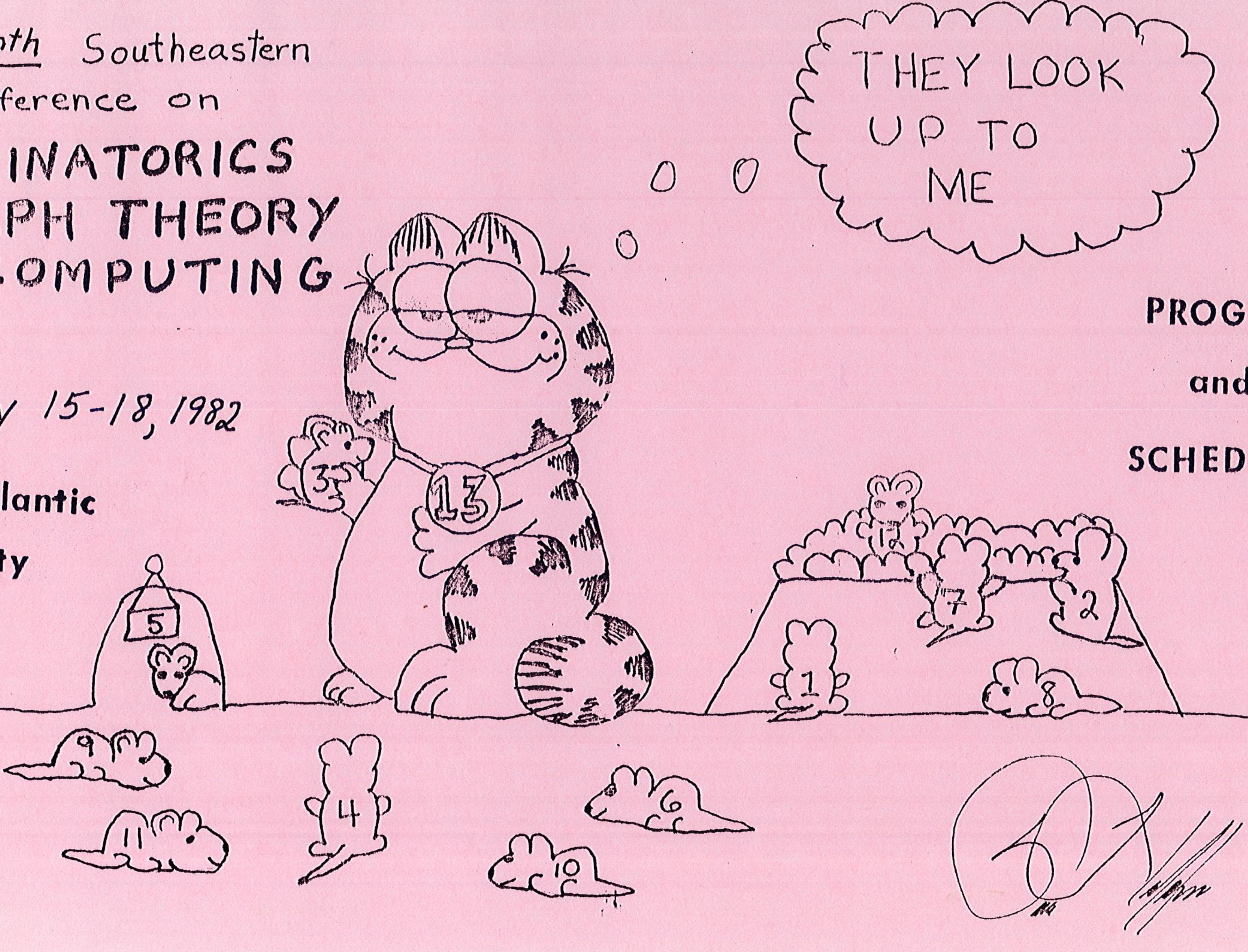
Thirteenth Southeastern Conference on

COMBINATORICS GRAPH THEORY + COMPUTING

February 15-18, 1982

Florida Atlantic University



PROGRAM and SCHEDULE

Conference participants are invited to the Cocktail Party Reception in the Board of Regents Room of the University Center from 6:00PM to 7:30PM on Monday, February 15, 1982. (The drink tickets are for accounting purposes--one ticket for a mixed drink, 1/2 ticket for a beer. Additional tickets may be requested.) Transportation will be available back to the motels after the Reception. The vans going to the motels at 5:45 will return to the University, leaving the motels at about 6:15.

Conference participants are also invited to a Beer Party, Tuesday February 16 from 6:00PM to 8:00PM at the home of Dr. and Mrs. John M. Freeman, 741 Azalea Street (but park on Aurelia!). Vans will leave the University Center at 5:45 and 6:15PM for the party, and will leave the motels at 6:30PM. Transportation will be provided back to the motels and the University Center after the party. It is about a one-mile walk from the University to the Freeman's home.

The Conference Banquet will be held at the Boca Teeca Country Club, Wednesday February 17. The cash bar opens at 6:30PM. Seating will be at 7:45 and service at 8:00PM. A van will leave for the banquet from the University Center at 6:20PM and vans will leave the motels at 6:30PM. It is more than a two-mile walk from the University Center to Boca Teeca.

There will be a Survivor's Party 6:00PM to 8:00PM Thursday evening, February 18 at the home of Dr. and Mrs. Frederick Hoffman, 4307 N.W. 5th Avenue. Conference participants who are still in the area are most welcome. Please let us know if you can come, and if you need transportation. It is a walk of less than two miles from the University to the Hoffman's home.

MONDAY

Time	GCN		GCS		207	
8:40	Registration (from 8AM)					
9:00	Opening session (V.P. Michels);				Deans Baker and Lindsey	
9:30	B O L L O B A S					
10:30			C O F F E E			
10:45	Havel	1	Hwang	5	Hamacher	9
11:05	Dinitz	2	Cozzens	6	Harker	10
11:25	Schatz	3	Alexander	7	Foulds	11
11:45	Rabinowitz	4	Sumner	8	Santoro	12
12:05						
12:25			L U N C H			
12:45						
1:35	B O L L O B A S					
2:40	Rosa	147	Farber	148	Edwards	149
3:00	Taft	13	Wayland	21	White	29
3:20	Freeman	14	Winkler	22	Holladay	30
3:40	Mills	15	Quintas	23	Tucker	31
4:00	M. Colbourn	16	Grossman	24	Pfaltz	32
4:20	Laskar	17	Johnson	25	Wachs	33
4:40	Hare	18	Buckley	26	Vince	34
5:00	Woolbright	19	Alameddine	27	Cummings	35
5:20	Anstee	20	Palka	28	Levinson	36

TUESDAY

GCN		GCS		207	
Chung	37	Luedeman	43	Hsu	49
Faudree	38	Arnow	44	Slater	50
S O S					
		C O F F E E			
Beck	39	Quilliot	45	Grace	51
Burr	40	Papaioannou	46	Z. Miller	52
Schelp	41	Niederhausen	47	Opatrny	53
Trotter	42	Klawe	48	Chinn	54
		L U N C H			
		S O S			
SUNLEY					
		Fisk	62	Spencer	70
Farley	55	Wallis	63	McKay	71
Liestman	56	Hobbs	64	Duke	72
Richards	57	Heinrich	65	I. Hartman	73
Matula	58	Bruen	66	Holton	74
Hammer	59	Parker	67	Entringer	75
Balas	60	Roth	6	Jackson	76
Baker	61	J. Brown	69	Bridgland	77

WEDNESDAY

GCN		GCS		207	
Lipman	78	Chvatal	84	Buhler	90
Proskurowski	79	Sbihi	85	Batten	91
P I P P E N G E R					
		C O F F E E			
Pulleybank	80	Reid	86	Payne	92
Parsons	81	Clark	87	Killgrove	93
Pittel	82	Klerlein	88	Dow	94
Rosenfeld	83	Thomason	89	P. Smith	95
Conference Photo: Outdoor Stage 12:05					
		L U N C H			
P I P P E N G E R					
Owings	150	Golumbic	151	Purdy	152
Roberts	96	Jung	104	Kreher	112
Luks	97	Guay	105	Abrham	113
Goldberg	98	Syslo	106	Kramer	114
Culik	99	Gupta	107	Billington	115
S.T. Hedetniemi	100	Ntafos	108	Magliveras	116
Shier	101	Perkel	109	Shrikhande	117
C. Colbourn	102	Varma	110	Healey	118
Lubiw	103	Urrutia	111	Mesner	119

THURSDAY

GCN		GCS		207	
Canfield	120	Plummer	129	Ross	138
Comer	121	Harris	130	Zeid	139
E R D Ő S					
		C O F F E E			
Nowakowski	122	Cook	131	Doob	140
Straley	123	Bloom	132	Fajtlowicz	141
Griggs	124	Cameron	133	Capobianco	142
Silverman	125	Kunze	134	Paul	143
D. Smith	126	Ecklund	135	Duffus	144
Wang	127	Hartung	136	Simmons	145
Edmonds	128	Schwimmer	137	Godsil	146

Coffee will be available in GCS, as will the book exhibits and the display of menus from many area restaurants.

- Monday, February 15 at 9:30AM and 1:35PM, Professor Bela Bollobas of the University of Cambridge and Louisiana State University will speak on, "Sorting in Few Rounds".
- Tuesday, February 16 at 9:30AM and 1:35PM, Professor Vera T. Sos of the Scientific University of Budapest and the Mathematical Institute of the Hungarian Academy of Sciences will speak on, "Intersection Theorems on Set Systems and on Structures".
- Tuesday, February 16 at 2:40PM, Dr. Judith S. Sunley of the National Science Foundation will speak to Conference participants about the Foundation, emphasizing aspects of interest to combinatorics. Dr. Sunley has indicated her willingness to speak individually with participants wishing further communication with NSF.
- Wednesday, February 17 at 9:30AM and 1:35PM, Dr. Nicholas Pippenger of IBM Research Laboratory, San Jose, will speak on, "Channel Graphs".
- Thursday, February 18 at 9:30AM Professor Paul Erdos of the Hungarian Academy of Sciences will speak on "Problems in the Theory of Designs".

Our Conference Proceedings are published by Utilitas Mathematica Publishing Co., P.O. Box 7, University Centre, Winnipeg, Manitoba, Canada, R3T 2N2 in its series Congressus Numerantium. Back issues and additional copies may be ordered from them. The publisher has asked us to announce that the Proceedings of the 12th Conference will be mailed this month. The delay is due to the lateness of last year's conference and the traditional two-month Canadian postal strike. If you participated in last year's conference and either forgot to pay your postage or changed your address you may take advantage of this delay to arrange to obtain your copy.

Transportation: We shall provide van transportation from Day's Inn and University Inn to the University Center at 8:00AM Monday through Thursday, making two trips if necessary. There will be van transportation to the motels at 5:45PM Monday through Wednesday from the University Center. There will also be van transportation from the reception at the Board of Regents Room on Monday evening. Transportation will be provided at 5:45PM and 6:15PM from the University Center and at 6:30PM from the motels to the Tuesday night beer party. The van will go back to the University Center and to the motels. A van will leave the University Center for the banquet at 6:20PM Wednesday, and vans will leave the motels at 6:30PM. Transportation will be provided back to the motels at the close of the banquet. The vans will be available for transportation back to the motels at the close of the Conference on Thursday. Although we aim to provide all needed transportation, we encourage car-pooling where possible.

Miscellaneous information

The phone number at the Conference desk is 393-3719.
There is a coin-operated xerox machine in the second floor lounge of the University Center.
The copy center (behind the police station, which is near the south end of the covered walkway) can xerox onto transparencies. Blank transparencies are available in the bookstore in the University Center and at the copy center.
GCN and GCS are the two halves of the Gold Coast Room of the University Center.
Room 207 is entered from the second floor lounge of the U.C. Room 232 (also off the lounge) is available to Conference participants, and has a blackboard.

AUTHOR INDEX TO ABSTRACTS

(This index includes all authors. The papers are listed on the schedule by the presenters.)

(1) THE COMBINATORIAL DISTANCE GEOMETRY APPROACH TO THE CALCULATION OF MOLECULAR CONFORMATION: A Challenge to Combinatorics from Biology

by Timothy F. Havel, Univ. of Calif., San Francisco

The conformational properties of macromolecules play a key role in the functioning of living cells. Thus a central problem in molecular biophysics is the prediction of conformation from covalent structure. Most approaches to this problem are based on minimization of an additive function of the interatomic distances designed to simulate the free energy of interaction between the atoms. Although sound in principle, the presence of many local minima in the energy surface versus the usual three dimensional coordinate systems has prevented its application in practice. By specifying the distances approximately as "contacts" or "noncontacts", it is possible to represent a molecular conformation quite precisely as an intersection graph of equal diameter spheres centered on each of the atoms. Weights may be assigned to all the possible edges of this graph commensurate with the interaction energies, and degree constraints are imposed by the packing densities attainable in three space. The corresponding optimum degree constrained subgraph, as computed by the BLOSSOM algorithm of Edmonds and Johnson, provides the necessary relaxation for a branch and bound search for the minimum energy conformation subject to the constraint that the solution be the intersection graph of unit spheres in three dimensional Euclidean space. An implementation of this algorithm now exists, but many improvements are possible, and necessary. The problems proposed include: a) a recognition algorithm for the intersection graphs of unit 3-spheres; b) incorporation of a subset of the constraints into a matching algorithm to obtain an improved relaxation.

(2) Continuous Maps on Block Designs

J.H. Dinitz and S.W. Margolis
University of Vermont

Let $D = (V,\mathcal{B})$ be a (v,k,λ) design. A partial function $f: V \to V$ which enjoys the property that $f^{-1}(b) \in \mathcal{B} \cup \{\phi\}$ for all blocks $b \in \mathcal{B}$, is called <u>continuous</u>. Clearly if $f \in \mathrm{Aut}(D)$, then f is continuous. However, there exist continuous maps on D which are not automorphisms. We show that the existence of such a map gives restrictions on the parameters of the design. For example, if f is a continuous map on D and $f \notin \mathrm{Aut}(D)$, then $r \leq \lambda^2$. We also show that if D is the complement of a Desarguesian projective space P(V), then every continuous map is induced by a (not necessarily one-to-one) semilinear transformation on V. This generalizes the Fundamental Theorem of Projective Geometry.

(5) AN ANOMALY IN KNOCKOUT TOURNAMENTS

F. K. Hwang
Bell Laboratories
Murray Hill, New Jersey 07974

A player is said to be stronger than another player if he has a better chance of beating the other player than vice versa and his chance of beating any third player is at least as good as that of the other player. Recently, Israel gave an example which shows that a stronger player can have a smaller probability of winning a knockout tournament than a weaker one when players are randomly assigned to starting positions. In this paper we prove that this anomaly cannot happen if the stronger player is a strongest player.

Circular Dimension, N-dimensional Arc Graphs (6)

Margaret Barry Cozzens, Northeastern University

A graph is a circular-arc graph if it is the intersection graph of a family of arcs on a circle. A graph G is a <u>n-dimensional arc graph</u> if it is the intersection graph of "patches" on a sphere, $S^{(n)}$, where a <u>patch</u> is the cartesian product of n arcs of $S^{(1)}$. Equivalently, it is the intersection of n circular-arc graphs. The <u>circular dimension</u> of a graph, denoted D(G), is the least integer k such that G is the intersection graph of patches on $S^{(k)}$. We show that determining the circular dimension of a graph is an NP-complete problem. However, for dimension 1, Tucker produced a polynomial time algorithm to determine if a graph is a circular-arc graph. Even though, in general, the circular dimension of a graph cannot be precisely determined, we give bounds on the circle dimension, using other parameters of the graph; eg: $D(G) \leq X'(\bar{G}) \leq \Delta(\bar{G}) + 1 = n - \delta(G)$.

(7) Bipartite Subgraphs and Triangle Free Graphs.

William Staton, Glenn Hopkins, and <u>Charles Alexander</u>, of the University of Mississippi.

Upper bounds are investigated for the number of edges which must be removed from a graph with n vertices in order that the subgraph remaining be bipartite. Exact values are determined for some small values of n.

Title: Weighted Min Cost Flows (9)

by H. Friesdorf, Bayer Leverkusen,
and *H. Hamacher, University of Florida

Abstract: The minimal ratio problem which is, in the context of combinatorial optimization, treated for shortest paths and for spanning trees is considered in a generalized form for network flow problems. The resulting problem of finding so-called *weighted minimal cost flows* has nice practical applications: for instance, flows with minimal average cost or flows minimizing cost with respect to possible penalties can be treated in this way.

We discuss two algorithms for determining weighted minimal cost flows: the negative circuit algorithm and the shortest augmenting circuit algorithm. The validity of both algorithms follows from a negative circuit theorem for weighted minimal cost flows.

A short discussion of the theoretical and computational complexity of the proposed algorithms is closing the paper.

(10) A Simultaneous Freight Network Equilibrium Model

Patrick T. Harker* Terry L. Friesz University of Pennsylvania

This paper describes a single optimization model for the prediction of flow on the freight transport network. This model incorporates the actions of both shippers and carriers, as well as being able to deal with both multiple modes and multiple commodities without the usual symmetry restrictions. The use of an augmented Lagrangian algorithm for its solution is described and numerical results are given.

(11) ENUMERATION OF PHYLOGENETIC TREES WITHOUT POINTS OF DEGREE TWO

*L.R. Foulds, University of Canterbury
R.W. Robinson, Newcastle University

Evolutionary trees of biology are represented by a special class of labelled trees, termed phylogenetic trees. These are characterized by having disjoint subsets of the labelling set assigned to the points of a tree, in such a way that no endpoint is assigned an empty set of labels. Evolutionary trees do not usually have points of degree two and so we make that restriction for the class of trees studied here. The exact and asymptotic numbers of phylogenetic trees are determined under the presence or absence of two additional conditions on the labelling. The optional constraints studied require nonempty label sets to be singletons and that only endpoints be labelled. This paper extends previous work by the authors on other cases including the restriction to binary trees.

(3)

A Note on Subset and Subspace Intersection Problems

James R. Schatz, Dept. of Defense, Ft. Meade, MD 20755

Let $M(n,k,r)$ denote the maximum number of k-subsets of an n-set such that any two subsets meet in at most r points. Similarly, let $S(n,k,r)$ denote the maximum number of k-dimensional subspaces of an n-dimensional vector space over GF(2) such that any two subspaces meet in a space of dimension at most r. We prove that for $m \geq 2$:

$$M(2m^2+2m+k, 2m, 1) = 2m+2, \quad -1 \leq k < \frac{m-3}{3} \qquad (1)$$

$$M(2m^2+4m+k, 2m+1, 1) = 2m+3, \quad 1 \leq k < \frac{m+2}{3} \qquad (2)$$

$$M(2^{2n-1}-1, 2^{n}-1, 1) = 2^{n}+1, \quad n \geq 3 \qquad (3)$$

$$S(2n-1, n, 1) = 2^{n}+1, \quad n \geq 3 \qquad (4)$$

Formulas (3) and (4) taken together are rather surprising since one involves arbitrary subsets while the other concerns subspaces.

De Bruijn Sequences and Hypergraphs Over Finite Alphabets

(4)

Joshua H. Rabinowitz

The MITRE Corporation* Bedford, MA 01730

This paper introduces a new graph theoretic technique to the study of nonlinear feedback shift registers (NLFSRs) and an application of this technique to the synthesis of de Bruijn sequences. Let $L = \{1,\cdots,k\}$. An (n-stage) NLFSR over L is a mapping $F: L^n \to L^n$ such that $F(x_0,\cdots,x_{n-1}) = (f(x_0,\cdots,x_{n-1}), x_0,\cdots,x_{n-2})$. Let S_k be the permutation group of L and B_k the set of partitions of L. A (reduced) truth table is a function $T: L^{n-1} \to S_k(B_k)$. Reduction is defined via the natural map $\pi: S_k \to B_k$. We show that there is a natural bijection between truth tables and bijective NLFSRs. Next, we introduce a hypergraph $P = P_{n,k}$ and a certain natural class of "admissible" subhypergraphs and show that there is a natural 1-1 correspondence between admissible subhypergraphs of P and reduced truth tables. Finally: if T is a reduced truth table corresponding to a spanning hypertree of P then any truth table with reduction T corresponds to an NLFSR which generates a de Bruijn (maximal length) sequence. Moreover, any admissible hypertree in P can be expanded to an admissible spanning hypertree so that such trees are easy to construct.

*(The work presented was performed as part of the independent research and development program of The MITRE Corporation).

Starter-Adder Techniques for Kirkman Squares and Kirkman Cubes of Small Sides

(147)

S.A. Vanstone, St. Jerome's College, University of Waterloo
*A. Rosa, McMaster University, Hamilton, Ontario

A Kirkman square [Kirkman cube] is a square [cubic] array whose nonempty cells contain the triples of a resolvable Steiner triple system such that the nonempty cells of each row and column [line] contain the triples of a parallel class. For a Kirkman cube, moreover, no projection on any plane is a Kirkman square.

We describe some computational experience in producing Kirkman squares and Kirkman cubes of low orders that are generated by a single parallel class of the underlying Steiner triple system.

(8)

Random Walks on Trees: Probabilistic Connectivity

Lynn Pearce, University of North Carolina at Charlotte Charlotte, NC 28223; David Sumner* and Steve Durham, University of South Carolina, Columbia, SC 29208

Random walks are investigated on trees with endpoints as absorbing states and the additional property that for every vertex v, and vertex x adjacent to v, the probability that a walk originating from v includes x is at least ϵ (ϵ-connected trees). It is shown that ϵ must be no more than $\frac{1}{2}$. Results are obtained about the structure of such trees. In particular, a characterization in terms of the degrees of vertices in the tree is given for trees where the probability described above is exactly ϵ. For $\frac{1}{2}$ - connected trees there is a characterization in terms of certain induced subgraphs.

(148)

Dominating Sets in Strongly Chordal Graphs

Martin Farber

University of Waterloo

A set S of vertices of a graph G is <u>dominating</u> if every vertex in V(G) - S is adjacent to some vertex in S.

In this talk we will introduce the class of strongly chordal graphs, an interesting subclass of chordal graphs which properly includes interval graphs and powers of trees, and present an efficient algorithm to locate minimum cardinality dominating sets in these graphs. We will also present several characterizations of the class of strongly chordal graphs, including a forbidden subgraph characterization.

(149)

ALGEBRAIC MEASURES ON TRANSITION GRAPHS

W.R. Edwards* and M. Samadzadeh-Hadidi, CMPS Dept., U. of Sw. Louisiana

The input semigroup of a transition graph or nondeterministic automaton expresses its ability to distinguish its input strings. From this semigroup new transition graphs can be constructed, with covering relations to the original graph.

(21)

BIPARTITE SCORE SETS

Keith Wayland, University of Puerto Rico

The question of what sets of integers may be the score sets of bipartite tournaments was posed recently by K. B. Reid. The main theorem of this paper establishes a sufficient condition for pairs of sets to be bipartite score sets. This simple condition yields an immediate affirmative answer for a large class of sets.

DISTRIBUTED ALGORITHMS FOR RANKING THE NODES OF A NETWORK

(12)

E. Korach(+), D. Rotem(+), *N. Santoro(++)

(+) University of Waterloo
(++) University of Ottawa

ABSTRACT
Ranking refers to the process of determining the rank of n uniquely numbered processors loosely coupled in an a synchronous network where no central controller exists and the number of processors is not known a priori. In this paper, we present efficient algorithms for the ranking process in acyclic networks, networks of abritrary topology, and circles of processors. These algorithms are fully distributed and, in order to be executed, require only local topological knowledge at each processor. The worst case as well as the average complexity of the proposed algorithms are analysed and discussed.

Tying Together Two Rigid Frameworks

(29)

Neil L. White, University of Florida

A bar-and-joint framework consists of rigid bars joined together at flexible joints. Let us take two such frameworks, each of which is rigid, and pick joints $a_1, a_2, \ldots, a_k$ on one and $b_1, b_2, \ldots, b_m$ on the other. For which bipartite graphs on $\{a_1,\ldots,a_k;\ b_1,\ldots,b_m\}$ do we get a rigid structure by adjoining a rigid bar between the two frameworks for each edge of the graph, assuming that the joints are in general position? This combinatorial problem is answered for frameworks in Euclidean space of arbitrary dimension.

(13)

Combinatorial sequences as sequences of divided powers

Earl J. Taft, Rutgers University

We identify certain types of sequences arising in combinatorics as sequences of divided powers in appropriate coalgebras. For example, a sequence $\{p_n(x)\}$ is of binomial type in $R[x]$, R the real numbers, if and only if $\left\{\frac{p_n(x)}{n!}\right\}$ is a sequence of divided-powers (SDP). This means that there is an associative operation Δ from $R[x]$ to $R[x] \otimes R[x]$ such that $\Delta\left(\frac{p_n(x)}{n!}\right) = \sum_{i+j=n} \frac{p_i(x)}{i!} \otimes \frac{p_j(x)}{j!}$. Δ is the algebra mapping with $\Delta x = 1 \otimes x + x \otimes 1$. The umbral algebra is the full linear dual of the coalgebra $R[x]$. We give two other examples which are q-analogues of classical combinatorial sequences. A sequence $\{p_n(x)\}$ is of q-binomial type if and only if $\left\{\frac{p_n(x)}{(n)_q!}\right\}$ is an SDP in the q-Eulerian coalgebra $R[x]$, where $\Delta x^n = \sum_{i+j=n} \binom{n}{i}_q x^i \otimes x^j$. A sequence $\{p_n(x)\}$ is of q-Eulerian type if and only if $\left\{\frac{p_n(x)}{(n)_q!}\right\}$ is an SDP in $R[x]$, where $\Delta x^n = x^n \otimes x^n$. In the latter example, $\Delta p_1(x) = 1 \otimes p_1(x) + p_1(x) \otimes x$, and $\Delta \frac{p_n(x)}{(n)_q!} = \sum_{i+j=n} \frac{p_i(x)}{(i)_q!} \otimes \frac{x^i p_j(x)}{(j)_q!}$, a generalization of the classical notion of SDP. Further examples require a generalization of the notion of SDP to noncommutative coalgebras. In most cases, our sequences of divided powers can be identified in coalgebras which also have the structure of a Hopf algebra.

ALTERNATING SIGN MATRICES AND DESCENDING PLANE PARTITIONS

(15) W. H. Mills,* David P. Robbins, Howard Rumsey, Jr.
Institute for Defense Analyses, Princeton, N.J. 08540

An *alternating sign matrix* is a square matrix such that (i) all entries are 1, —1, or 0, (ii) every row and column has sum 1, and (iii) in every row and column the non-zero entries alternate in sign.

We have discovered striking numerical evidence of a connection between these matrices and the descending plane partitions introduced by Andrews, but we have been unable to prove the existence of such a connection. However this evidence did suggest a method of proving the Andrews Conjecture on descending plane partitions, which in turn suggested a method of proving the Macdonald Conjecture on cyclically symmetric plane partitions.

In this paper we discuss alternating sign matrices and descending plane partitions, and present several conjectures and a few theorems about them.

A Recursive Construction for 1-Rotational Steiner 2-Designs

Charles J. Colbourn and Marlene J. Colbourn,* (16)
University of Saskatchewan

Given 1-rotational Steiner 2-designs S(2,k,m(k-1)+1) and S(2,k,v+1), where m is relatively prime to (k-1)!, we construct a 1-rotational S(2,k,mv+1). Applications of this recursive construction to the existence problem for 1-rotational Steiner 2-designs are discussed.

ON MENDELSOHN (k,t,λ^*)-SYSTEMS

Renu Laskar*, Greg A. McClanahan (Clemson, S.C.) (17)

A Steiner k-tuple system is a pair (P,B), where P is a finite set of v points and B is a collection of k-element subsets of P, called blocks, such that every (k-1)-element subset of P belongs to exactly one block. The concept of a Mendelsohn (k,t,λ^*)-system (MS) is introduced: it is a pair (P,B_M), where P is a finite set of v points and B_M is a collection of ordered k-element subsets of P, called blocks, such that every ordered t-element subset of P occurs in exactly λ^* blocks of B_M. The block (1,2,...,k) contains the k ordered t-tuples (1,2,...,t-1,t), (2,3,...,t,t+1), (k-t+1,...,k-1,k). Necessary conditions for existence of MS are given for the general setting and some particular results are obtained for the case k=4, t=3, λ^*=1. Such systems are called Mendelsohn Quadruple Systems (MQS). Theorem 1. If a MQS of order v exists, then v is congruent to 0, 1, or 2, modulo 4, and the number of blocks b = (v(v-1)(v-2))/4. The nonexistence of a MQS of order 5 is established. Theorem 2. There exists a MQS of order v iff there exists a latin cube of order v satisfying a^3=a and bc(abc)=a, for every a,b,c. Theorem 3. There exists a MQS of order v for every even v.

(22) REALIZABILITY OF ALMOST ALL DEGREE SETS BY k - TREES

Richard A. Duke, Georgia Institute of Technology and
Peter M. Winkler*, Emory University

A k-tree is a k-uniform hypergraph constructed by the successive addition of edges containing one new vertex and k-1 vertices of an existing edge. We show for any k ≥ 2 that with only finitely many exceptions, any finite set of positive integers containing "1" is the set of degrees of the vertices of some k-tree.

The proof makes use of a classical theorem of linear inequalities.

DEGREE DISTRIBUTIONS FOR RANDOM 4-TREES AND SKELETONS OF SYMMETRY WEIGHTED (1,4)-TREES (23)

Louis V. Quintas and Joshua Yarmish, Pace University, New York, NY

We present some observations based on the explicit computations that we have obtained of the number of unlabeled 4-trees on up to 25 points having a specified degree distribution. Because of their applicability in chemistry we also discuss the degree distributions of the 4-tree skeletons of (1,4)-trees that are weighted in accordance with the reciprocal of their symmetry numbers, where the symmetry number is the order of the automorphism group of the (1,4)-tree. In particular, we note that using the equiprobable distribution for the 4-trees and the symmetry weighted distribution for the (1,4)-trees the expected degree distribution of a random 4-tree and the expected degree distribution of the skeleton of a symmetry weighted (1,4)-tree are both "nicely" approximated by a linear function of the number of points of the tree.

The relation of these distributions for small order 4-trees, i.e., 25 or less points, to distributions for large order 4-trees obtained by other authors by other methods is commented upon.

On Edge-partitioned Graphs with Degree Restrictions (24)

Jerrold W. Grossman, Oakland Univ., Rochester, MI

Let m > n ≥ 2 . It is shown that there is no multigraph whose edges can be 2-colored uniquely so that each vertex is incident with fewer than n edges of one color and fewer than m edges of the other color. The proof is based on the existence of highly symmetric 2-edge-colored multigraphs which contain no color-alternating cycles.

Describing Simple Blocks (25)

R. H. Johnson, Sonoma State University

A graph which is determined up to isomorphism by its degree sequence is called, following Harary, simple. A description (characterization) of simple blocks is given. This description completes the project of describing simple graphs and settles some conjectures raised at an earlier Southeastern Conference.

FACTORIAL POWERS OF PENUMBRAL OPERATORS

John M. Freeman, Florida Atlantic University

The penumbral algebra is the algebra of operators on polynomials for which deg Ap(x) ≤ deg p(x). For penumbral A we define upper and lower factorial powers as

$$A^{(n)} = A\phi(A)\cdots\phi^{n-1}(A) \quad \text{and} \quad A_{(n)} = A\phi_-(A)\cdots\phi_-^{n-1}(A)$$

where ϕ and ϕ_- are the twist and anti-twist homomorphisms. A related operation, the factorial product, A!B is introduced. These notions are used to derive polynomial expansions. The examples of Euler, Laguerre, Abel, and Hermite polynomials are among those treated.

On Some Classes of Context-free Array Grammars

(30) Kenneth Holladay, University of Miami

An Array is a two-dimensional generalization of a string. Both sides of each rewriting rule of an isotonic array grammar have the same shape. Isotonic array grammars can be restricted by 3 progressively stricter requirements that are all analogs of the context-free restriction for string grammars. We give examples to show that corresponding classes of array languages are different. All 3 kinds of grammar have Greibach normal forms. We discuss the extent to which pumping lemmas hold.

On the genus of a group and its quotients

Thomas W. Tucker, Colgate University, Hamilton, NY 13346 (31)

The genus of a finite group A, denoted $\gamma(A)$, is the minimum genus over all surfaces that contain an imbedded Cayley graph for A. If B is a subgroup of A, then L.Babai has shown that any Cayley graph for A contracts to a Cayley graph for B and hence that $\gamma(B) \le \gamma(A)$. It is natural to conjecture the same inequality for a quotient group of A. It is shown why such a result is expected to be far more difficult to prove. Connections to the symmetric genus of A, the least genus of any surface upon which A acts, are discussed. The author's progress on this problem is presented, together with some stronger conjectures about the genus of a quotient group and their possible role in a general scheme to compute the genus of a group.

SPACES OF GRAPHS

(32) John L. Pfaltz, Univ. of Virginia

We will define an infinite discrete space whose element, or points, are entire directed graphs. The elements of this space are partially ordered by morphisms which map one graph onto another. Such spaces, although "lattice-like", are not true lattices. In particular, there are no sup or inf operators.

We will show however that the space of trees ordered by convex homomorphisms is "modular". This permits us to define two n-point trees T_1 and T_2 to be "adjacent" in the space if there exists an (n-1)-point tree T' and two convex homomorphisms f_1 and f_2 such that $f_1:T_1 \to T'$ and $f_2:T_2 \to T'$. Several interesting observations concerning random trees follow.

ON DIRECTED (k,t,λ^*)-SYSTEMS (18)

Greg A. McClanahan, Renu Laskar, William R. Hare* (Clemson, S.C.)

A Steiner k-tuple system is a pair (P,B), where P is a finite set of v points and B is a collection of k-element subsets of P, called blocks, such that every (k-1)-element subset of P belongs to exactly one block. The concept of a directed (k,t,λ^*)-system (DS) is introduced; it is a pair (P,B_D), where P is a finite set of v points, and B_D is a collection of ordered k-element subsets of P, called blocks, such that every ordered t-element subset of P occurs in exactly λ^* blocks of B_D. The ordering of blocks means that the block (1,2,...,k) contains the $\binom{k}{t}$ ordered t-tuples (1,...,t-1,t), (1,2,...,t-1,t+1),...,(k-t+1,...,k-1,k). Some necessary conditions for existence of DS are given for the general situation and several particular results are obtained for the case $k=4$, $t=3$, $\lambda^*=1$. Such systems are called directed quadruple systems (DQS). Theorem 1. If a DQS of order v exists, then v is even and the number of blocks is $b = (v(v-1)(v-2))/4$. Theorem 2. If v is congruent to 2 or 4, modulo 6, then there exists a DQS of order v. An example of a DQS of order 6 is given.

A Necessary and Sufficient Condition for Decomposing a Collection of Sets into Systems of Distinct Representatives (19)

David E. Woolbright, Columbus College

If $\mathcal{L} = \{L_1, L_2, \ldots, L_n\}$ is a collection of finite sets, then a system of distinct representatives for $\mathcal{L}$ is an n-tuple, $(x_1, x_2, \ldots, x_n)$, such that $x_i \in L_i$ for $1 \leq i \leq n$ and $x_i \neq x_j$ if $i \neq j$. If $|L_i| = m$ for all $L_i \in \mathcal{L}$, then a decomposition of $\mathcal{L}$ into systems of distinct representatives is a collection of m SDR's for $\mathcal{L}$, $\{(x_{11}, x_{12}, \ldots, x_{1n}), \ldots, (x_{m1}, x_{m2}, \ldots, x_{mn})\}$, such that $\bigcup_{i=1,m} x_{ij} = L_j$ for $1 \leq j \leq n$. This paper gives a necessary and sufficient condition for the existence of such decompositions.

New results on a subclass of balanced matrices. (20)

R.P.Anstee* and Martin Farber, University of Waterloo.

We discuss some properties of a subclass of balanced matrices called totally balanced matrices, introduced by Lovasz. These are (0,1)-matrices which do not contain the edge-vertex incidence matrix of any cycle of size greater than 2. A number of characterizations of these matrices are discussed, some of which yield efficient recognition algorithms. An interesting completion algorithm is presented which takes an arbitrary totally balanced matrix and completes it to a maximal one by adding columns.

The Common Neighbor Distribution of a Graph (26)

by Fred Buckley, St. John's University, Staten Island, N.Y. 10301

ABSTRACT

We define the common neighbor distribution (cnd) of a graph G to be $(n_0, n_1, n_2, \ldots, n_{n-2})$, where n_i denotes the number of pairs of vertices having i common neighbors. The mean number of common neighbors is

$$\mu_n(G) = \sum_{i=0}^{n-2} i\, n_i \Big/ \binom{n}{2}.$$

By using cnd(G) and $\mu_n(G)$ for several classes of graphs, one can easily establish various combinatorial identities. If all of the (nonzero) terms of cnd(G) are consecutive terms and are equal, then we have a uniform common neighbor distribution (ucnd). Graphs having ucnd's are examined. Finally, we consider relationships between cnd(G) and the degree sequence of G, the line graph of G, and hamiltonicity.

SELF-CENTRED MAXIMAL PLANAR GRAPHS (27)

A.F. Alameddine
University of Petroleum & Minerals
and
University of Waterloo

Let G be a maximal planar graph of order p with degree sequence $d_1 \leq d_2 \leq \ldots \leq d_p$. The center of a graph is the sub-graph induced by the set of vertices that have minimum distance to the most distant vertices. Here we consider graphs whose centers contain all their vertices. Here a class of such graphs will be characterized and enumerated when the d_i's are re-stricted.

VERTEX - DEGREES OF RANDOM GRAPHS (28)

Zbigniew Palka, A. Mickiewicz University & University of Florida

We shall be concerned with the discrete probability space G(n,p) consisting of the graphs with a fixed set of n labeled vertices in which each of $\binom{n}{2}$ possible edges occurs with the same probability p independently of all other edges. An element of G(n,p) will be denoted by $G_{n,p}$.

There are many papers which are concerned with the asymptotic properties (i.e. as $n \to \infty$) of degrees in a random graph $G_{n,p}$ (e.g., the distribution of the number of vertices of given degree, degree sequences, distribution of the maximum and minimum degree). In some of them the graphs $G_{n,p}$ with a fixed p are considered whereas in others, p is a function on n tending to zero as $n \to \infty$.

This paper will be a review of some recent (and not so recent) results in the second area (i.e., when $p = p(n) \to 0$), including a discussion of some open questions.

LEXICOGRAPHICALLY SHELLABLE POSETS (33)

Michelle Wachs*, University of Miami
Anders Bjorner, University of Stockholm

Shellability is a property of posets and simplicial complexes which has important consequences both inside and outside of combinatorics. An edge labeling technique, called lexicographical shellability which was introduced by A. Bjorner, has proved very useful in establishing the shellability of many classes of posets. Here we discuss some variations of this technique. One variation allows us to prove that Bruhat posets are shellable. Another is used in showing that certain fundamental poset operations preserve shellability. Results on the lexicographical shellability of semimodular posets and face lattices of simplicial complexes are also presented.

A NON-SHELLABLE 3-SPHERE (34)

Andrew Vince University of Florida

Shellability of simplicial complexes has been a useful concept in polyhedral theory, in piecewise linear topology and more recently in connection with Cohen Macauley rings. We initiate a study of shellability in a graph theoretic context. A correspondence is established between n-colored graphs and (n-1)-dimensional simplicial complexes. Via this correspondence, results on shellability of graphs have implications for problems in combinatorial topology. For example, it is known that all 2-spheres and convex d-spheres are shellable. It has been conjectured that all 3-spheres are shellable. We provide a counterexample.

INEQUALITY DEFINED COMMA-FREE CODES (35)

L.J. Cummings, University of Waterloo

Cooma-free codes with block length n atr synchronizable codes with bounded synchronization delay 2n-1. In 1965 W.L. Eastman gave a recursive construction of isomorphism classes of maximum comma-free codes for every odd n. Eastman's construction yields sets of codewords where each set is defined by inequalities between adjacent symbols in a codeword. We give necessary and sufficient conditions for a string of inequalities to define a comma-free code and discuss which collections of strings determine maximum comma-free codes.

Planar Cayley Diagrams: Accumulation Points (36)

H. Levinson
Rutgers University

Two results are proved. First, if a Cayley diagram Γ has an embedding in the sphere, then the minimum number of accumulation points of vertices of Γ is either 0, 1, 2, or ∞. Second, every planar Cayley diagram has a quasi point-symmetric or quasi weakly point-symmetric planar embedding. The "quasi" refers to the possibility of undirected edges representing generators of order two.

(37) ON UNAVOIDABLE GRAPHS

F.R.K. Chung, Bell Laboratories
P. Erdős, Math Inst. of the Hungarian Academy of Sciences

How many edges can be in a graph which is forced to be contained in every graph on n vertices and e edges?

In this paper, we obtained bounds which are in many cases asymptotically best possible.

(38) TREE RAMSEY MINIMAL GRAPHS

by

R. J. Faudree, Memphis State University, and
John Sheehan, Aberdeen University

For graphs G and H, $R^T(G,H)$ is the set of trees T such that $T \to (G,H)$ but $T' \not\to (G,H)$ for any proper subgraph $T' \subset T$. All pairs (G,H) such that $R^T(G,H) \neq \phi$ will be determined, in fact an explicit tree in $R^T(G,H)$ will be exhibited when $R^T(G,H) \neq \phi$. For a large class of graphs, $|R^T(G,H|$ will be shown to be infinite. For G a star and H a path, $R^T(G,H)$ will be completely determined.

(39) SIZE RAMSEY NUMBERS OF PATHS, TREES AND CYCLES

József Beck, Math Inst. of Hungary and SUNY at Stony Brook

We prove the existence of a graph of $c \cdot n$ edges such that any 2-coloring of its edges yields a monochromatic path of length n. We have results in the same spirit for cycles and trees as well.

(45) Discrete Pursuit Game

Alain Quilliot, University of South Carolina

On a finite graph $G = (X,E)$, 2 players A and B move in turn along the edge of G; A (pursuer) trying to catch B (evader). We give a characterization of those of the graphs for which pursuer A has a winning strategy in this game.

We extend this problem in the cases where there are several pursuers, or when the "speeds" of A and B are not the same, and we connect this problem with some fixed point problems on graph. We also deal with a discretization of the classical pursuit game on a compact metric space. $\varepsilon > 0$ being a positive number, A and B move in turn on E by leaps with length at most ε. This defines a game $J_\varepsilon(E)$ and we characterize the spaces where pursuer A can catch evader B in a "short enough" discrete time.

We give some examples of irregularities which may occur in the study of these games.

(41) Upper Bound Sequences and Sets for Partially Ordered Sets

John K. Luedeman, Clemson Univ. & F. R. McMorris, Bowling Green State University

Let $(P,\leq)$ be a finite partially ordered set (poset). For $x \in P$ define $U(x) = \{y \in P | y \geq x\}$. Let $P = \{p_1,\dots,p_n\}$ and denote $|U(p_i)|$ by a_i. The sequence $a_1 \leq a_2 \leq \dots \leq a_n$ is an <u>upper bound</u> (UB) <u>sequence</u> if there exists some poset $(P,\leq)$ with $|U(p_i)| = a_i$. In this situation we say that P realizes the UB-sequence $a_1 \leq a_2 \leq \dots \leq a_n$. We prove the following results:

(1) The sequence $a_1 \leq a_2 \leq \dots \leq a_n$ of positive integers satisfying $a_k \leq k$ for $k = 1,2,\dots,n$ is a UB-sequence iff $a_1 \leq a_2 \leq \dots \leq a_{n-1}$ is a UB-sequence.

(2) The sequence $a_1 \leq a_2 \leq \dots \leq a_n$ is a UB-sequence iff $a_k \leq k$ for $k = 1,2,\dots,n$.

(3) We characterize linearly ordered posets and binary tree semilattices in terms of their UB-sequences.

Given a poset P with UB-sequence $a_1 \leq \dots \leq a_n$, the set $S = \{a_1,\dots,a_n\}$ is called the <u>upper bound</u> (UB) set of P. We prove the following results:

(4) The set $S = \{a_1,\dots,a_k\}$ with $a_1 < a_2 < \dots < a_k$ is a UB-set iff $a_1 = 1$. Moreover, the minimum number of elements in a poset whose UB-set is S is a_k.

(5) Given $S = \{a_1,\dots,a_k\}$ with $1 = a_1 < \dots < a_k$ there is a poset P with m elements whose UB-set is S for all $m \geq a_k$.

(44) An Investigation of the Move-Ahead-k Rules

Aaron M. Tenenbaum and David M. Arnow

Department of Computer and Information Science, Brooklyn College

A set of $n-1$ sequential search algorithms which dynamically reorganize a list of n records is investigated. Each algorithm, $A(k)$, in the set is characterized by a distinct integer k, between 1 and $n-1$ inclusive, which indicates the number of positions that a requested record is moved forward upon retrieval. $A(1)$ is the well-known transposition algorithm, while $A(n-1)$ is the well-known move-to-front algorithm. If it can be shown that the expected costs of the algorithms $A(k)$ are linearly ordered, with $A(1)$ having least expected cost, then the long conjectured superiority of the transposition heuristic $(A(1))$ would be indicated. Some partial results supporting such a linear ordering for a restricted class of probability distributions of record requests are presented.

(49) A note on elegant graphs

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The harmonious graphs introduced and studied by Graham and Sloane are a modular version of the better known graceful graphs. They are also known to have applications to error correcting codes and networks of transmitting stations. Work on these graphs has been further assisted by considering a restricted notion, that of 'strongly harmonious graphs' and this, in turn, suggests another variant 'elegant graphs' which are the exact modular additive analogue of graceful graphs. We discuss some recent results on elegant graphs and show how they are obtained.

(50) On k-graceful graphs

Peter J. Slater
The University of Alabama in Huntsville

The concept of a k-graceful graph is presented as follows. For a graph G with vertex set $V(G)$ and edge set $E(G)$ with $|E(G)| = e$ and any function $h : V(G) \to N_0$ (the set of nonnegative integers), the induced function $g_h : E(G) \to N_0$ is defined by $g_h(u,v) = |h(u) - h(v)|$. Function $h : V(G) \to N_0$ is a <u>numbering</u> of G if both h and g_h are one-to-one. Graph G is <u>k-graceful</u> if there exists a numbering $h : V(G) \to \{0,1,\cdots,k+e-1\}$ with $g_h(E(G)) = \{k,k+1,\cdots,k+e-1\}$. An introductory study of k-graceful graphs is made.

(51) NEW PROOF TECHNIQUES FOR GRACEFULLY LABELLING GRAPHS

Thom Grace, University of Illinois at Chicago Circle

A labelling of a simple graph G with e edges is an assignment of distinct labels from $\{0, 1, \dots, e\}$ to the vertices of G which gives a label to each edge of G by taking the positive difference of its endpoints. If the edge labels thus induced are distinct, the labelling is called graceful. Graceful labellings have an extensive literature, yet most proofs have been of a constructive or enumerative nature. In this paper, it is shown that the unused vertex label in any graceful labelling of a polygon with one pendant point for each polygon vertex must be even. An induction-type argument is used to establish this result (first conjectured by R. Frucht). In addition, a process is described which gracefully labels any tree which contains an infinitely long path. This establishes an infinite analog of the long-standing Ringel-Kotzig conjecture; other variations are discussed.

NP-COMPLETE AND UNDECIDABLE PROBLEMS IN EUCLIDEAN RAMSEY THEORY

40

Stefan A. Burr, City College, C.U.N.Y.

Consider the following decision problem: INSTANCE: A set of points in the Euclidean plane. PROBLEM: Can the points be 3-colored so that no two points at unit distance have the same color? It is shown that this problem is NP-complete for finite sets, and that it is undecidable for infinite sets. Indeed, a doubly-periodic countable set is constructed such that the above problem is undecidable for sets formed from it by adding a finite set of points.

41 THE SMALLEST RAMSEY NUMBER FOR A TREE

P. Erdös, Hungarian Academy of Sciences
R. J. Faudree, C. C. Rousseau, R. H. Schelp*
Memphis State University

Canonical examples demonstrate that if T_n is a tree on n vertices then the Ramsey number of T_n is bounded below by $\{4n/3 - 1\}$. We show this lower bound is the Ramsey number for a certain family of trees. Let $B_{k,\ell}$ denote the tree (called a broom) obtained by identifying the end vertex of a P_ℓ with the central vertex of a $K_{1,\ell}$, these graphs being otherwise disjoint. We show the Ramsey number $r(B_{k,\ell}) = 4k + \{3(\ell-2k)/2\} - 1$ when $\ell \geq 2k$. In particular this gives, for each $n = k+\ell$ and appropriate choices of k and ℓ, trees T_n such that $r(T_n) = \{4n/3 - 1\}$. We also show for $1 \leq \ell < 2k$ that $K_{2k+\ell+4} \to (B_{k,\ell}, B_{k,\ell})$.

42 ON REGRESSIVE K-CHAINS: A RAMSEY-THEORETIC EXTREMAL PROBLEM FOR PARTIALLY ORDERED SETS

Douglas B. West, Princeton University
* William T. Trotter, Jr., University of South Carolina

A regression is a function g from a partially ordered set to itself such that $g(x) \leq x$ for all x. A regressive k-chain is a chain of k elements $x_1 < x_2 < \dots < x_k$ such that $g(x_1) \leq g(x_2) \leq \dots \leq g(x_k)$. For positive integers w and k, we define f(w,k) as the smallest number such that every regression on every partial order with size at least f(w,k) and largest antichain at most w has a regressive k-chain. The existence of f(w,k) is an easy consequence of Ramsey's theorem. In this paper, we determine the following explicit formula: $f(w,k) = (w+1)^{k-1}$.

46 BIASED POSITIONAL GAMES

A. Papaioannou*, L.S.U. and PP.MMS Cambridge, England and B. Bollobás

Biased positional games were first studied by Lehman (1964) and further results are due to Edmonds (1965) and Chvátal and Erdős (1978). A biased positional game is played by two players on the complete graph with n vertices. Each move of the first player consists of claiming one previously unclaimed edge, each move of the second player consists of claiming k previously unclaimed edges. The first players goal is to claim all the edges of some graph theoretical structure and the second players goal is to foil the first.

Erdős and Chvátal solved the Hamiltonian cycle game in 1978 for k = 1. Moreover they conjectured that there is a winning strategy for the first player in the general case. In this paper, a proof of the Erdős-Chvátal conjecture will be described.

HOW MANY PATHS CROSS AT LEAST s GIVEN LATTICE POINTS? 47

H. Niederhausen, Dept. of Statistics, Univ. of Toronto

The roadmap of Chess County shows a perfect grid. Gas rationing allows you to buy only 10 liters at each gas station, but you don't know where they are. What are your chances to get at least 50 liters on any shortest way from the Southwest to the Northeast corner of Chess County (conditional on the position of the gas stations, of course)? There is a nice answer to this question if the stations all lie on one line. We give a new proof and extensions of Takács' (1971) results. An algorithm is available for the general case. This problem and its continuous analog come up in a goodness-of-fit test (Matching Test).

48 EQUAL JUSTICE FOR UNEQUAL SHARES OF THE CAKE

by
Maria Klawe, IBM Research, 5600 Cottle Rd., San Jose, CA 95193,
and
Andrew Yao, U.C. Berkeley, Berkeley, CA 94720.

Many solutions have been presented for the classical problem of dividing a cake among n participants (with possibly different measures) in such a way that each participant is guaranteed to receive at least 1/n-th of the cake according to his measure. Recent research has focused on the problem of determining the minimum number of cuts such cake-cutting protocols must use.

Here we present some results on the following variant of this problem. Given a real number p between 0 and 1, what is the minimum number of cuts needed by a cake-cutting protocol which divides the cake between two participants A and B, such that the fractions of the cake received by A and B are guaranteed to be at least p and 1-p according to their respective measures?

62 Automorphisms of Graphs

Steve Fisk

Bowdoin College

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In this talk we generalize Frucht's theorem on the representation of a group as the automorphism group of a graph. Define a group-graph to be a graph whose vertices are the elements of a group, for which left and right multiplication induce automorphisms of the graph. For any graph X we construct a group-graph Aut(X) whose group is the automorphism group of X. Our main result is that any group-graph is isomorphic to the group-graph Aut(X) for some graph X. We also show that any group-graph is a Cayley graph. We conclude by giving some examples.

52 Lattice Embeddings and the Bandwidth Problem

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Miami University

J. Opatrny
Dept. of Computer Science
Concordia University

A labelling of a graph is an injection $\ell: V(G) \to \mathbb{Z}^+$ from the point set of G to the positive integers. The bandwidth of G is by definition $\min_\ell \max(|\ell(v) - \ell(w)|)$, where the maximum is taken over all pairs v,w of adjacent points of G. We develop a geometric method, based on embedding V(G) in the lattice points of $\mathbb{R}^2$, for obtaining estimates on the bandwidth of graphs. This method is applied in finding an exact formula for the bandwidth of "regular" refinements of caterpillar graphs.

53 Bandwidth of Directed Graphs

J. Opatrny

Concordia University
Montreal, Canada

We define the bandwidth of a directed graph H, to be the number

$$\min_f \{\max_{(u,v)\in E(H)}\{f(v) - f(u)\} - \min_{(u,v)\in E(H)}\{f(v) - f(u)\} + 1\}$$

where f is a one-to-one mapping from V(H) to integers.

The relation of the bandwidth of an undirected graph G to the bandwidth of orientations of G is studied. Several bounds on the bandwidth of directed graphs in terms of graph invariants are given. The above definition of bandwidth is compared to the definition of the bandwidth of acyclic directed graphs given in a paper by Garey, Graham, Johnson and Knuth.

The Bandwidth of the Corona and the Composition of Graphs. 54

Phyllis Zweig Chinn, Humboldt State University, Arcata, CA 95521

The *bandwidth* of a graph, B(G) is min max $\{|f(u) - f(v)|\}$ where the maximum is taken over all adjacent pairs of vertices and the minimum is taken over all 1:1 assignments of integers to the vertices of G. The *composition* G(H) is the graph obtained by replacing each vertex u of G by a copy H_u of H, taking the join $H_u + H_v$ whenever u is adjacent to v in G. G *corona* H, G∘H, is formed by taking one copy of G and |G| copies of H, joining the i^{th} vertex of G to each vertex in the i^{th} copy of H.

Upper bounds for B(G(H)) and B(G∘H) are given in terms of B(G) and B(H) and the exact bandwidth given for several families of coronas.

TURAN'S THEOREM FOR UNCROWDED GRAPHS 70

Joel Spencer, SUNY at Stony Brook

The classical theorem of Turan may be "improved" if we make additional assumptions that the graph G is "uncrowded" in some sense, for example, triangle free.

OPEN IRREDUNDANT SETS IN GRAPHS (55)

Arthur M. Farley University of Oregon; Eugene, OR 97403

In this paper we introduce the notion of open irredundancy. An open irredundant set in a graph G is a subset of vertices OIR such that, for each vertex v in OIR, the intersection of the open neighborhood of v with the union of the closed neighborhoods of the other vertices in OIR is not empty. Consideration of open irredundant sets is motivated by certain problems in broadcast-based communication networks. We compare the min and maximum orders of maximal open irredundant sets in G with those of maximal independent, minimal dominating, and maximal irredundant sets in G.

Optimal Gossiping Schemes with Conference Calls (56)

*Arthur Liestman; Simon Fraser University *
Dana Richards; Indiana U. - Purdue U. at Indianapolis

Each of n gossipers has unique information which is to be exchanged by telephone calls so that all gossipers will know all information. The graph on n vertices induced by the calls is known to involve 2n-4 calls and contains a C_4 subgraph. If the gossipers can use k-person conference calls it is also known how many calls are necessary and sufficient. We have investigated the structure of the hypergraph induced by an optimal calling scheme. We give a rather involved conjectured characterization of these hypergraphs for which the C_4 result, for two-person calls, becomes a corollary. An important intermediate conjecture is: If the hypergraph of calls is optimal and there is an m and p such that $n = mk + p(k-1) - d$, $0 \le d \le m-2 \le k-2$, then the hypergraph contains two sets of m edges which are perpendicular and further, is isomorphic to a canonical hypergraph up to d "don't care" vertices.

Traversing Unknown Graphs (57)

*Dana Richards, Charles Swart;
Indiana U. - Purdue U. at Indianapolis

Consider an automaton moving between vertices, along directed edges of an unknown graph. All vertices are unlabelled and so all vertices of degree d are indistinguishable. The outgoing edges are labelled 1,2,...,d; however incoming edges are not visible to the automaton. The automaton's goal is to traverse every edge of the graph given no prior information, except that the graph is strongly-connected (a necessary requirement). We give a Turing Machine which can traverse any graph but does not know when to stop. The solution is related to universal traversal sequences. Further, we then allow the automaton to leave movable pebbles behind, as markers. We show that a Turing Machine with one pebble can determine the given graph and halt. We conjecture that our algorithm makes a number of moves (on the graph) which is polynomial in the size of the graph. We discuss the relationship of this problem to current models of computation. Various results for less powerful automata are given.

(58) Improved Bounds on the Chromatic Number of a Random Graph

David W. Matula, S.M.U., Dallas, Texas

It is well known for any $\varepsilon > 0$ and sufficiently large n that all but a vanishingly small fraction of n-vertex graphs have chromatic number at least $(1-\varepsilon)n/(2 \log_2 n)$ and at most $(1+\varepsilon)n/\log_2 n$. The lower bound derives from an upper bound on the largest clique size and the upper bound on the chromatic number comes from analysis of a naïve sequential coloring algorithm. We show that all but a vanishingly small fraction of n vertex graphs must have chromatic number at most $(1+\varepsilon)n/(\frac{4}{3} \log_2 n)$. This narrows the bounds from a factor of 2 to a factor of 3/2, and also proves that the naïve algorithm is not asymptotically optimal.

ROOM SQUARES HAVE MINIMAL SUPERSQUARES (63)

W.D. Wallis, University of Newcastle

It is known that if a Room square of side r is a subsquare of a Room square of side s, then s must be at least 3r+2. We shall show that this lower bound can always be attained provided r>1.

(64)

A. Hobbs*, A. Kotzig, and J. Zaks, Latin squares with high homogeneity. Presented by Arthur M. Hobbs.

Given a latin square L with cells containing the symbols a, b, ..., d, for each symbol a, we define a graph G_a having the cells of L containing a as its vertices and having two vertices joined by an edge iff the symbols in those cells form a diagonal of an order 2 latin subsquare of L. If G_a is regular of degree h for every symbol a in L, we say that L is h-homogeneous. We show that there is a unique (n-3)-homogeneous latin square of order n for $n \in \{3, 4, 6, 8, 12, 16\}$, and that (n-3)-homogeneous latin squares do not exist for any other orders n. Further, we show that, if k is an odd positive integer, there are (n-k)-homogeneous latin squares of orders k, 2k, and 4k, and, if k is an even positive integer, there are (n-k+1)-homogeneous latin squares of orders k, 2k, and 4k.

DOUBLY DIAGONAL ORTHOGONAL LATIN SQUARES (65)

Katherine Heinrich* and A.J.W.Hilton
Simon Fraser University

A pair of doubly diagonal orthogonal latin squares of order n, DDOLS(n), is a pair of orthogonal latin squares of order n with the property that each square has a transversal on both the front diagonal (the cells $\{(i,i): 1 \le i \le n\}$) and the back diagonal (the cells $\{(i,n+1-i): 1 \le i \le n\}$). We show that for all n except n = 2,3,6,10,12,14,15,18 and 26, there exists a pair of DDOLS(n). Obviously these do not exist when n = 2,3 and 6.

Intersection sets in block designs (66)

A.A. Bruen, The University of Western Ontario, London, Canada

Let S be a set of points in a design such that each block meets S. Various questions and results concerning the possible size of S are discussed.

(74) A TEN POINT THEOREM FOR 3-CONNECTED CUBIC GRAPHS

D.A. Holton, Vanderbilt University and
University of Melbourne

It is a well-known result of Dirac that in a k-connected graph any k points lie on a cycle. For 3-connected cubic graphs Holton, McKay, Plummer and Thomassen have proved that any 9 points lie on a cycle. This result has recently been extended by Ellingham, Holton and Little who have shown that in a 3-connected cubic graph G any 10 points lie on a cycle unless the 10 points are on a configuration in G which is isomorphic to the Petersen graph.

This paper will give a more precise statement of the last theorem and discuss the proof techniques used in both the 9 and 10 point results. Consideration will also be given to the case of a k-connected k-regular graph.

Maximum bipartite subgraphs of regular graphs with large girth

Brendan D. McKay (71)
C. S. Department, Vanderbilt University, Nashville, TN 37235

For a graph G, define $b(G)$ to be the maximum number of edges of a bipartite subgraph of G. For $n, g, r \ge 3$ define $B(n,g,r) = \min(b(G)/(nr/2))$, where the minimum is over all regular graphs with order n, degree r and girth at least g. Bondy and Locke have proved that, for example, $B(n,g,3) \ge \frac{7}{9}$ for $n \ge 6$ and $B(n,g,3) \ge \frac{4}{5}$ for $g \ge 4$. In this paper we prove that there are constants $\alpha_r < 1$ such that $\liminf_{n\to\infty} B(n,g,r) \le \alpha_r$ for any fixed g. Furthermore, $\alpha_r \to \frac{1}{2}$ as $r \to \infty$. The technique used is a model for random graphs developed by Bender, Canfield and Bollobás.

SUBGRAPHS IN WHICH EACH PAIR OF EDGES LIE IN A SHORT COMMON CYCLE (72)

Richard Duke *
Georgia Institute of Technology

Paul Erdős
Hungarian Academy of Sciences

We show first that for each positive constant c there exists a positive constant c' such that for sufficiently large n every graph with n vertices and cn^2 edges contains a subgraph H with $c'n^2$ edges which is such that each pair of edges of H are contained in a common cycle of H of length 4 or 6, with each pair of edges which share a common vertex being in a cycle of length 4.

A generalization of this result is obtained for hypergraphs with "cycle" being replaced by "k-cycle", where the latter term denotes a k-graph having no "separating edge" (a notion introduced by Lovász in his definition of a "k-forest") and minimal with respect to this property.

LONG CYCLES GENERATE THE CYCLE SPACE OF A GRAPH

Irith Ben-Arroyo Hartman (73)
University of Waterloo

The cycle space of a graph G is the vector space of all edge sets of Eulerian subgraphs of G over GF(2). A set C of cycles in a graph G is said to generate the cycle space if every cycle C of G can be written as a sum of cycles C_i in C.

Let G be a 2-connected graph in which the degree of every vertex is at least d. We give an outline of a proof that the cycles of length at least d + 1 generate the cycle space of G, unless $G \cong K_{d+1}$ and d is odd.

Another result states: if G is 2-connected with chromatic number at least k, then the cycles of length at least k generate the cycle space of G, unless $G \cong K_k$ and k is even. We deduce several corollaries, among them a stronger version of a theorem of Erdős and Hajnal.

Boolean Techniques for Matroidal Decomposition of Independence Systems and Applications to Graphs (58)

Claude Benzaken and Peter L. Hammer *

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University of Waterloo,
Waterloo, Ontario, Canada

The problem of decomposing an independence system into the set-theoretic union of matroids is considered in the first part of this paper and a Boolean procedure is proposed for finding the prime matroidal components of such a decomposition. The second part of the paper deals with the special case of the independence system of all stable sets of a graph, characterizes the graphs whose family of stable sets is the set-theoretic union of two matroids, produces a class of perfect graphs of matroidal number k and gives for graphs an accelerated version of the general decomposition technique.

BOX REPRESENTATIONS OF A FAMILY OF ELLIPSOIDS (60)

Egon Balas, Carnegie-Mellon University

The following problem often arises in a practical context: given a family $\mathcal{E}$ of n-dimensional ellipsoids, find a point contained in a maximum number of ellipsoids. The combinatorial aspect of this problem involves the finding of a maximum clique of the intersection graph $G(\mathcal{E}) = (V(\mathcal{E}), E(\mathcal{E}))$ of $\mathcal{E}$. We define a valid box-representation of $\mathcal{E}$ as a family $\mathcal{B}$ of n-dimensional boxes such that the intersection graph $G(\mathcal{B})$ of $\mathcal{B}$ satisfies $V(\mathcal{B}) = V(\mathcal{E})$ and $E(\mathcal{B}) \supseteq E(\mathcal{E})$. $\mathcal{B}$ is optimal if $G(\mathcal{B})$ has a minimum number of edges. While finding a maximum clique of $G(\mathcal{E})$ is an NP-complete problem, every clique of $G(\mathcal{E})$ is contained in some clique of $G(\mathcal{B})$; and since $G(\mathcal{B})$ is the intersection of n interval graphs, the cliques of $G(\mathcal{B})$ can be listed in time polynomial in $|V|$ (for fixed n). We give several results that can be used to approximate an optimal box representation of $\mathcal{E}$.

The Analysis of Network Optimization Algorithms: An Overview (61)

*Edward K. Baker, University of Miami
Bruce L. Golden, University of Maryland

In this paper we study the analysis of algorithms, especially with regard to network optimization. We focus on three key topics -- exact algorithms, approximate algorithms, and computer implementation of algorithms. The tone is largely tutorial. In the first section, we discuss the notions of efficiency, polynomial boundedness, and NP-completeness and we use *linear programming* to illustrate that the difference between "good" and "bad" algorithms is sometimes illusory. In the second section, we address the importance of heuristics and describe what is meant by worst-case behavior, average-case behavior, statistical analysis, and empirical analysis of heuristics. We also mention some related future research directions. In the third section of our paper, we examine several issues involved in computer experiments aimed at testing and demonstrating the efficiency of an algorithm. Performance indicators such as CPU time, numerical accuracy, storage requirements, number of iterations, portability, etc. are discussed.

A TRY FOR THREE ORDER-10 ORTHOGONAL LATIN SQUARES (67)

E. T. Parker*and John Wesley Brown,
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We start with direct product of groups of orders 5 and 2. Then we turn the latin 2-subsquares on main diagonals of the 4 latin 5-subsquares. This yields a latin 10-square with $2816 = 2^8.11$ transversals, compared with some 850 for random squares. This case seemed worthy of computation to decide whether it is extendible to an orthogonal triple. The considerable symmetry made it possible to reduce the search to one of 11 transversals required in an orthogonal mate. The CDC Cyber computer at the University of Illinois was programmed to place two transversals through cell one, two through the second, etc. Time averaged 270 seconds for each of 11 cases. The extension to an orthogonal triple does not exist.

Slightly Generalized Association Schemes, The Rands One-Factor Schemes, and a Possible Application to the Existence Question for an Oval in a Plane of Order Ten (68)

Robert Roth, Department of Mathematics and Computer Science,
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The subsets determined by two partitions of an n-set into parts of size k intersect in various "patterns" which sometimes (but not often) determine the classes of an association scheme. This does occur for $k = 2$ and I will discuss how here in the case of $n = 12$ the Delsarte inequalities provide some information about the structure of an oval in a plane of order 10.

On Projective Planes of Order 12 with Collineation Group of Order 27

Julia M. Nowlin Brown, York University (69)

Abstract Let π be a projective plane of order 12 which has a collineation group of order 27. We are able to determine the isomorphism class of the collineation group, the number and sizes of the point orbits, the number and sizes of the line orbits, the number and sizes of the flag orbits, the action of the group on each orbit, and the incidence numbers. (An incidence number is the number of points of a point orbit incident with a line of a line orbit.) Up to classification by the above properties, there are four possible types of projective plane of order 12 with collineation group of order 27. The existence of a plane of any one of these types can be shown to be equivalent to the existence of a 111-tuple of elements from the group which satisfies 81 conditions. Each of these 81 conditions asserts that some set of elements defined in terms of the 111-tuple and the (known) stabilizers of points includes all elements of G. Some of the above results generalize to planes of order $p^2 + p$ (where p is an odd prime).

LONGEST PATHS IN LOCALLY HAMILTONIAN GRAPHS (75)

Roger Entringer* and Sharon MacKendrick, University of New Mexico

A graph is a locally hamiltonian iff the subgraph induced by the neighborhood of any vertex is hamiltonian. We give bounds for the length of a longest path in a connected locally hamiltonian graph.

MINIMUM k-GEODETICALLY CONNECTED GRAPHS (76)

Douglas E. Jackson*, Eastern New Mexico University
Roger C. Entringer, University of New Mexico

A graph G is said to be k-geodetically connected if and only if G is connected and the removal of at least k vertices is required to increase the distance between any pair of vertices. Bollobás and Eldridge found the minimum possible number of edges in any k-geodetically connected graph of diameter 2 on n vertices and characterized those graphs having the minimum number of edges when $n \geq 2k$. We extend these results to the case $n < 2k$ and discuss possible extensions to graphs of larger diameter.

ULTRAGEODETIC GRAPHS

Michael F. Bridgland - Louisiana State University (77)

Motivated by the rich structure of geodetic graphs of diameter two, we consider the class U of two-connected geodetic graphs G with arbitrary diameter $d(G)$ that are "packed" with pyramids [2] in this sense: for each maximal clique M, the distance-preserving pyramids with base M and altitude $d(G)$ exhaust $V(G)$. We obtain two characterizations; the first indicates the similarity of the graphs in U to the strongly geodetic graphs of Bosák, Kotzig, and Znám [1] (which turned out to be just the Moore graphs), and the second is an analogue of Singleton's characterization of Moore graphs:

(1) A two-connected graph G is in U iff there is a unique chordless x,y-path of length less than or equal to $d(G)$ for each pair of vertices x,y in $V(G)$.
(2) A graph G with augmented girth $g^{\#}(G)$ [2] is in U iff G has no subgraph isomorphic to $K_4 - e$ and $g^{\#}(G) = 2 \cdot d(G) + 1$.

In view of (1), we call the graphs in U ultrageodetic. Some obvious examples are pyramids, geodetic graphs of diameter two, and Moore graphs; indeed, an ultrageodetic graph is a Moore graph iff every maximal clique is an edge or the entire vertex-set. We restrict a construction of geodetic graphs due to Plesník [3] to obtain an infinite family of ultrageodetic graphs with diameter $3k + 1$ for each $k \geq 1$.

1. Bosák, J., Kotzig, A., and Znám, S., Strongly geodetic graphs, J. Comb. Theory 5 (1968), 170-176.
2. Bridgland, M., Pyramidal subgraphs of geodetic graphs, Proc. 12th Southeastern CGTC Conference, 1981 (to appear).
3. Plesník, J., A construction of geodetic blocks, Acta Fac. Rer. Nat. Univ. Com. Math. 36 (1980), 47-60.

A Technique for Finding Approximate Inverses of a Continous One-Way Transformation (78)

Marc J. Lipman, Indiana U.-Purdue U. at Fort Wayne

It is easy to calculate local variations in the Earth's gravitational field at the ocean's surface from variations in the topography of the ocean floor. But even under fairly restrictive assumptions it is difficult to invert the proceedure and solve for the topography given the gravity. However, given the gravity it is possible to use the easy calculation to find approximate values for the topography, accurate to within limits determined by the errors in the gravitational data.

FAST ENUMERATION, RANKING, AND UNRANKING OF BINARY TREES (79)

Ekaputra Laiman and Andrzej Proskurowski
Department of Computer and Information Science
University of Oregon

A linear representation of binary trees proposed earlier is analyzed with respect to the complexity of ranking and unranking operations. Simple algorithms are given for performing these operations in time proportional to the order of the binary tree, provided a table of values of certain integer function is precomputed. Also, the complexity of generating representations of all binary trees of given order is discussed. An algorithm is given, which updates the representation of a tree to yield the representation of the next tree in a time averaging to a constant over all generated binary trees. The larger the order of the generated trees the smaller the constant.

(84) TWO RESULTS ON PERFECT GRAPHS

V. Chvátal, McGill University

The first result characterizes those ordered graphs for which a minimum coloring of each induced subgraph can be obtained by a greedy heuristic. The second result shows that a certain "semi-strong perfect graph conjecture", whose validity would imply Lovász's Perfect Graph Theorem, is implied by Berge's Strong Perfect Graph Conjecture.

On h-perfect graphs

(85) N. Sbihi, University of Waterloo

A graph is said to be h-perfect if its stable set polytope (i.e. the convex hull of the incidence vectors of its independent set of vertices) is defined by the constraints corresponding to cliques and odd-holes. Series-parallel graphs and bipartite graphs are h-perfect. We prove here that the graph obtained by substituting bipartite graphs for edges of a series-parallel graph is also h-perfect. We do this by making use of LP duality and we also define an operation of reduction which preserves h-perfection which allows us to prove that the graph obtained by identifying two vertices of a bipartite graph is h-perfect.

Channel Graphs

Nicholas Pippenger, IBM Research, San Jose, CA 95193

Channel graphs, which have their origin in the study of telecommunication networks, are graphs endowed with a pair of distinguished vertices, the <u>source</u> and <u>target.</u> Each vertex and edge in the graph is either <u>available</u> or not, these alternatives occuring at random according to some probability distribution. The principal questions are whether there exists a path from the source to the target consisting entirely of available vertices and edges and, if so, how to find one. Results concerning these questions, which have extensive connections with combinatorics, graph theory and computing, will be surveyed in this lecture.

(90) The Affine Line Conjecture

T. C. Brown, Simon Fraser University, and
J. P. Buhler, Reed College

Conjecture: For every finite field F and $\varepsilon > 0$ there is an $n = n(F,\varepsilon)$ such that if V is an affine space over F with dim $V \geq n$ and A is a subset of V with $|A| > \varepsilon|V|$ then A contains an affine line (i.e. a one-dimensional affine subspace). We show this when F is the field with three elements. More generally we show that if F has odd characteristic then there is an $n' = n'(F,\varepsilon)$ such that if V is an affine space over F with dim $V \geq n'$ and A is a subset of V with $|A| > \varepsilon|V|$ then A contains three elements of the form x, x+y, x+2y, where $y \neq 0$.

When F is the 2-element field an even stronger statement is true: for all k and $\varepsilon > 0$ there is an $n'' = n''(k,\varepsilon)$ such that if V is an affine space over the two-element field with dim $V \geq n''$ and A is a subset of V with $|A| > \varepsilon|V|$ then A contains a k-dimensional affine subspace.

A PROBLEM IN TRANSVERSAL THEORY (91)

Lynn M. Batten, University of Winnipeg

The well-known Rado-Hall theorems in transversal theory are based on a rank function defined on independence systems. We propose a weaker notion of rank function defined on independence systems, and indicate how this may lead to more general theorems. Some partial results are obtained.

Halin Graphs and the Travelling Salesman Problem (80)

G. Cornuéjols
Carnegie-Mellon University

D. Naddef
Université Scientifique et Médicale de Grenoble

W.R. Pulleyblank *
University of Waterloo

A Halin graph $H = T \cup C$ is obtained by imbedding a tree T having no degree two nodes in the plane, and then adding a cycle C to join the leaves of T in such a way that the resulting graph is planar. These graphs are edge minimal 3-connected, hamiltonian, and in general have large numbers of hamilton cycles. We show that for arbitrary real edge costs the travelling salesman problem can be polynomially solved for such a graph, and we give an explicit linear description of the travelling salesman polytope (the convex hull of the incidence vectors of the hamilton cycles) for such a graph.

Wrapped Coverings and Their Realization. (81)

J.L. Gross, B. Jackson, T.D. Parsons and T. Pisanski - Penn State

In two earlier papers, Jackson, Parsons and Pisanski introduced the notion of a wrapped covering of graphs, showed how it was related to the topological theory of current graphs and voltage graphs, and applied wrapped coverings to obtain many graph embeddings. The authors now show how to construct wrapped coverings directly from current graphs with permutation currents, and they discuss the problem, first raised by Gross, of which wrapped coverings are topologically realizable.

On the probable behavior of some algorithms for finding the stability number of a graph. (82)

Boris Pittel , The Ohio State University

A class of f-driven algorithms due to Chvátal for finding the stability number of a graph are studied. It is shown that, for almost all graphs, the computation time is $\exp(c \ln^2 n)$, where n is the number of vertices, and $0.25/\ln 2 \leq c \leq 0.5/\ln 2$. If, however, each edge exists with probability δ/n, independently on other edges, then asymptotically almost certainly the computation time is exponential. Still, for all large enough δ's, these algorithms perform noticeably better than a naive algorithm. All the results are extended on random graphs with a fixed number of edges.

APPLICATION OF DUALITY TO THE SHANNON CAPACITY OF GRAPHS

(83) M. Rosenfeld, Simon Fraser University and
Ben Gurion University of the Negev

L. Lovász has developed a most powerful method for obtaining upper bounds (the Lovász bound) for the capacity of graphs. He proved two most useful formulas for $\nu(G)$ - the Lovász bound. In this note we show that one can be obtained from the other by dualizing a suitable convex program.

(86) REMARKS ON COHESION AND INDUCED EDGE CONNECTIVITY

K.B. Reid, The Johns Hopkins University and Louisiana State University

The cohesion of a vertex x, denoted $\mu(x)$, in a connected graph G is the minimum number of edges of G whose deletion results in a subgraph of G in which x is a cut vertex. The induced edge connectivity of x, denoted $\lambda(x)$, is the edge connectivity of G-x. Lipman and Ringeisen (Congressus Numerantium XXIV, Utilitas Mathematica Publ., Inc. (1979), 713-720) reported some work on when $\mu(x) = \lambda(x)$, but that work involved the parameter λ, the edge connectivity of G. In this note necessary and sufficient conditions are obtained for $\mu(x) = \lambda(x)$ as well as for when $\mu(x)$ is equal to the cohesion of x in G + xy, where $y \in V(G)$ and $xy \notin E(G)$.

(87) SMALLEST MAXIMALLY NONHAMILTONIAN GRAPHS

L.H. Clark * and R.C. Entringer
Louisiana State University and University of New Mexico

A graph G is maximally nonhamiltonian if G is not hamiltonian but G + e is hamiltonian for any edge e in G^c. Bollobás posed the problem of finding the least number of edges , f(n) , in a maximally nonhamiltonian graph of order n . We prove the following.

Theorem. (i) $f(n) = 3n/2$, for all even $n \geq 36$,
(ii) $f(n) = (3n+1)/2$ or $(3n+3)/2$, for all odd $n \geq 55$.

(88) On the Reverse of a Hamiltonian Cycle

J.B. Klerlein * and A.G. Starling

Western Carolina University

The reverse of the hamiltonian cycle $(v_1, v_2, \ldots, v_n)$ is the hamiltonian cycle $(v_1, v_n, v_{n-1}, \ldots, v_2)$. Given the hamiltonian cycles of the complete graph K_n in lexicographic order, we attempt to describe a procedure to select a collection H of hamiltonian cycles with the property that a cycle is in H if and only if its reverse is not.

EVERY TOURNAMENT CONTAINS EVERY ORIENTED HAMILTON PATH

(89) Andrew Thomason, Louisiana State University

It is well known that every tournament contains a directed hamilton path. We give here a proof of a conjecture of Rosenfeld, that, except for a few very small cases, every tournament contains every orientation of a hamilton path.

Subquadrangles of Kantor's quadrangles in characteristic 2. (92)

S. E. Payne *, Miami University, and C.C. Maneri, Wright State University.

For $q = 2^e$, e odd, the generalized quadrangle S of order (q^2, q) constructed by Kantor has a family of subquadrangles of order (q, q) all isomorphic to the self-polar one S' constructed by Tits from the oval $\{(1, c, c^4) \mid c \in GF(2^e)\} \cup \{(0, 0, 1)\}$ in $PG(2, 2^e)$. The embedding of S' in S has a variety of consequences for S' and also shows that certain theorems of Thas characterizing the classical examples with these parameters are essentially the strongest that one could hope for.

Completions of quadrangles in projective planes, revisited II

R. B. Killgrove *, St. Lawrence U., M. Cates, Cal State U, L. A. (93)

This time we want to give the orbits of the quadrangles which singly generate the translation plane of order nine and likewise for the dual plane as we (Killgrove, Kiel) did for the self-dual plane (non-Desarguesian) plane of order nine. The orbits for the quadrangles generating the subplanes are already established. Also we show how certain quadrangles will not generate any planes of order nine. Further, a particularly symmetric configuration already found in the self-dual (Hughes plane also occurs in the translation plane. We call it a Fano cluster and it arises as follows: let A, B, C, D be the points for the quadrangle and E,F,G be the vertices of the diagonal triangle when these are not collinear, then should A,E,F,G and B,E,F,G and C,E,F,G and D,E,F,G all generate subplanes of order two (Fano configurations), then this set of points and lines forms a confined configuration not closed under either join or intersection which we dub a Fano cluster.

(94) Extending Partial Projective Planes

Stephen Dow, University of Florida

A partial projective plane Σ of order n is an incidence structure on $n^2 + n + 1$ points such that each line contains exactly n + 1 points and any two lines meet in a unique point. In this paper it is shown that, if $b > n^2 - n$ and each point is on at least $n - \sqrt{n+12} + 4$ lines, then Σ can be embedded in a projective plane of order n. This fact allows us to improve a result of McCarthy and Vanstone.

(95) MINIMAL PACKINGS OF PARTIAL PLANES

H.P. Smith, University of Victoria, Victoria, B.C.

Let $(\mathcal{P}, \mathcal{L})$ be a partial plane on v points, each line in $\mathcal{L}$ containing k points, $k \geq 3$. Assume that no pair of distinct lines intersects in more than one point and that if S is any k-subset of $\mathcal{P}$ then $S \cap L > 1$ for some line in $\mathcal{L}$. Alternatively, the plane can be regarded as a packing of the graph K_v by edge-disjoint copies of K_k. Let $\rho(k,v)$ be the minimum cardinality of $\mathcal{L}$ over all such planes. Precise values of ρ for all v are known only for k = 3. We exhibit some bounds for k > 3.

(150) POLYA'S THEOREM FOR COSETS

James O. Owings, Jr.
University of Maryland, College Park

Polya's Counting Theorem finds a practical use for the cycle index of a permutation group; in fact, it more or less justifies the definition of cycle index. In this note, we show how the cycle index of a coset of a normal subgroup of a permutation group can be used to solve certain counting problems. Our theorem is based on a generalization of Burnside's Lemma.

T-Colorings of Graphs and the Channel Assignment Problem

(96)

Fred S. Roberts
Rutgers University

In this paper we study some graph-theoretical problems which arise from the need to make more efficient use of the spectrum of radio frequencies in communications. In particular, suppose T is a set of nonnegative integers and $G = (V,E)$ is a graph. A T-coloring of G is a function f which assigns to each x in $V(G)$ a positive integer or channel such that

$$\{x,y\} \in E(G) \rightarrow |f(x)-f(y)| \notin T.$$

T-colorings were introduced by Hale in a 1980 paper in Proceedings of the IEEE. The order of a T-coloring f is the smallest number of distinct values $f(x)$ and the span is the maximum $|f(x)-f(y)|$ over all vertices x and y. We present results on the problems of how to compute a minimum order and minimum span T-coloring.

On a Moderately Exponential Graph Isomorphism Test

(97) Eugene M. Luks, Bucknell University

Recently, Zemlyachenko announced the first "moderately exponential" graph isomorphism test, that is, its running time is bounded by $\exp(n^{1-\epsilon})$ for some $\epsilon > 0$, where n is the number of vertices. His algorithm uses techniques of the author's which were instrumental in demonstrating a polynomial bound for graphs of bounded valence. Together with results of Babai on the orders of primitive permutation groups, this led to an $\exp(n^{2/3 + o(1)})$ bound for general graphs. We now describe an essential improvement in the bounded valence algorithm, removing a factor of d(= valence) from the exponent in the time bound. This, in turn, improves the bound for general graphs to $\exp(n^{1/2 + o(1)})$.

TOLERANCE GRAPHS (151)

Martin C. Golumbic
Bell Laboratories
Murray Hill, N.J.

Clyde L. Monma
Bell Laboratories
West Longbranch, N.J.

In this talk we introduce three new classes of perfect graphs: tolerance graphs, bounded tolerance graphs, and proper tolerance graphs. Several results are given which describe the structure of these graphs. In particular, we prove the following proper inclusions between classes of graphs:

proper interval graphs ⊂ proper tolerance graphs
⊂ bounded tolerance graphs ⊂ tolerance graphs

and

interval graphs ⊂ bounded tolerance graphs
⊂ complements of comparability graphs

A number of open problems will also be discussed.

(104) On 3-skein Isomorphisms of Graphs

R. L. Hemminger, Vanderbilt University
H. A. Jung,* Technische Universität Berlin
A. K. Kelmans, Profsouznaya Str., Moscow.

A graph consisting of n openly disjoint paths joining two vertices is called an n-skein. A bijection between the edge sets $E(G)$ and $E(G^*)$ of graphs G and G^* is called an n-skein isomorphism if it induces a bijection between the sets of n-skeins contained in G and G^*, respectively.

Whitney proved in [Congruent graphs and the connectivity of graphs, Amer. J. Math 54 (1932), 150-168] that circuit isomorphisms of 3-connected finite graphs are induced by isomorphisms.

The main result of this paper is the following.

Theorem. Each 3-skein isomorphism from a 3-connected graph with at least 5 vertices onto a graph without isolated vertices is induced by an isomorphism.

The Theorem in the present form was proved by Kelmans and independently by Hemminger and Jung. Previously, it was proved for 4-connected graphs by Halin and Jung [Note on isomorphisms of graphs, J. London Math. Soc. 42 (1967), 254-256. MR 34 #7402] and for 3-connected graphs containing at least one 4-skein by Hemminger and Jung [to appear in J. Combinational Theory].

A sufficient condition for a graph to be the union of a fixed number of complete subgraphs (105)

M.D. Guay, University of Maine

A graph G is defined to be star-complete at a vertex v of G if the subgraph of G induced by the star determined by v is complete. A graph G is star-complete if it is star-complete at each of its vertices. The main result is stated as follows: Let G be a graph of order $\geq n \geq 4$ which is star-complete except for some subset Q of n vertices. If $\langle G - Q \rangle$ is connected, then G is the union of $[\frac{1}{2} n^2] + 1$ or fewer complete subgraphs.

The Most Covered Region of an Arrangement of Lines.

George B. Purdy,* Texas A and M University, and Roger Entringer, University of New Mexico. (152)

The authors discuss the following question: Given an arrangement of n lines in the real euclidean plane, if we ask of each bounded region: How often is it also bounded if only three of its sides are used? We then discuss the maximum of this over all of the regions in the arrangement. A previous result of ours, to appear in The Israel Journal implies that the maximum is at most cn^3.

We show that the minimum is about cn^2.

We also discuss the more general, motivating question of how to best place points in the plane so as to minimize the number of convex quadrilaterals formed, and we clarify the old Erdős-Szekeres construction.

(112) t-wise Designs on the Complete Graph with Symmetric Automorphism Group

L. Chouinard, E. Kramer and, D. Kreher (Univ. of Nebraska-Lincoln 68588)

A t-wise balanced design $(X, \mathcal{B})$ with parameters t-(v,K,λ), is a set X and family of subsets $\mathcal{B}$ from X, where if $B \in \mathcal{B}$ then $|B| \in K$, and such that each t-subset of X is in exactly λ members of B. Furthermore, to avoid trivial designs we require $t < k < v$ for all $k \in K$, and that $\mathcal{B}$ does not contain all the k-subsets of X for any k. Let p be the number of vertices in a finite undirected complete graph. Let X be the set of all $v = \binom{p}{2}$ edges of this graph and, let S_p be the symmetric group on the vertices of this graph. We determine all t-$(\binom{p}{2},K,\lambda)$-wise designs preserved by S_p with $\lambda = 1$ and $\lambda = 2$.

(113) INEQUALITIES FOR PERFECT SYSTEMS OF DIFFERENCE SETS

J. Abrham,* University of Toronto, and
A. Kotzig, Université de Montréal.

Let $c, m, p_1, \ldots, p_m$ be positive integers. Let $S_i = \{x_{0i} < x_{1i} < \ldots < x_{p_i,i}\}$ be sequences of integers and let D_i be their sets of differences: $D_i = \{x_{ji} - x_{ki},\ 0 \leq k < j \leq p_i\}$, $i = 1, \ldots, m$. Then $\{D_1, \ldots, D_m\}$ is a perfect systems of difference sets (PSDS) starting with c if $\bigcup_{i=1}^{m} D_i = \{c, c+1, \ldots, c-1+\frac{1}{2}\sum_{i=1}^{m} p_i(p_i+1)\}$. Each D_i is called a component of the given PSDS and the size of D_i is p_i. The following inequality can be proved to hold for every PSDS: $c_2 + c_3 + c_4 \geq c + \frac{1}{2}\sum_{k=4}^{\infty} k(k-3)(c_{2k-1} + c_{2k})$ where c_r, $r \geq 2$, denotes the number of components of size r. This inequality can be used to obtain bounds for the number of "small" comonents (of size ≤ 4), and for the maximum size and the average size of components.

"On Graph Certificates"

Derek Corneil, University of Toronto

(98) Mark Goldberg,* Clarkson College

A graph certificate or canonical form is a short unique (up to isomorphism) representation of the graph. Thus two graphs are isomorphic iff their certificates are identifal. In this paper we demonstrate an $O(c^n)$ graph isomorphism algorithm which also produces a certificate of the graph. It is noted that other approaches using Luks' results which yeild faster algorithms do not seem to be capable of producing certificates.

TRANSITION GRAPHS OF COMPUTER AUTOMATA AND SEQUENTIAL PROGRAMS (99)

Karel Culik, Dept. of Comp. Sci., Wayne State Univ., Detroit, MI 48202

An automaton A = ⟨In, St, trans⟩ (without initial and final state) is called a concrete computer automaton if its inputs from In are computer instructions (assignments, input/output instructions, procedure calls, tests), its states from St are memory states, i.e., $St = Var^{Values}$ where Var is the set of all variables occuring in instructions, and Values is the set of values concerned, and trans: In×St→St is its transition function (by tests and by output instructions states are not changed). The transition graph of A is TG = ⟨St, T, τ⟩ where T={(st,st*)∈ St×St; there exists in ∈ In such that trans(in,st)=st*} and $\tau: T \to (2^{In} - \{\emptyset\})$.

A sequence of instructions from In, $P=(in_1, in_2, \ldots, in_k = STOP)$ where k>1 and STOP∉In, is called a (sequential) program for A. P is accepted by A if there exists $st_1 \in St$ such that the complete computation CComp(A, $P, st_1) = (st_1, st_2, \ldots, st_k)$ is defined, i.e. trans $(in_{i-1}, st_{i-1}) = st_i$ is defined for i=2,3,...,k. The domain of P is the set DomP={$st_1 \in St$; CComp(A,P,st_1) is defined} and the abstract function computed by P on A is $f_{A,P}(st_1) = st_k$ for each $st_1 \in$ DomP. Let SL_A, F_A be the set of all sequential programs accepted by A, the abstract functions computed by all $P \in SL_A$, respectively. P and P* are function equivalent if $f_{A,P} = f_{A,P*}$.

The sets SL_A and F_A are studied using the following concepts: $P \in SL_A$ is called total, if DomP=St, and is a mono-program if |DomP|=1; St∈DomP is a fixedpoint of P if $f_{A,P}$ (st)=st; $P \in SL_A$ is an identity program if each st∈DomP is a fixedpoint of P, etc.

A Linear Algorithm for the Grundy (Coloring) Number of a Tree (100)

S. M. Hedetniemi, S. T. Hedetniemi,* and T. Beyer
Univ. of Oregon

The Grundy number of a graph G = (V,E), denoted Γ(G), is the largest number of integers which can be assigned to the vertices of G such that every vertex u is assigned the smallest integer which is not assigned to any vertex adjacent to u. The Grundy number Γ(G) is related to the chromatic number χ(G) and the achromatic number ψ(G) of a graph by the inequalities: $\chi(G) \le \Gamma(G) \le \psi(G)$. Although the problem of determining Γ(G) (also χ(G) and ψ(G)), for an arbitrary graph G is NP-complete, a linear algorithm is presented for determining Γ(T) for any tree T. The problem of determining ψ(T) for any tree T, however, is suspected to be NP-complete.

(106) GRAPHS RELATED TO EXTERIOR RECTANGULAR DISSECTIONS

Maciej M. SYSLO, Systems and Computing, W.S.U., Pullman, WA

A rectangular dissection is defined as a partition of a rectangle into a family of nonempty and mutually non-overlapping rectangles. The intersection graph of such a dissection is called an adjacency graph. A dissection is non-proper if it consists of two other rectangular dissections, and is called exterior if each rectangle of the partition shares at least one side with the main rectangle.

In this talk we will restrict our attention only to exterior dissections and present: (i) a charecterization of adjacency graphs, (ii) a characterization of skeletons, and (iii) a proof that every exterior rectangular dissection is non-proper.

A DECOMPOSITION THEOREM FOR BALANCED HYPERGRAPHS (107)

Ram Prakash Gupta

College of the Virgin Islands

Let H = (X, $\mathcal{E}$) be a balanced hypergraph. Then, for any integer k ≥ 1, there exists a decomposition of X,

$$X = X_1 \cup X_2 \cup \ldots \cup X_k \quad , \quad X_i \cap X_j = \emptyset, \quad i \neq j,$$

such that for every edge $E \in \mathcal{E}$,

$$\left|\{j: E \cap X_j \neq \emptyset, \ 1 \le j \le k\}\right| = \min\{k, |E|\}.$$

This generalizes an earlier theorem for bipartite graphs (Discrete Mathematics, vol. 23, No. 3, 1978, pp. 229-233)

"On Finding Legal Paths in the Presence of Impossible Paths" (108)

H.S. Ihm and S.C. Ntafos,* University of Texas at Dallas

In this paper we consider the problem of finding a legal source to sink (s-t) path in an acyclic digraph G = (V,E), for which a set of impossible paths is specified. This problem arises in program testing. An $O(L*d_{max})$ algorithm is presented, where L is the length of the input string describing the impossible paths of G, and d_{max} is the maximum degree in G. Also, some extensions of the algorithm are discussed.

(114) ON THREE-DESIGNS OF SMALL ORDER

Haim Hanani, Technion, Israel Inst. of Tech., Haifa, Israel
Alan Hartman, U. of Waterloo, Waterloo, Ontario, Canada
*Earl S. Kramer, U. of Waterloo (visiting from U. of Nebraska)

For positive integers $t \le k \le v$ and λ we define a t-design, denoted $B_t[k,\lambda;v]$, to be a pair (X,B) where X is a set of points and B is a family (B_i) of (not necessarily distinct) subsets of X, called blocks, which satisfy the following conditions

(i) |X| = v, the order of the design,

(ii) $|B_i| = k$ for every block $B_i \in B$, and

(iii) every t-subset of X is contained in precisely λ blocks.

The purpose of this paper is to investigate the existence of 3-designs with $3 \le k \le v \le 32$ and λ > 0.

Wilson has shown that there exists a constant N(t,k,v) such that designs $B_t[k,\lambda;v]$ exist provided λ > N(t,k,v) and λ satisfies the trivial necessary conditions. We show that N(3,k,v) = 0 for most of the cases under consideration and we give a numerical upper bound on N(3,k,v) for all $3 \le k \le v \le 32$. We give explicit constructions for all the designs needed.

A list of parameters of unknown designs is available for distribution.

(115) Further Constructions of Irreducible Designs

Elizabeth J. Billington (Visiting Department of Computer Science
University of Queensland University of Manitoba)

A balanced incomplete block design with parameters (v, tb, tr, k, tλ), that is, a quasi-t-multiple design, is said to be irreducible if it contains no (v, sb, sr, k, sλ) - design with s < t. In [1] constructions were given for irreducible quasi-2-multiple designs, starting from any (v, b, r, k, 1)-design with k > 2, (except of course from the Fano plane), and for irreducible quasi-3-multiple designs, starting from any finite affine or projective plane.

Here we extend this by constructing irreducible (v, 3b, 3r, k, 3)-designs from any (v, b, r, k, 1)-design with k > 2. The case of 'higher' irreducible designs is also considered.

[1] Elizabeth J. Billington, Construction of some irreducible designs, Proc. IX Australian Conference in Combinatorial Math., Lecture Notes in Mathematics (Springer-Verlag), to appear.

(116) Some new 5-designs

Spyros S. Magliveras, University of Nebraska, Lincoln, NE 68588

A t-design, or t-(v,k,λ) design, is a pair (X,$\mathfrak{B}$) where $\mathfrak{B}$ is a collection of distinct k-subsets (called blocks) from a v-set X, such that each t-subset of X is in exactly λ blocks of $\mathfrak{B}$. By using the group $P\Gamma L_2(2^5)$ in its 4-homogeneous representation on the projective line $GF(2^5) \cup \{\infty\}$ we construct over one hundred new, non-trivial 5-designs with v=33 and k ≥ 6. There are two other known 5-designs with odd v discovered by E.S. Kramer in 1975 and 1981 respectively.

ALGORITHMS FOR DETERMINING FUNDAMENTAL CYCLES IN A GRAPH

(101)

S. A. Jordan, D. R. Shier*, Clemson University

Three algorithms are presented for determining a set of fundamental cycles in an undirected graph. These algorithms all make use of a compact representation of the graph and generate fundamental cycles at the same time as a spanning tree is constructed. The three algorithms differ mainly in the type of spanning tree produced and in the mechanism for tracing out fundamental cycles relative to this tree. Computational experience with these algorithms indicates that two of the algorithms are especially effective in practice.

Steiner Trees in Outerplanar Graphs (102)

Joseph A. Wald and Charles J. Colbourn*,
University of Saskatchewan

It is known that finding a Steiner tree in a graph is NP-complete, even if the graph is planar. We show by contrast that Steiner trees in outerplanar graphs can be found in linear time. Applications of this result in network design are described.

(103) Good Algorithms for Gamma-Free Matrices.

Professor Anna Lubiw, University of Waterloo, Waterloo, Ontario

A gamma-free matrix is a 0-1 matrix which has no 2 x 2 sub-matrix whose only 0 is in the lower right corner. Algorithms are given to solve certain min-max problems on 0-1 matrices when these matrices are gamma-free. Some natural constructions and applications of gamma-free matrices are discussed. An algorithm is given to rearrange if possible the rows and columns of a 0-1 matrix to be gamma-free. The class of matrices admitting such rearrangement is exactly the class of totally balanced matrices. I am indebted in this work to related work of Martin Farber on strongly chordal graphs.

(109) POLYGONAL GRAPHS OF VALENCY FOUR

Manley Perkel, Wright State University, Dayton, Ohio 45435

A polygonal graph is a pair (H,E) consisting of an undirected, regular, connected graph H of girth m, together with a set E of m-gons of H distinguished by the fact that every path of length 2 of H lies in a unique element of E.

Let $Aut^*(H)$ denote the strict automorphism group of H, i.e. those automorphisms which fix the set E. In this paper we investigate the case where m=5 and the valency of H is 4, under the assumption that $Aut^*(H)$ is transitive on paths of length 2 of H. As a result we prove that if H contains a dodecahedral subgraph (i.e. a subgraph isomorphic with the vertices and edges of a dodecahedron) whose pentagons are in E, then H is isomorphic to the vertices and edges of the 4-dimensional polytope known as the 120-cell.

DECOMPOSING COMPLETE SYMMETRIC DIGRAPHS INTO CIRCUITS OF LENGTH $2.p^{\alpha}$

(110)

Dr. Badri Varma, Bayero Univ., Kano (Nigeria)

Hartnell has shown that the complete symmetric digraph DK_{xy} can be decomposed into circuits of length 2x, where x and y are both odd and $y \geq 5$. Using one of our earlier results, Hartnell's result can easily be extended to include the case when y = 3. This result along with some new and some earlier known results, is used to show that the necessary conditions for the decomposition of DK_n into circuits of length $2.p^{\alpha}$ where p is prime and α any positive integer are also sufficient.

(111) V-Critical Dichromatic Tournaments

by V. Neumann-Lara, University of Mexico & J. Urrutia, Metropolitan University of Mexico.

The dichromatic number of a digraph D is the minimum number of colors to color V(D) in such a way that no directed cycle of D is monochromatic. In this paper we construct for each $k \geq 3$ an infinite family of vertex-critical (v-critical) k-dichromatic regular tournaments.

(117) S-QUASI-SYMMETRIC DESIGNS

Mohan Shrikhande*
Central Michigan University
Navin Singhi
Tata Institute of Fundamental Research

Let D be a balanced incomplete block design (v,b,r,k,λ) with two-block intersection numbers x and $y (x < y < k)$. Stanton and Kalbfleisch call D s-quasi-symmetric if each block of D intersects s other blocks in x points and the remaining b-s-1 blocks in y points each. We show that for any fixed s, if $v > s^2 + 1$, then D is a strongly resolvable design with parameters

$$v = \frac{n^2 y}{t^2},\ b = \frac{n^2}{t^2}\frac{(n^2 y - t^2)}{(n-1)},\ r = \frac{(n^2 y - t^2)}{(n-1)t},\ k = \frac{ny}{t},$$

$$\lambda = \frac{(ny - t)}{(n-1)},\ x = \frac{ny(t-1)}{(n-1)t},\ y = y\ (n = s+1,\ 1 \leq t \leq s).$$

This generalizes results of Stanton and Kalbfleisch and also of Caromony and Tan.

Healey, Paul, American University, Washington DC, 20016

(118) Necessary Conditions for the Existance of Nested Balanced Incomplete Block Designs.

It is possible to construct combinatoric designs in which t disjoint blocks from a $(v;k,\lambda)$BIBD are used as blocks of a $(v;tk,\lambda_1)$BIBD, and the construction proceedure can repeat. Peerce called these nested designs. Particular manifestations of nested designs have been studied as Resolvable BIBDs, Bridge Tournament Designs, Whist Tournaments, and so on.

This paper shows that this species of design has a simple description which allows the parameters of the (v,k_i,λ_i)BIBDs to be obtained from the formulae: $k_i = k_1 t_2 t_3 \ldots t_i$; $\lambda_i = \tau(k_i - 1)/a_n$ $b_i = \tau v(v-1)/a_n k_i$ and $r = \tau(v-1)/a_n$ where $a_n = \gcd\{k_1 - 1, t_2 - 1, \ldots t_n - 1\}$ and τ is a new parameter which summarizes the pair-incidence requirements. This leads to the very familiar looking, but rather surprising, necessary conditions that $\tau(v-1) \equiv 0 \pmod{a_n}$ and $\tau v(v-1) \equiv 0 \pmod{k_n a_n}$

(119) A NEGLECTED THEOREM ON DISSECTION OF TRIANGLES AND A RELATED COUNTING PROBLEM

Dale M. Mesner, University of Nebraska-Lincoln

The following theorem, which does not seem to be well known, was used in a historically important paper on polygonal area. THEOREM 1 (P. Gerwien, 1833). For a positive integer n let distinct collinear points $B_0,\ldots,B_n$ s.t. $B_0B_1 = \ldots = B_{n-1}B_n$ be joined to a point A not on the same line, forming triangles $AB_{i-1}B_i$, i=1,...,n. Then if lines are drawn through each of $B_1,\ldots,B_{n-1}$ parallel to $AB_0,\ldots,AB_n$, triangles $AB_{i-1}B_i$ will be simultaneously equidecomposed. That is, the n triangles are dissected into the same parts (up to congruence). Gerwien's proof is inadequate and a new one is given. The number of parts, say N, in each triangle is independent of the shapes of the triangles, and we enumerate the parts. THEOREM 2. Let $\phi(n)$ be the Euler totient function. Then

$$N = N(n) = \tfrac{1}{2}\left(1 + \sum_{j=1}^{n} \sum_{k=1}^{j} \phi(k)\right).$$

Asymptotic behavior of the Dickman-de Bruijn function (120)

E. R. Canfield University of Georgia

The Dickman-de Bruijn function, which is the solution of a certain Volterra integral equation, arises in the study of the function $\psi(x,y)$, which counts the integers $\leq x$ having all their prime divisors $\leq y$. Over 30 years ago de Bruijn gave an asymptotic formula for this function. We reprove his result, using discrete methods.

(121) CHROMATIC POLYGROUPS

Stephen D. Comer (The Citadel)

Given a set C (of colors) and an involution ϵ of C, a color scheme is a system $V = \langle V, \{C_a : a \in C\}\rangle$ such that (i) $\{C_a : a \in C\}$ partitions $\{(x,y) \in V^2 : x \neq y\}$, (ii) $C_a^{\smile} = C_{\epsilon(a)}$ for each $a \in C$, (iii) each color is present on some edge eminating from each vertex, and (iv) for $a,b,c \in C$ with $(x,y) \in C_c$, the existence of an (a,b)-path from x to y is independent of the choice of x and y. A multivalued algebra M_V (called a chromatic polygroup) can be associated with a color scheme V in a natural way. These polygroups play an important role in the study of algebraic logic and also seem to be useful to classify combinatorial objects. A sufficient condition is given for a polygroup to be isomorphic to an M_V. As an application, there are chromatic systems that cannot be derived from double cosets of finite groups.

Tri-Weaving (122)

R. Nowakowski*, D. Skillicorn; Dalhousie University, Halifax, N.S.

Most weaving consists of interlacing two sets of parallel strands. Throughout the ages, weavers and now mathematicians have tried to classify and enumerate patterns that have certain "desirable" properties. Other varieties of weaving do exist. We consider the analogous problems that arise when three sets of parallel strands (at 120° to one another) are used. In one aspect the "desirable" patterns are related to "greenhouse" arrays."

(129) On proximity to paths and cycles in 3-connected graphs

M. D. Plummer, Vanderbilt University and
W. R. Pulleyblank, University of Waterloo

A path P in graph G is a <u>k-path</u> if every point of G is at distance no more than k from some point of P. The <u>k-path problem</u> is: given a graph G and an integer k, does G contain a k-path. We show this problem to be NP-complete for (a) the class of all cubic 3-connected planar graphs and for (b) the class of all cubic 3-connected bipartite graphs. Similar results hold when the term "k-path" is replaced by "k-cycle".

For each of the classes (a) and (b) we also construct, given a k, examples which have no k-path.

Relationships to Barnette's conjecture are discussed. In particular, it is shown that assertions (A) and (B) are equivalent:

(A) (Barnette's conjecture). Every cubic 3-connected bipartite planar graph has a Hamiltonian cycle.

(B) Given any cubic 3-connected bipartite planar graph G and any line e in G, G-e has a Hamiltonian cycle.

SORTING ALGORITHMS BY STRING REVERSALS

(130) Andrew Harris - Louisiana State University

The paper concerns another variant of the usual sorting problem, initially posed by Harry Dwighter in 1975.

Speaking intuitively, given a pile of n pancakes of distinct sizes stacked on a plate in arbitrary order, and a spatula which may be inserted into the pile to flip the pancakes above, what is the minimum number of flips necessary to guarantee that the pancakes may be arranged in size order?

It is easily seen that in the worst case at least n flips are needed, and 2n-3 are sufficient. The lower and upper bounds were improved by Garey and Johnson to n+1 and 2n-6.

The present paper will improve considerably both the upper and lower bounds for this problem. Results are also obtained in the analogous problem in which the sides of each pancake are distinguishable.

(131) More On Minimal Perfect Hash Tables

Curtis Cook*, Oregon State University
R. R. Oldehoeft, Colorado State University

Cichelli has presented a simple method for constructing minimal perfect hash tables of identifiers for small static word sets. The hash function value for a word is computed as the sum of the length of the word and the values associated with the first and last letters of the word. Cichelli's backtracking algorithm considers one word at a time and performs an exhaustive search to find the letter value assignments. In considering heuristics to improve his algorithm, we were led to develop a letter oriented algorithm that handles more than one word per iteration and that frequently outperforms Cichelli's. We also investigated the impact of relaxing the minimality requirement and allowing blank spaces in the constructed hash table. This substantially reduced the execution time of the algorithm. This relaxation and partitioning data sets are shown to be two useful schemes for handling large data sets.

(138) ON A GENERALIZED MATCHING PROBLEM ARISING IN ESTIMATING THE EIGENVALUE VARIATION OF TWO MATRICES

L. Elsner, Universität Bielefeld
Charles R. Johnson, University of Maryland
Jeffrey A. Ross*, University of South Carolina
J. Schönheim, Tel Aviv University

It is shown that if G is a graph having vertices $P_1, P_2, \ldots, P_n$, $Q_1, Q_2, \ldots, Q_n$ and satisfying some conditions, then there is a permutation σ of $\{1,2,\ldots,n\}$ such that for $i = 1,2,\ldots,n$ there is a path connecting P_i with $Q_{\sigma(i)}$ having length at most $\lceil \frac{n}{2} \rceil$. This is used to prove a theorem having applications in eigenvalue variation estimation.

THE SPECTRUM OF A SKEW SYMMETRIC ADJACENCY MATRIX OF AN ORIENTED GRAPH (139)

Ashraf Zeid; Mechanical Engineering Department, Michigan State University

For an oriented graph G, with weigted edges, and having no loops or multiple edges, an associated skew symmetric adjacency matrix $S(G) = [s_{ij}]$ is defined.

G is a simplified form of a graphical model of a class of dynamic systems; S(G) is the state matrix.

The following results were obtained:

1. An expression for the characteristic polynomial of S(G) in terms of the structure of G.
2. A recurrence formula for computing the characteristic polynomial of S(G) based on the characteristic polynomials of subgraphs of G.

It is shown that if G is a tree, and $\tilde{G}$ is the tree obtained by relaxing all the directions on the edges of G, then the spectrum of the adjacency matrix of $\tilde{G}$ and the imaginary part of the spectrum of G are identical. This information is used to obtain bounds on the eigenvalue with the largest modulus of the oriented graph G.

(140) On graphs whose eigenvalues do not exceed $(2 + \sqrt{5})^{1/2}$

Dragos Cvetkovic	Michael Doob*	Ivan Gutman
Univ. of Belgrade	Univ. of Manitoba	Univ. of Kragujevac

Let $\Lambda(G)$ denote the largest eigenvalue of a graph G. J. Smith has described all graphs with $\Lambda(G) \leq 2$, and, in addition, A. J. Hoffman has given the limit points of those $\Lambda(G)$ that are attained. In this paper we describe all graphs G with $\Lambda(G) \leq (2 + \sqrt{5})^{1/2} = \sqrt{\tau} + 1/\sqrt{\tau}$ (τ is the golden mean). We also derive all graphs with $\Lambda(G) > 2$ that are minimal with respect to that property; this allows the computation of several gaps in the possible values of $\Lambda(G)$.

SCHEDULING DESIGNS FOR A LEAGUE TOURNAMENT

Tina H. Straley, Kennesaw College, Marietta, Georgia 30061

Let T be a set of teams of size mn, partitioned into m n-sets $C_1, C_2, \ldots, C_m$, called conferences. The designs constructed for a tournament on T satisfy the following: each pair of teams which are from the same C_i meet p times; each pair of teams which are from distinct C_i's meet q times; the games of the schedule fill a rectangular array which is the union of subarrays A and B of the same number of columns; A is subdivided into subrectangles containing maximal collection of games in which no team appears more than once; B is subdivided so that the entries of a union of subrectangles forms a maximal collection of intraconference games in which no team appears more than once. Two different designs are presented and sufficient conditions given for their existence. In one, each play period consists either entirely of intraconference games or entirely of interconference games. In the other, $p > q$ and each play period consists either entirely of intraconference games or of both intraconference and interconference games.

ON PACKING SUMS OF VECTORS

Jerrold R. Griggs, University of South Carolina

Suppose $a_1, \ldots, a_n$ are vectors of length at least one in m-dimensional space. Suppose S is an m-dimensional sphere of diameter d. Let $f_m(n,d)$ denote the maximum, over all choices of the a_i and S, of the number of sums

$$\sum_{i \in I} a_i$$

which lie in S, where I ranges over all subsets of $\{1, \ldots, n\}$. For example, the original Littlewood-Offord problem shows that

$$f_2(m,1) = \binom{n}{\lfloor n/2 \rfloor}.$$

Here we describe some recent results and open problems about $f_m(n,d)$.

Topologies on Finite Sets IV

H. Levinson and Ruth Silverman*

Rutgers University and W. Va. College of Graduate Studies

Each maximal chain in the lattice of all topologies on a finite set is shown to be of a certain cannonical form. Using this form, the total number of maximal chains is counted, as well as those of each given possible length. The problem of finding the shortest path between two arbitrary points in the lattice of all topologies on a finite set is also investigated.

ON GRACEFUL DIGRAPHS AND A PROBLEM IN NETWORK ADDRESSING

Gary S. Bloom, The City College / C.U.N.Y., New York, NY 10031

D. Frank Hsu, Fordham University, Bronx, NY 10458

Recent work on labelling graphs has led to a new class of these graphs, which give an efficient designation of communication activities in an arbitrary network. This problem has led to a number of interesting problems in graph theory and additive number theory. We briefly survey some of these developments.

Path Partitions

Kathie Cameron, University of Waterloo

A path partition is a partition of the nodes of a digraph into node-sets of directed paths. A set of nodes in a digraph is called independent if no two of them are joined by an edge. In 1960, T. Gallai and A.N. Milgram proved that for every digraph G, there exists an independent set J of nodes and a path partition P such that $|J| = |P|$. A very simple good algorithm for finding such a J and P will be presented.

We will also present a conjecture generalizing the Gallai-Milgram theorem, and prove the conjecture for acyclic digraphs.

COMMON SUBSEQUENCES AND SUPERSEQUENCES OF FINITE SETS OF WORDS

M. Kunze and G. Thierrin

The University of Western Ontario, London, Canada

Given a finite language L over an alphabet Σ, we consider the set inf(L) of maximal common subsequences of the words in L, and dually the set sup(L) of minimal common supersequences of L. The special case $|L| = 2$ is of practical interest, e.g., in data processing for error correction in strings. Some complications of this special case can be avoided by allowing L to be an arbitrary finite language. We prove that the operators inf and sup are compatible with the concatenation of languages and, in a certain sense, also with the union of languages. If L consists of incomparable elements with respect to embedding order, then inf(sup(L)) = L while sup(inf(L)) = L is not true in general. Furthermore, we characterize the class of languages satisfying both of those equations, and we investigate the closure operations induced by the operators inf and sup on finite languages.

GRAPHS OF DIAMETER 2 WITH CYCLIC DEFECT

S. Fajtlowicz, University of Houston

Let G be a graph of diameter 2 with the adjacency matrix A. A graph D is a defect of G if its adjacency matrix satisfies the equation

$$A^2 + A + (D-1)I = J + K$$

where every entry of J is 1.

Erdős, Hoffman and myself showed that C_4 is the only graph of diameter 2, with K being the adjacency matrix of a matching.

We shall show that there is a unique graph of diameter 2 with K being the adjacency matrix of a cycle.

On Some Quadratic Graph Equations

M. Capobianco, M. Karasinski and M. Randazzo

St. John's University

The three graph equations (1) $G^2 + \bar{G} = H$, (2) $G^2 = \bar{G}$, and (3) $G^2 + G = H$ are studied. The properties required of H for (1) and (3) to have solutions are examined, and theorems are proved which eliminate several classes of graphs. An interesting interplay between (1) and (2) is exhibited, and equations (1) and (3) are seen to be very different. The study of equation (2) leads to the concept of a distance-balanced graph. A graph is D(i,j)-balanced if the number of pairs of points that are a distance i or less apart equals the number of pairs of points which are a distance j or more apart. Solutions of (2) must be D(1,3)-balanced.

Chromatic numbers for linear lattice paths in Z^n

J. L. Paul, Univ. of Cincinnati

A linear lattice path in euclidean n-space is the intersection of a connected subset of a straight line in n-space with the lattice points having integer coordinates. For a given positive integer k the problem of determining the minimum number of colors needed to color the lattice points so as to avoid monochromatic paths of length k is considered. Some related problems will also be discussed for lattice paths with diagonal steps which are not required to lie along a straight line.

(126)

THE NUMBER OF ORTHOGONAL CONJUGATES OF A QUASIGROUP

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A finite quasigroup (Q,·) has 6 (not necessarily distinct) conjugates. Lindner and Steedley (Algebra Universalis 5 (1975), 191-196) showed that the number of distinct conjugates of (Q,·) must be 1,2,3, or 6. They showed further that for each $x \in \{1,2,3,6\}$ there exist quasigroups of every order $n \geq 4$ with exactly x distinct conjugates.

There are several recent results involving quasigroups orthogonal to certain of their conjugates. We investigate the number of conjugates of (Q,·) which are orthogonal to (Q,·) and show that for every $x \in \{1,2,3,6\}$ and every $y < x$ there exists a quasigroup orthogonal to exactly y of those conjugates. Conjugate orthogonal latin square graphs are used to describe such relationships. For the case x=6 and $y < x$, we show that for all but finitely many n there is a quasigroup of order n orthogonal to exactly y conjugates. For $x < 6$ an infinite class of examples is given in each case.

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A CONVEXITY INEQUALITY ON THE PERMANENT OF DOUBLY STOCHASTIC MATRICES

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In this paper we investigate the convexity property of the permanent function on the set Ω_n of all $n \times n$ doubly stochastic matrices. We propose the conjecture that for all A in Ω_n and all reals α, $\frac{1}{2} \leq \alpha \leq 1$, $\mathrm{per}(\alpha J_n + (1-\alpha)A) \leq \alpha\ \mathrm{per}\ (J_n) + (1-\alpha)\ \mathrm{per}\ (A)$, where J_n is the matrix in Ω_n with all entries equal to $\frac{1}{n}$. We prove this conjecture for n = 3 and discuss the case for n = 4.

On ordering the Rows and Columns of a Matrix M so that no Edge of D heads South-West, where D is a Digraph whose Nodes are Positions in M.

Jack Edmonds, University of Waterloo (128)

On the Towers of Hanoi and the Generalized Towers of Hanoi Problems

(135)

by Paul Cull and E. F. Ecklund, Jr.*

In the generalized towers of Hanoi problem on is given t towers (1,...,t) and n disks (1,...,n) of distinct sizes. (Without loss of generality we assume that the size of disk i = i.) Initially the disks are stacked on tower 1 in order of size (disk n on the bottom, disk 1 on the top). The problem is to move the stack of disks to tower t, where the disks are to be moved one at a time in such a way that a disk is never placed on top of a smaller disk.

We prove the following:

Theorem 1. The space complexity for the towers of Hanoi problem (t = 3) is of order n.

Theorem 2. There is a unique solution to the generalized towers of Hanoi problem if and only if $n = \binom{k}{t-2}$ for some $k \geq t-2$, and that solution is achieved by moving $\binom{k-1}{t-2}$ disks to tower 2, moving the remaining $\binom{k-1}{t-3}$ disks to tower t, and then moving the $\binom{k-1}{t-2}$ disks from tower 2 to tower t.

(136) ON LOGICAL CONNECTIVES I

Paul Hartung* and Seymour Schwimmer Bloomsburg State College

By a logical connective, we mean an operator that connects two statements and yields a truth value. Familiar examples of logical connectives are "implies" and "or". The authors have developed an algorithm to produce all logical connectives for n-valued logics. In the algorithm the Euclidean Algorithm is employed. Hartung will discuss the ordinary logic (2-valued, or true and false) case and the 3-valued case. In the algorithm, a primitive connective is defined and a combinatorial procedure is given to get to each of the other connectives, via the Euclidean Algorithm.

(137) ON LOGICAL CONNECTIVES II

Seymour Schwimmer* and Paul Hartung Bloomsburg State College

Schwimmer will illustrate the algorithm of the previous talk for a 5-valued logic. This algorithm also shows that there is a binary operator for sets of order n which generates all groups of order n.

Graph Products and Unique Coloring (144)

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It has been conjectured that for finite, simple graphs G and H, the chromatic numbers of $G \times H$, $\chi(G \times H)$, is $\min(\chi(G), \chi(H))$. Here $G \times H$ denotes the cartesian product of G and H: vertices of $G \times H$ are adjacent if there is adjacency in each component. The conjecture is verified in certain instances with the following fact: if $\chi(H) \geq n + 1$ and H is connected then $K_n \times H$ is uniquely n-colorable. Also, it is shown that the conjecture would follow from several statements concerning unique colorability. For instance, if for all uniquely n-colorable graphs X and Y, $X \times Y$ has precisely two n-colorings, then the chromatic number of a product is the minimum of the chromatic numbers of its components.

(145) The Ordered Chromatic Number of Planar Maps*

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Given an ordering φ of the regions of a planar map $\mathfrak{M}$, we define the ordered chromatic number induced by φ, $\chi_\varphi(\mathfrak{M})$, to be the least number of colors that a parsimonious painter -- i.e., one who only opens a new tube of paint when a region of the map cannot be properly colored in its order with any of the tubes already opened -- would have to have in his palette to properly color $\mathfrak{M}$. The minimum of $\chi_\varphi(\mathfrak{M})$, over all possible orderings of the regions of $\mathfrak{M}$, is simply the face chromatic number of $\mathfrak{M}$, denoted by $\chi^*(\mathfrak{M})$. The four color theorem can be interpreted as saying that the maximum (over all planar maps) of $\chi^*(\mathfrak{M})$ is four.

We define the maximum of $\chi_\varphi(\mathfrak{M})$ over all possible orderings of the regions of $\mathfrak{M}$ to be the ordered hromatic number of $\mathfrak{M}$; denoted by $\chi^+(\mathfrak{M})$. The first (surprising ?) result is that the maximum (over all planar maps) of $\chi^+(\mathfrak{M})$ is unbounded, i.e., that while four colors in the palette will suffice for our parsimonious painter for some ordering of any planar map, that no finite number will suffice for all orderings (and maps). Accordingly, we pose the question of determining the least number of regions, N_n, that a planar map can have for which some ordering in a parsimonious proper coloring requires the use of n colors. As a partial answer we construct maps having asymptotically r^n regions, where r is the root of $x^5 - x^3 - x^2 - x + 1 = 0$ between 1 and 1.4, and define orderings of the regions that force the use of n colors; i.e., $N_n \leq r^n$.

Determining sharp bounds for N_n remains an open question.

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(146)

ANOTHER GRAPH POLYNOMIAL WITH REAL ZEROS

C. Godsil (SFU, Burnaby)

Let S be a subset of the edges of the connected graph G, where G has n vertices. Let f_k be the number of spanning trees in G which have exactly k edges belonging to S. We show that the zeros of the polynomial $\Sigma f_k x^k$ are real, thereby answering a question raised by Richard Stanley and simultaneously providing a second proof of his result that the sequence $\{f_k/\binom{n}{k}\}_{k=0}^{\infty}$ is log-concave.