously traceable graph is fifteen. Also, it is shown that the minimum size of a 1-traceable non-homogeneously traceable graph is $\lceil 5n \rceil + 1$.

## INGRID: A Software tool for Extremal Graph Theory Research (134)

R. Dutton* and R. Brigham
University of Central Florida

INGRID, for INteractive GRaph Invariant Delimiter, is a computer program which allows a user to specify a value or range of values for one or more of up to 35 graph invariants. Embedded in the system is a data base of some 200 theorems which are selectively invoked and give the user values and/or ranges of values for the remaining graph invariants. The system is then poised to accept further invariant specifications, to cancel the effects of the last input, or to start over from the beginning. Facilities also exist which aid the user in determining the sequence of theorems causing some particular result. Although INGRID is not a "theorem proving" program it has pointed out relationships which are not part of its data base and at least some of these have not been found in the literature.

## Algorithms for Pension Sums (135)

T.D. Lambert*, W.D. Hoskins, R.G. Stanton; Univ. of Manitoba
P. Ponzo; University of Waterloo

Pensions at the University of Manitoba are computed from an individual's best five salary years. These best five years need not be contiguous or calendar years but must be disjoint. Algorithms for calculating such pension sums will be discussed, and generalisations of the problem considered.

## COLOR SCHEMES FORBIDDING MONOCHROME TRIANGLES (141)

Stephen D Comer, The Citadel, Charleston, SC 29409

Given n colors, denoted 1,2,...,n, a *color scheme* $\mathcal{C} = (V, C_1, \ldots, C_n)$ is an edge coloring of a complete (undirected) graph with vertex set V such that (i) each color is present on some edge eminating from each vertex, and (ii) for colors i,j,and k, $C_i \cap (C_j | C_k) \neq \emptyset$ implies $C_i \subseteq C_j | C_k$. A color scheme $\mathcal{C}$ *forbids* a colored triangle (i,j,k) if $C_i \cap (C_j | C_k) = \emptyset$.

PROBLEM. For which positive integers n does there exist a color scheme $\mathcal{C}$ (using n colors) which forbids exactly monochrome triangles? Partial results concerning this problem will be discussed.

## Anscombe's Paradox and the Rule of Three-Fourths (142)

Carl Wagner, University of Tennessee-Knoxville

That the resolution of a set of proposals by majority rule may result in outcomes with which a majority of voters disagree in a majority of cases seems first to have been noted by Anscombe (Analysis 36:4 (1976), pp. 161-168). We show that if the prevailing coalitions across all proposals comprise, on average, at least three-fourths of those voting, then the set of voters disagreeing with a majority of the outcomes cannot comprise a majority. In particular, the rule requiring ratification of amendments to the U.S. Constitution by at least three-fourths of the states guarantees that the set of states whose legislatures have rejected a majority of the amendments thus adopted can never constitute a majority.

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## PREFERENCES AMONG PREFERENCES OR NESTED CYCLIC STOCHASTIC INEQUALITIES (143)

Walter W. Funkenbusch*, Michigan Technological University
Donald G. Saari, Northwestern University

Cyclic stochastic inequalities have been known since the time of Condorcet for voting anomalies and since Steinhaus and Trybula for independent random variables. These are the inequalities where, with a "reasonable" probabilistic definition for "better than", set A is better than B, B is better than C, but set C is better than set A. Using the expositional approach of describing these inequalities in terms of games of chance, we show that cyclic stochastic inequalities for sums of independent random variables can be nested to any desired order; and that such nestings are not probabilistic anomalies. As part of the proof, it will be indicated that the sum of two cyclic inequalities can result in a cyclic inequality in the opposite direction. Also, for any integer $n > 2$, we describe two elementary methods to define n independent random variables which have the cyclic property.

## The Frequency Distribution of Pascal's Triangle Modulo p. (144)

Dale M. Mesner, Univ. of Nebraska

We consider the array of residues modulo a prime p of binomial coefficients $\binom{n}{r}$, $0 \leq r \leq n < N$. Elementary methods are used to investigate the frequency distribution of these $N(N+1)/2$ residues.