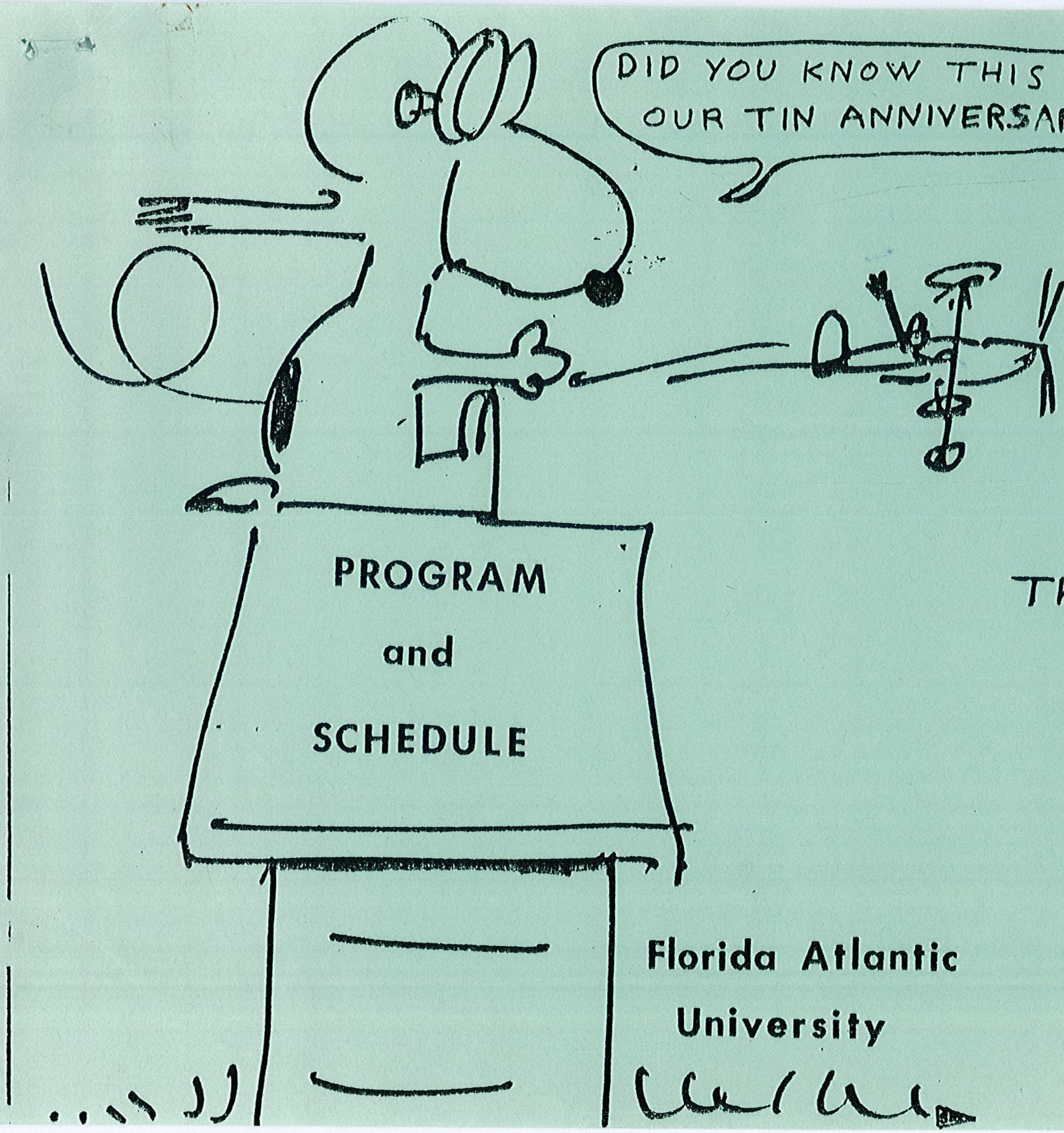
Ruth Ebel 032300611



Florida Atlantic University

The TENTH Southeastern Conference on

COMBINATORICS GRAPH THEORY + COMPUTING

April 2-6, 1979

GCRN and GCRS are the two halves of the Gold Coast Room of the University Center.

Room 207 is entered from the second floor lounge of the University Center.

Coffee available in Gold Coast Room South.

CONFERENCE COCKTAIL PARTY at the Patio and Oceanview Bars of Howard Johnson's Ocean Resort Monday, April 2, 1979 at 6:00PM.

CONFERENCE BANQUET in Windsor Ballroom of the Howard Johnson's Ocean Resort Tuesday, April 3, 1979. Seating at 7:00PM, service at 7:15PM. Cash bar opens at 6:00PM

NOTE: Conference bus will leave University Center at 5:30PM Monday and Thursday, 4PM Wednesday (van at 5:20), 5:10PM Tuesday, and 1:00PM Friday. There will be limited transportation between University Center and Howard Johnson's Monday through Thursday, leaving the University Center at 12:10PM and leaving Howard Johnson's at 1:15PM.

All Conference participants are welcome at informal gatherings to be held at Hoffman's, 4307 N.W. 5th Avenue, 6-8PM Wednesday, and at Freeman's, 741 Azalea St., 6-8PM, Thursday.

MONDAY

Time	GCRN		GCRS		207	
8:15	Registration (from 8AM)					
8:40						
9:00	Opening remarks; welcome by Pres. Creech					
9:30	SELFRIDGE					
10:30	COFFEE					
10:45	Dymacek	1	Lamprey	5	Jones	9
11:05	Little	2	Jenkyns	6	Jamison-Waldner	10
11:25	Varma	3	Opatrny	7	Kahn	11
11:45	Dillon	4	Buckley	8	Killgrove	12
12:00			LUNCH			
1:30	MILLS					
2:40	SELFRIDGE					
3:10	COFFEE				Meeting on W. Coast Conference	
3:40	Hales	13	Cockayne	18	Nemeth	23
4:00	Levinson	14	Parsons	19	Sumner	24
4:20	Kou	15	Doob	20	Hsu	25
4:40	Liu	16	Greenwell	21	Cadogan	26
5:00	Geldmacher	17	Cook	22	Holladay	27

TUESDAY

Time	GCRN		GCRS		207	
8:15	Registration					
8:40	Gupta	28	Ealy	34	Abrham	40
9:00	Lipman	29	Payne	35	Navlakha	41
9:30	GRAHAM					
10:30	COFFEE					
10:45	Mitchell	30	Rothschild	36	Cameron	42
11:05	Hedetniemi	31	Ebert	37	Rabinovitch	43
11:25	Hartnell	32	Laskar	38	Trotter	44
11:45	Chinn	33	Funkenbusch	39	Zaslavsky	45
12:00			LUNCH			
1:30	HALL					
2:40	SPENCER					
3:40	COFFEE					
4:00	Grossman	46	Kramer	49	Zimmer	52
4:20	Fajtlowicz	47	Magliveras	50	Babai	53
4:40	Polimeni	48	Friedlander	51	C. Colbourn	54
5:00						

WEDNESDAY

Time	GCRN		GCRS		207	
8:15	Registration					
8:40	Bramwell	55	Ko	61	DeWitt	67
9:00	Bastida	56	Baker	62	Thomas	68
9:30	NASH-WILLIAMS					
10:30	COFFEE					
10:45	Alter	57	Hare	63	Hadlock	69
11:05	Matula	58	Reid	64	Simmons	70
11:25	Eberly	59	Rogers	65	Alspach	71
11:45	Hartung	60	Silverman	66	Starling	72
12:00			LUNCH			
1:30	STANTON					
2:40	NASH-WILLIAMS					
3:40	COFFEE				Informal	
4:00	SIGMA				Problem	
4:20	XI				Session	
4:40	TALK				↓	
5:00						

THURSDAY

Time	GCRN		GCRS		207	
8:15	Registration					
8:40	J.M. Freeman	73	Slater	79	Tannenbaum	85
9:00	Ihrig	74	Quintas	80	Calvillo	86
9:30	SIMMONS					
10:30	COFFEE					
10:45	Niederhausen	75	Berman, D.	81	Dewdney	87
11:05	Anderson	76	Kleiman	82	Lee	88
11:25	Mesner	77	Frye	83	Edmonds	89
11:45	Williams	78	Schwenk	84	Garey	90
12:00			LUNCH			
1:30	STANTON				12:20	
2:40	MULLIN					
3:40	COFFEE					
4:00	Heinrich	91	Rosenberg	94	Edwards	98
4:20	Deza	92	Ringeisen	95	Foldes	99
4:40	M. Colbourn	93	K. Berman	96	Leiss	100
5:00	Schellenberg	93.5	Maurer	97	Wachs	101

FRIDAY

GCRN		GCRS		207	
Registration					
J.W. Freeman	102	Kimball	107	Cot	112
Davis	103	Hutchinson	108	Bennett	113
ERDÖS					
MULLIN					
COFFEE					
Rosa	104	Chung	109	Snider	114
Dinitz	105	Tupitsyn	110	Deo	115
Caccetta	106	Rao	111	Riesenkönig	116
				Seymour	117

MONDAY, April 2, Dr. John L. Selfridge will speak on "Design Aspects of Prime Factors of Consecutive Integers."
MONDAY, April 2, Dr. William H. Mills will speak on "The Construction of Balanced Incomplete Block Designs."

TUESDAY, April 3, Dr. Ronald L. Graham will speak on "Ulam Decompositions of Graphs."
TUESDAY, April 3, Dr. Marshall Hall, Jr. will speak on "Applications of Coding Theory to Block Designs."
TUESDAY, April 3, Dr. Joel Spencer will speak on "Ramsey's Theorem for Spaces."

WEDNESDAY, April 4, Dr. Crispin St. J. A. Nash-Williams will speak on "Marriage in Infinite Societies."
WEDNESDAY, April 4, and THURSDAY, April 5, Dr. Ralph G. Stanton will speak on "Covering Problems."

THURSDAY, April 5, Dr. Gustavus J. Simmons will speak on "On a Theory for the General Encryption/Decryption Channel."
THURSDAY, April 5, Dr. Ronald C. Mullin will speak on "A Generalization of the Singular Direct Product."

FRIDAY, April 6, Dr. Paul Erdös will speak on "Problems and Solutions--a Ten Year Review"
FRIDAY, April 6, Dr. Ronald C. Mullin will speak on "What Cayley Did."

On Wednesday, April 4, Dr. Gustavus J. Simmons will give a public lecture, sponsored by the Florida Atlantic Club of Sigma Xi, on "The Current Revolution in Cryptography, or 'I wonder what he meant by that.'"

There will be a meeting at 3:15 P.M. Monday in Room 207 of people interested in planning an annual West Coast Combinatorics conference. Anyone interested is invited to attend as long as they don't stop coming to this one. If you cannot attend the meeting, you may contact Phyllis Chinn for information.

AUTHOR INDEX TO ABSTRACTS

(This index includes all authors. The papers are listed on the schedule by the presenters.)

LATE PAPERS

ABSTRACTS OF SOME OF THE HOUR AND HALF-HOUR TALKS

On a Theory for the General Encryption/Decryption Channel*

Gustavus J. Simmons
Sandia Laboratories, Albuquerque, New Mexico 87185†

A revolution of truly enormous significance is occurring in the science of secure communications. That such a revolution is occurring and some appreciation of its effects have been generally recognized - even by the popular press - but the nature of the changes have been generally misunderstood, even by the discoverers of some of the new systems. For example, there is a widespread misconception that the new cryptosystems are unbreakable, i.e., cryptosecure, while existing systems are not, when in fact both depend on the same mathematical principles for security.

What is true is that all cryptosystems currently in use are symmetric in the sense that either the same piece of information (key) is held in secret by both communicants or else that each holds one from a pair of related keys, where either key is easily derivable from the other. These secret keys are used in the encryption process to introduce uncertainty (to the unauthorized receiver) which can be removed in the process of decryption by an authorized receiver using his copy of the key or the "inverse key." This means, of course, that if the information held by either is compromised, that further secure communications are impossible. In the new cryptosystems the information held by the transmitter and receiver is not only different but in addition a "computationally-complex" problem relates the two keys so that it is computationally infeasible to derive one from the other. A novel feature of an asymmetric encryption system is that it, unlike a symmetric one, permits secure communication even though the transmitter's encryption information has been compromised. Equally important for many applications the asymmetric system permits the communicant whose encryption information has been protected to authenticate himself, i.e., to prevent anyone from impersonating the authorized transmitter, to receivers in high risk or exposed positions where it must be assumed that the decryption information has been compromised.

If encryption is viewed as a function from messages to ciphers, then for asymmetric encryption to be practical it must be "easy" to find pairs of such inverse functions which are individually "hard" to invert. The whole concept of asymmetric encryption is a "belling-the-cat" proposal until such time as someone exhibits a family of pairs of such trap-door or one-way functions.

* This article sponsored by the U. S. Department of Energy under Contract AT(29-1)-789.
† A. U. S. Department of Energy Facility.

The approach thus far to constructing such functions has been to take some problem of known or generally accepted computational complexity such as factoring a product of very large prime factors, the general knap-sack or sub-set sum problem, finding the logarithm of an element in a large field with respect to a primitive element, etc., and then devise an encryption scheme dependent on this problem. This means of course that solving the "hard" problem implies breaking the encryption scheme. What is hoped is that for some such scheme that the converse will also be true, namely, that decryption will be equivalent to doing the hard problem.

This paper attempts to formulate a theory for the general encryption/decryption channel to make clear the exact nature of the current revolution in the science and to provide a sharp mathematical model for further analysis.

APPLICATION OF CODING THEORY TO DESIGNS

Marshall Hall, Jr., California Institute of Technology

The highly developed theory of error-correcting codes and, in particular, the MacWilliams identities enable us to get information on designs not previously available. For example, a plane of order ten cannot have a collineation of order five.

RAMSEY'S THEOREM FOR SPACES

Joel Spencer, SUNY Stony Brook

G.-C. Rota conjectured that Ramsey's Theorem holds when the notions of cardinality and subset are replaced by dimension and subspace over a fixed finite field. To be precise: If F is a finite field and $n,r,k > 0$ there exists m' so that for $m > m'$ if $\dim(V) = m$ (V a vector space over F) and the k-spaces $W \subset V$ are r-colored there exists an n-space $U \subset V$ all of whose k-spaces are the same color. This conjecture was proven by Ron Graham, Klaus Leeb and Bruce Rothschild in 1972.

A short proof of the above result is given.

A GENERALIZATION OF THE SINGULAR DIRECT PRODUCT

R.C. Mullin

This one hour talk discusses a generalization of the singular direct product. The new product can be used to create several types of objects such as quasigroups, balanced incomplete block designs, Room squares and pairwise balanced designs.

WHAT CAYLEY DID!

R.C. Mullin

This half hour talk deals with the construction of A. Cayley which uses Room squares to construct Steiner triple systems. Generalizations of the construction are given.

(1) Steinhaus Graphs

Wayne M. Dymacek, Department of Defense

The Steinhaus graph G, with n vertices generated by the sequence of zeros and ones, $a_{1,2},\dots,a_{1,n}$, is the graph whose adjacency matrix is $A(G)=(a_{i,j})$, where $a_{i,i}=0$ for $i=1(1)n$, $a_{j,i}=a_{i,j}$, and for $1<i<j$, $a_{i,j}=a_{i-1,j-1}+a_{i-1,j} \pmod 2$.

We first establish that Steinhaus graphs are either connected or totally disconnected. Also, for $n\geq 5$, there are only three Steinhaus trees with n vertices.

A Steinhaus graph whose adjacency matrix is symmetric with respect to both diagonals is called doubly symmetric. We show that each eulerian Steinhaus graph is doubly symmetric. The number of sequences that generate doubly symmetric and eulerian Steinhaus graphs with n vertices is $2^{[n/2]}$ and $2^{[(n-1)/3]}$, respectively.

The complement of a Steinhaus graph has at most two components. If the complement has two components, the smaller, S, is complete with $|S|\leq[(n+2)/3]$ and the larger has diameter at most two. Furthermore, if $|S|\geq 3$, then the vertices of S are in arithmetic progression with common difference 2^m-1 for some $m>1$.

(2) COMPLETE BIPARTITE GRAPHS AND THE HADAMARD CONJECTURE

Charles H.C. Little* and David J. Thuente, Indiana University-Purdue University

An nxn(1,-1)-matrix H is Hadamard if $HH^T = nI$. The Hadamard conjecture states that there exists an nxn Hadamard matrix for all $n\equiv 0 \bmod 4$. We show that this conjecture can be restated as a problem about the circuits of length 4 in a complete bipartite graph, and also as a problem about the 1-factors of a complete bipartite graph.

(3) ON ALMOST STEINER TRIPLE SYSTEMS

Brian Alspach and Badri N. Varma*, Simon Fraser University

We show that the edge set of the complete graph K_n, $n \equiv 5 \pmod 6$, can be partitioned into one K_5 and K_3's.

(4) THE WATERLOO PROBLEM

John F. Dillon, Department of Defense

The (V,k,λ) - difference set D satisfying in the group G the equation

$$D^{(-1)}D = k + \lambda G^*$$

is said to have a Waterloo decomposition

$$D = A + B$$

if the subsets A and B constitute a difference family satisfying

$$A^{(-1)}A + B^{(-1)}B = k + \frac{\lambda}{2} G^*.$$

Such a decomposition yields balanced weighing designs and relative difference sets among other interesting combinatorial configurations.

In this paper we survey the problem of determining the difference sets having such a decomposition and present a solution to the problem for Singer difference sets.

(5) Primitivity of Products of Graphs

Roger Lamprey,* Georgia State University
Bruce Barnes, National Science Foundation

A digraph G=(V,E) is primitive if there exists a positive integer k such that for all $v,v'\in V$ there exists a walk from v to v' of length exactly k. The minimum such k is called the index of primitivity of G. This paper gives necessary and sufficient conditions for the conjunction of two digraphs to be primitive, determining the index of primitivity of this product. Conditions for the cartesian product of two digraphs to be primitive are also established and a bound on the index of primitivity is set. A formula is developed for calculating the index of primitivity of the cartesian product of two directed cycles.

(6) GREEDY BUT NOT-SO-STUPID ALGORITHMS

T. A. Jenkyns, Brock University

Combinatorial Optimization problems frequently take the following form: Given a finite set X, a non-negative weight-function on X, and a family A of subsets of X; find a member of A whose total weight is as large as possible. The greedy algorithm for finding an approximate solution examines the elements of X one at a time in order of decreasing weights and takes those it can. The effectiveness of several heuristics combining partial enumeration of the subsets of X with this algorithm is examined and a lower bound on the effectiveness is given.

(7) ON BANDWIDTH OF GRAPHS

J. Chvatalova, University of Waterloo and Vanier College
J. Opatrny*, Concordia University

In this paper some results on bandwidth of finite and infinite graphs are presented. The bandwidth of a graph G is defined to be

$$\phi(G) = \min(\max(|f(u)-f(v)|))$$

where the minimum is taken over all one-to-one mappings f from V(G) to integers, and the maximum is taken over all edges (u,v) in E(G).

First, several results concerning characterisation of infinite graphs with finite bandwidth are presented.

We then show that the difference between $\phi(G)$, $\phi(G-e)$ where e is an edge of G is not bounded by any constant.

(8) SELF-CENTERED GRAPHS WITH A GIVEN RADIUS

by Fred Buckley, Math and Science Division, St. John's University, Staten Island, N.Y. 10301

A graph G is called self-centered if all of its vertices are in the center of G. The center of a connected graph lies in a block. We show that a graph with n vertices, diameter $r>1$, and only one block must have at least $\lceil(nr-2r-1)/(r-1)\rceil$ edges. We show that if $n\geq 2r>2$ then there exists a connected self-centered graph having n vertices, radius r, and k edges if and only if $\lceil(nr-2r-1)/(r-1)\rceil \leq k \leq (n^2-4rn+5n+4r^2-6r)/2$.

(9) GRACELESSNESS

Rhys Price Jones, Indiana Univ.-Purdue Univ. at Fort Wayne

An attempt is made to link two ruler problems: The damaged ruler problem of H. E. Dudeney, and the Golomb ruler problem.

We present a conjecture and invite the mathematical community to tackle some difficult problems.

(10) A CONVEXITY CHARACTERIZATION OF ORDERED SETS

Robert Jamison-Waldner, LSU-Baton Rouge

After the ordinary notion of convexity in real linear spaces, the most common occurrence of "convex sets" is in partially ordered (PO) sets. A subset K of a PO-set X is order convex provided that whenever K contains a pair of comparable points, K contains all points between them. This defines an alignment on X--that is, a family of abstract "convex" sets which is closed under arbitrary intersections and nested unions. It is possible to characterize those alignments on a set X which arise from orderings of X in terms of natural convexity properties. It is also possible to provide a characterization in terms of "forbidden" subspaces as in the Kuratowski planar graph theorem. For the case of total orders, there are only a finite number of forbidden subspaces since an alignment is determined by a total order if this is the case for every 4 point subspace.

(11) Characterization of some Classical Circle Geometries

by Jeff Kahn, Ohio State University

If a Möbius, Laguerre or Minkowski plane satisfies the "bundle theorem", then it can be embedded in some three-dimensional projective space. This answers a question raised by van der Waerden and Smid in 1935.

(12) Arcs, Conics, and Ovals

R.B. Killgrove,* SUNY, Geneseo; D.I. Kiel, C.L.Koch, Cal. State L.A.

We define generalized parabola for ANY affine plane and show in Moulton Plane one such bounds a region NOT convex But star-shaped with respect to its vertex. In projective setting we call this set an algebraic conic and note no oval in the translation plane of order nine is an algebraic conic, while the ovals in Desarguesian plane are always algebraic conics and conversely, but the ovals in the Hughes plane of order nine are algebraic conics with some but not all of its points as vertex of generalized parabola. Moreover, every multiplicative loop whose parabola is an oval is commutative in the known planes of order nine. If time permits, other results in finite planes will be mentioned.

(13)

THE BINDING NUMBER OF TWO STRONG CARTESIAN PRODUCTS OF GRAPHS

R. Stanton Hales, Pomona College

In this paper we calculate the binding numbers of $L_m * K_2$, the strong cartesian product of the m-path and the 2-clique, and of $C_m * K_2$, the strong cartesian product of the m-cycle and the 2-clique. This answers two open problems posed by Kane and Mohanty in a paper, "Product Graphs and Binding Number," to appear jointly with this work.

Planarity of Cayley Diagrams: Weak Point-symmetry

(14)

H. Levinson
Rutgers University

A Cayley graph is called weakly point-symmetric planar if it has a planar embedding in which the clockwise or the counterclockwise succession of directed edges is the same (mod cyclic permutations) at each point, and both clockwise and counterclockwise cases occur. All presentations of groups which have weakly point-symmetric planar Cayley graphs are found. An algorithm is given to determine if a given presentation with solvable word problem has a weakly point-symmetric planar Cayley graph, and the set of all groups with such Cayley graphs is characterized.

(15)

A FAST ALGORITHM FOR STEINER TREES

L. Kou,* G. Markowsky and L. Berman
IBM Thomas J. Watson Research Center, Yorktown Heights, N.Y.

Given an undirected distance graph $G=(V,E,d)$ and a set S, where V is the set of vertices in G, E is the set of edges in G, d is a distance function which maps E into the set of nonnegative numbers and $S \subseteq V$ is a subset of the vertices of V, the Steiner tree problem is to find a tree of G that spans S with minimal total distance on its edges. In this paper, we analyze a heuristic algorithm for the Steiner tree problem. The heuristic algorithm has a worst case time complexity of $O(|S||V|^2)$ on a random access computer and it guarantees to output a tree that spans S with total distance on its edges no more than $2(1-\frac{1}{\ell})$ times that of the optimal tree, where ℓ is the number of leaves in the optimal tree.

ON THE DELETION OF NONPLANAR EDGES OF A GRAPH

(16,17)

P. C. LIU, ENVIRONPLAN, INC.
R. C. GELDMACHER, STEVENS INSTITUTE OF TECHNOLOGY

ABSTRACT The practical usefulness, to applied mathematicians, computer scientists, and engineers, of the concept of NP-completeness has become more and more widely documented [1] [2]. To paraphrase Garey and Johnson [2], " - if your boss has asked you to find an efficient algorithm for a part of a new system, and you can show the problem to be NP-complete, you can tell him that you can't find an algorithm but neither can this long list of famous people." The work described in this paper is a case in point. It was motivated by the engineering problem of efficiently laying out LSI digital networks and is the culmination of many, many, many, hours spent in trying to invent a polynomial-time algorithm for identifying a set of nonplanar edges of a graph. The paper outlines the original proof of NP-completeness of the nonplanar edge deletion problem [3] and then gives the details of a reorganization of the proof following a suggestion by D.S. Johnson [4]. The proof consists of a reduction of the vertex cover problem to the Hamilton path problem for graphs with no triangles, and a reduction of the later problem to the nonplanar edge problem.

[1] R.E. TARJAN, "Complexity of Combinatorial Algorithms", Siam Review 20, 3 (1978), 457-491. [2] M.R. GAREY, and D.S. JOHNSON, Computers and Intractability: A Guide to the Theory of NP-completeness, H. Freeman and Sons, San Francisco, 1979. [3] Peter C. Liu, "Analysis of Nonplanar Graphs", Ph. D. Thesis, Department of Electrical Engineering, Stevens Institute of Technology, April, 1976. [4] D.S. Johnson, Private Communication, October, 1976.

DISJOINT CLIQUES IN COSET GRAPHS

(18) E.J. Cockayne, University of Victoria, B.C. Canada

Sufficient conditions for the existence of disjoint cliques are given for two types of graphs formed from a finite group X, the cosets of a subgroup H and a subset $L \subseteq X$.

Permutation Action Graphs

(19) T.D. Parsons, The Pennsylvania State University

Let X be a nonempty set and F a multiset of permutations chosen from the symmetric group S_X on X. The permutation action graph (p.a.g.) G = (X,F) has vertex set V(G) = X and arcs the ordered pairs (x,f) for $x \in X$ and $f \in F$. It is a directed pseudograph (loops and multiple arcs may occur). When F is a set, we say G is proper. Every Cayley graph is a proper p.a.g., and every Schreier coset graph is a p.a.g. If $F = F^{-1}$, we may regard G as a pseudograph with undirected edges {x,xf}. It is easy to see which F result in no loops or multiple edges. Graph-theoretic properties correspond to group-theoretic ones much as in the case of Cayley graphs (of which the p.a.g.'s are obvious generalizations). Of course, p.a.g.'s need not be point symmetric, but it is easy to construct many which are (in addition to the Cayley graphs). The elements t of the automorphism group Aut G may be characterized by a simple condition on the sets $ft^{-1}F^{-1}t$ for $f \in F$. To each graph H we may associate the p.a.g. (V(H), Aut H). Therefore the problem of characterizing those permutation groups which are the automorphism groups of graphs can be regarded as the problem of characterizing a certain family of p.a.g.'s. It is felt that, like Cayley graphs, p.a.g.'s deserve study.

GENERALIZED LINE GRAPHS

(20) Michael Doob University of Manitoba

The concept of a generalized line graph is due to A. J. Hoffman, who appended cocktail party graphs to line graphs in order to construct and characterize graphs with bounded least eigenvalue. In this presentation several new characterizations of generalized line graphs are given: (1) a Krausz type clique characterization, (2) a Beineke type forbidden subgraph characterization, and (3) a spectral characterization. A "root graph" is constructed which yields information on the structure of the automorphism group of the generalized line graph as well as several new spectral theorems.

Subgraphs of Critical Graphs of Diameter k

(21) D. L. Greenwell, Auburn University

The purpose of this note is to give a construction to show that any graph is an induced subgraph of some critical graph of diameter k for any $k > 1$.

(22)

GRAPH FOLDING

Curtis R. Cook,* Oregon State University, Corvallis, Oregon 97331
Anthony B. Evans, Washington State University, Pullman, WA. 99164

A graph fold is the special case of a graph homomorphism where the two identified vertices are both adjacent to a common vertex. Like homomorphisms, folds are related to the chromatic number and we obtain an Interpolation Theorem for folds. If $\chi(G) = n$, then G is absolutely n-chromatic if every fold preserves the chromatic number. Every nontrivial bipartite graph is absolutely 2-chromatic. Given $m \geq 4$, we give a construction of a three chromatic graph that folds onto K_m and conjecture that this is the smallest such graph.

ON THE DISTRIBUTION OF THE PERMANENT OF CYCLIC (0,1) MATRICES

(23) Evi Nemeth,* SUNY College of Technology, Utica, NY
Jennifer Seberry, Univ. of Sydney, New South Wales, Australia
Michael Shu, University of Manitoba

Some results are obtained on the permanent of cyclic (0,1) matrices which support the conjecture that for such matrices of prime order p, the number of distinct values the permanent attains is $p + 1 + \epsilon$ where ϵ is small compared with p. We obtain formulas for the permanent of cyclic (0,1) matrices in several cases.

" A Determinant Method for a Special 1-Circulant Matrix "

(24) John Sumner, University of Miami

Consider a v x v matrix M with integral entries and suppose the first row has the form $[r_0, r_1, r_2, \ldots r_{\frac{v}{2}-1}, r_{\frac{v}{2}}, r_{\frac{v}{2}-1}, \ldots r_1]$ when v is even and has the form $[r_0, r_1, \ldots r_{\frac{v-1}{2}}, r_{\frac{v-1}{2}}, \ldots r_2, r_1]$ when v is odd. Suppose that M is a 1-circulant matrix. This paper is concerned with the extraction of the determinant of M. The determinant is obtained (up to an "unknown" square). Under suitable conditions it is shown that the term $r_0 - 2r_1 + 2r_2 - \cdots \pm r_{\frac{v}{2}-1} \pm r_{v/2}$ is the square of an integer when v is even. A similar condition is obtained when v is odd.

A Special Family of Permutation Matrices

(25) Derbiau F. Hsu, University of Michigan

A special family of permutation matrices, called I-matrices is defined. It is proved that J^*_{2n} $(J_{ii} = 0,\ J_{ij} = 1,\ \forall i \neq j)$ can be decomposed into a sum of $2n - 1$ I-matrices if $2n + 1$ is an odd prime. This decomposition is called a 3-dimension König decomposition of J^*_{2n}. This result is then used to study directed graphs and factorization of complete directed graphs. Numbers of I-matrices of even orders less than or equal to 12 are given and the general enumeration problem is discussed. We introduce a structure on the set of I-matrices of given order via a set of transformations (DC, DR, t and t*). Examples are given.

(26)

SUBSETS OF POSITIVE INTEGERS: THEIR CARDINALITY AND MAXIMALITY PROPERTIES

O.S. Bellamy and C.C. Cadogan,* University of West Indies

Related subsets of positive integers are generated from the integer 8 by successive iterations. At each stage the cardinality of the subsets and the maximum integer of each subset are determined.

On 3-Irreducible Animals

(27) Kenneth Holladay, University of Miami

By animal, I shall mean a face-connected simply connected set of cells of the equilateral triangle tessellation of the plane. Only animals with area a multiple of 3 are allowed. A 3-irreducible is an animal that can not be split into two animals both with area a multiple of 3. All 3-irreducibles are contained in maximal 3-irreducibles and all maximal 3-irreducibles are described. They are "essentially" linear. They have two ends and a backbone running between the ends. This backbone may have turns and twists. All 3-irreducibles are generated as words in a certain language. The language is not context-free because the animal must still be checked for self-intersections after it has been generated.

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THE COVER INDEX AND CRITICAL GRAPHS: AN ANALOGUE OF VIZING'S ADJACENCY LEMMA

Ram Prakash Gupta, Ohio State University

Let $G = (V,E)$ be a simple graph. A set of edges F, $F \subseteq E$, is called a cover of G if for every vertex $x \in V$, F contains at least one edge incident with x. The cover index $\mathcal{X}(G)$ of G is the maximum number k such that G possesses k mutually disjoint covers. For $x \in V$, let $d_G(x)$ denote the degree of x in G and let $\delta(G) = \min_x d_G(x)$ be the minimum degree of G. It is known that for any simple graph G, $\mathcal{X}(G) = \delta(G)$ or $\mathcal{X}(G) = \delta(G) - 1$. A graph G is called $\mathcal{X}$-critical if $\mathcal{X}(G) = \delta(G) - 1$ and for any nonadjacent $x, y \in V$, $\mathcal{X}(G + xy) = \delta(G)$, where $G + xy$ denote the graph obtained from G by joining x and y by a new edge xy. Our main result may be stated as follows:

Theorem: Let G be any $\mathcal{X}$-critical graph. Then, for any vertex $x \in V$, at least one of the following two conditions holds:

1) x is adjacent to at least $2(d_G(x) - \delta(G) + 1)$ vertices of degree precisely $\delta(G)$;
2) every vertex y, nonadjacent to x, is adjacent to at least $2(d_G(y) - \delta(G) + 1)$ vertices of degree $\delta(G)$.

Corollary: Every $\mathcal{X}$-critical graph G contains at least three vertices of degree $\delta(G)$.

29

COHESION IN ALLIANCE GRAPHS

Marc J. Lipman* and Richard D. Ringelsen
Indiana University-Purdue University at Fort Wayne

For a set of countries with specified alliances, it is reasonable to ask, "How close can a particular country come to severing the alliance connections between 2 groups of countries?" One possible answer can be quantified as follows: Represent the countries as vertices of a graph, G, with the alliances as the edges. For a vertex, x, of G, the desired number is $\mu(x)$ = the minimum number of edges which can be deleted from G so that x is a cutpoint for the remaining graph. The cohesion of G, denoted μ, is the minimum value of $\mu(x)$ among all vertices of G. Let $\lambda(x)$ denote the edge connectivity of the subgraph induced from G by deleting x.

Theorem: μ = the minimum value of $\lambda(x)$ among all vertices of G. Thus μ belongs to the connectivity pair (l,μ) as defined by Beineke and Harary.

34

The Elation Groups of the Classical Generalized Quadrangles

Clifton E. Ealy Jr.; Miami University

Abstract: Let $Q=(P,L,I)$ be a generalized quadrangle. For each point u of Q, let

$st_P(u)=\{x \in P \mid x \text{ is collinear with } u\}$

(a point of Q is collinear with itself and, hence, $u \in st_P(u)$). Hence, for each point u of Q, define

$E(u)=\{\alpha \mid \alpha \text{ is a collineation of } Q \text{ and } \alpha \text{ is the identity on } st_P(u)\}$;

we call $E(u)$ the elation group of Q at u. In this paper, we calculate the elation groups of the classical generalized quadrangles.

GENERALIZED QUADRANGLES AS AMALGAMATIONS OF DESARGUESIAN PLANES: THE MULTIPLICATIVE CASE

S. E. Payne,* Miami University (Ohio); R. B. Killgrove, SUNY at Geneseo; D. I. Kiel, California State at Los Angeles

35

Any generalized quadrangle of order s that is an amalgamation of desarguesian planes can be coordinatized by a pair (α,β) of permutations of the elements of $GF(s)$, where s must be a power of 2 and the pair (α,β) must satisfy a condition referred to as admissibility. The determination of all admissible pairs seems hopelessly difficult, and in this note the case where α and β are both multiplicative maps is investigated. A combination of theoretical results and computer searches leads to the following: All generalized quadrangles of order $s = 2^e < 128$ coordinatized by admissible pairs (α,β) in which both α and β are multiplicative must be among the previously known examples.

40

CONSTRUCTION OF PLANAR EULERIAN MULTIGRAPHS

J. Abrham* (University of Toronto and CRMA Université de Montréal) and A. Kotzig (CRMA Université de Montréal).

It is shown that every planar eulerian multigraph contains an Euler tour which has the property that its transitions through the vertices of the multigraph never intersect. Using this a simple characteristic property is obtained which can be used for the construction of planar eulerian multigraphs. An indication is given as to the generalization of these results to the case of eulerian multigraphs realized on more general surfaces.

"On Alternating Eulerian Circuits and Complete Graphs"

41

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A characterization of complete graphs with respect to their properties regarding alternating and balanced alternating eulerian circuits is presented in this paper. After giving some basic properties of alternating eulerian circuits, we show that the conly complete graphs whose edges may be oriented such that they possess an alternating eulerian circuit are the ones on n vertices, where $n=4m+1$ and m is a natural number. An algorithm which orients the edges of such a graph such that the directed graph possesses a balanced alternating eulerian circuit is given, and its correctness is proved.

(30) THE ALGORITHMIC COMPLEXITY OF UNICYCLIC GRAPHS

S. Hedetniemi and S. Mitchell*, University of Oregon

In this paper we present an algorithmic complexity comparison of algorithms on unicyclic graphs with algorithms on trees. We show that the addition of one edge which creates a single cycle does not increase the complexity of algorithms to solve various problems on unicyclic graphs by more than a constant (for the problem) multiplicative factor. We present algorithms for solving: independence, matching, domination, recognition, shortest paths, isomorphism, centre and expected distance problems.

(31) TOWARDS A THEORY ON CENTRES OF RECURSIVE GRAPHS

S. Hedetniemi* and S. Mitchell, University of Oregon

We review the results for centres of the recursive graphs -- trees, maximal outerplanar graphs and unicyclic graphs. We present new results which determine all possible subgraphs which can be centres of cacti and C_4-trees. We generalize some of these subgraphs from C_4-trees to general C_n-trees. A C_n-tree is a graph which is essentially a tree of cycles of length n, where two cycles are either disjoint or joined together by an edge.

The Optimum Defense against Random Subversions in a Network

B.L. Hartnell, Memorial Univ. of Newfoundland, Corner Brook Campus

(32) A classic problem encountered by any underground resistance movement is the question of establishing a communications network among the members of the resistance which minimizes the effects of treachery or subversion of a particular member or members, followed by the consequent betrayal of other members. One of the aspects of this problem we shall examine is defending against k random subversions.

(33) MULTIPLE-MESSAGE BROADCASTING IN COMPLETE GRAPHS

Phyllis Chinn* Humboldt State University
Stephen Hedetniemi and Sandra Mitchell, University of Oregon

Broadcasting is the information dissemination process whereby a set of messages is relayed from one member to all other members of a communication network. Define P(m,t) to be the maximum number of people to whom m messages can be broadcast in t time units. We determine all values of P(m,t) for $1 < t < 8$, and all values for m = 1,2. We present a conjecture for the exact expression for P(m,t) for arbitrary m and t.

Characterizing Subspaces by Ineresection Cardinalities

A.A. Bruen, U. West. Ontario and B.L. Rothschild*, U.C.L.A.

(36) We consider generalizations of the following question: Is a subspace of a projective or affine space characterized by the cardinalities of intersections with all hyperplanes? Earlier work on the problem is extended in the affirmative direction, and some counterexamples are found for certain cases.

Bounds for msp Spreads

Gary L. Ebert, University of Delaware

(37) A maximal strictly partial (msp) spread of PG(3,q) is a collection of pairwise skew lines which is not a spread and is not properly contained in any other collection of pairwise skew lines. Bruen has shown that if W is a msp spread of PG(3,q), then $|W| > q + \sqrt{q} + 1$. However, the smallest known examples of msp spreads are of size $\frac{1}{2}(q^2 + q + 2)$, and these occur whenever 3 does not divide q + 1. In this paper new lower bounds for msp spreads are determined.

(38) "Partial d-space"

Renu Laskar, Clemson University

This paper introduces the concept of a partial d-space, extending the concepts of an (r,k,t) partial geometry (which may be called partial plane) [Bose] and of a partial geometry of dimension three due to Laskar and Dunbar (which may be called partial 3-space).

Counting of Normal-Facial Point Collineations in a Cubical Lattice

W. Wm. Funkenbusch, Michigan Technological University

(39) A tabulation of the frequency of occurrence of Normal-Facial Collineations and sets of Normal Facial Collineations of P points in a Cubical Lattice of side N. The table numbers were obtained by elementary combinatorics. The count tends to get quite tedious and tricky. Although never carried through to completion some preliminary thought was given to two other counting methods: one a computer counting idea, the other making use of Polya's Counting Theorem. The tabulated results might find some application in escape and capture problems in atomic piles and in crystal structure studies -- in particular in regard to probabilistic considerations concerning the initial formation and the future growth of aggregates of particles by random processes. Just for fun, this leads to the creation of a playable, very "wild", and hopefully enjoyable game of Three Dimensional Poker.

Algorithms for Optimum Antichains

(42)

Kathleen Cameron, University of Waterloo

We will present algorithms for finding a largest antichain (independent set) and a largest weight antichain in a partially ordered set.

A Characterization of Posets with Rank Equal to Dimension

S. B. Maurer, Princton U., I. Rabinovitch* and W. T. Trotter, Jr., U. of S. Carolina

(43)

A realizer R of a poset (X,P) is a set of linear orders whose intersection is P. A realizer is minimal if no proper subset of it is also a realizer. The dimension of (X,P) is the cardinality of a smallest (minimal) realizer of P. The rank of (X,P) is the cardinality of a largest minimal realizer. For every (X,P) with $|X| \geq 4$ we have

$$\frac{|X|}{2} \leq \text{Dim }(X,P) \leq \text{Rank }(X,P) \leq [\frac{|X|^2}{4}]$$

In this paper we provide a characterization of posets for which every minimal realizer has the same size, i.e., dimension equals rank.

A Characterization of Rank-Degenerate Posets

S. B. Maurer, Princton U., I. Rabinovitch and W. T. Trotter* Jr., U. of S. Carolina

(44)

In this paper we introduce a graph theoretic concept for the computation of the rank of a poset. The digraph of nonforced pairs N_p of a poset (X,P) has as directed edges the ordered pairs (x,y) where x|y in P, z > x in P implies z > y in P, and z > y in P implies z < x in P. A subgraph $H \subseteq N_p$ is said to be weakly unipathic when the existence of edge disjoint directed paths in H from a vertex x to a vertex y implies that x > y in P.

For most posets, the exceptional ones being rank-degenerate, the rank of the poset is the maximum number of edges in a weakly unipathic subgraph of the graph of nonforced pairs. In this paper we show that a poset is rank-degenerate if and only if it is a subposet $\underline{m} \oplus \underline{3} \oplus \underline{n}$ for some $m \geq 1$, $n \geq 1$ and that the rank of a rank-degenerate poset is equal to its width.

(45) Hyperplanes, Graphs and Matroids

Thomas Zaslavsky, The Ohio State University

Several properties of an arrangement of hyperplanes in Euclidean space depend only on its associated matroid. Among them are the number of regions and the number of bounded regions. Thus minimal arrangements, and the bounded part of an arrangement, can be investigated matroidally. In addition the numbers found for arrangements can be interpreted for graphs in terms of acyclic orientations.

A conjecture on the ramsey number of unicyclic graphs of odd girth

Jerrold W. Grossman, Oakland University, Rochester, Michigan

(46) The ramsey number $r(G)$ of a graph G is the least number p such that if every line of the complete graph K_p is colored either red or blue, then either the red subgraph or the blue subgraph of K_p contains a copy of G. Let G be a connected unicyclic graph with $n \geq 4$ points, whose cycle has odd length. We present evidence for the following CONJECTURE: The ramsey number of G is $2n-1$. It is easy to show that $r(G) \geq 2n-1$. If the cycle of G is a triangle and G has diameter at most 3, then $r(G) = 2n-1$. If G consists of a pentagon with a star emanating from one vertex, then $r(G) = 2n-1$. Other special cases are also considered.

THE HAJO'S RATIO, RAMSEY NUMBERS AND BUTTERFLIES

(47) S. Fajtlowicz, University of Houston

Let $\sigma = \sigma(G)$ denote the largest integer n such that G contains a subdivision of the complete graph with n vertices. Hajo's conjectured that every graph with chromatic number χ satisfies, $\chi \leq \sigma$. This conjecture was recently disproved by P. Catlin.

We shall show that the ratio $\frac{\chi}{\sigma}$ may be arbitrarily large. Using Érdös constructions of Ramsey-type graphs one can show that χ may be almost as large as σ^2.

We shall also discuss some connections between the above concepts and Butterflies, i.e. graphs with maximum degree p containing no complete graphs K_q and having independence ratio $\frac{2}{p+q}$.

SOME RAMSEY TYPE RESULTS ON TREES VERSUS COMPLETE GRAPHS

Gary Chartrand, Michigan State Univ. and Western Michigan Univ.
Ronald J. Gould, San Jose State Univ. and Western Michigan Univ.
*Albert D. Polimeni, Florida Atlantic Univ. and SUNY at Fredonia

(48) For an arbitrary tree T of order m and an arbitrary positive integer n, Chvátal proved that the ramsey number $r(T,K_n) = 1 + (m-1)(n-1)$, i.e., for any coloring of the edges of $K_{1+(m-1)(n-1)}$ with two colors, there exists a monochromatic tree T or a monochromatic K_n. Chvátal's theorem is extended by showing that, in certain cases, the result still follows if $K_{1+(m-1)(n-1)}$ is replaced by an appropriate proper spanning subgraph of $K_{1+(m-1)(n-1)}$.

t-Designs From the Large Mathieu Group. I: The t-Designs

Earl S. Kramer,* Spyros S. Magliveras, and Dale M. Mesner
University of Nebraska, Lincoln, NE 68588

(49) A t-design $(X, \mathfrak{B})$ is a v-set X together with a family $\mathfrak{B}$ of k-subsets from X, called blocks, such that each subset of X of size t is contained in exactly λ members of $\mathfrak{B}$. A t-design with the above parameters is also called a t-(v,k,λ) design. Here, we allow repeated k-sets in $\mathfrak{B}$, i.e. $\mathfrak{B}$ is a multiset. We describe the action of the Mathieu groups M_n, n = 24,23,22, on the power sets of the respective X (Chang Choi and John H. Conway have done this for M_{24}), and then determine all of the quadruples of parameters t,n,k,λ with $2 \leq t < k \leq n/2$ for which there is a t-(n,k,λ) design with M_n as automorphism group. Among the many new t-designs found there is, for example, an 11-(24,12,6) design which is the union of three orbits of 12-sets under M_{24}, two of which are repeated six times.

t-Designs From the Large Mathieu Groups II: The Group Action

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(50) A t-design $(X, \mathfrak{B})$ is a v-set X together with a family $\mathfrak{B}$ of k-subsets from X, called blocks, such that each subset of X of size t is contained in exactly λ members of $\mathfrak{B}$. A t-design with the above parameters is also called a t-(v,k,λ) design. Here, we allow repeated k-sets in $\mathfrak{B}$, i.e. $\mathfrak{B}$ is a multiset. We describe the action of the Mathieu groups M_n, n = 24,23,22, on the power sets of the respective X (Chang Choi and John H. Conway have done this for M_{24}), and then determine all of the quadruples of parameters t,n,k,λ with $2 \leq t < k \leq n/2$ for which there is a t-(n,k,λ) design with M_n as automorphism group. Among the many new t-designs found there is, for example, an 11-(24,12,6) design which is the union of three orbits of 12-sets under M_{24}, two of which are repeated six times.

REGULAR PARTITIONS OF GROUPS

(51)

by

Richard J. Friedlander,* University of Missouri - St. Louis
Basil Gordon, University of California, Los Angeles

A group G of order n is said to be R-sequenceable if its non-identity elments can be arranged in a sequence $a_1, a_2, \ldots, a_{n-1}$ such that the quotients $a_1^{-1}a_2, a_2^{-1}a_3, \ldots, a_{n-1}^{-1}a_1$ are all distinct. This condition is equivalent to the existence of a complete mapping of G which fixes the identity element and permutes the remaining elements cyclically. As a natural generalization we seek to determine which finite groups of order n have the property that, given any regular partition of n-1, there exists a complete mapping that fixes the identity and has that partition as its cycle structure. We have settled this problem for all abelian groups of order 15 or less, as well as for all elementary abelian p-groups, p prime. We conjecture that any finite abelian group having either trivial or non-cyclic Sylow 2-subgroup possesses such a mapping.

ON ISOMORPHISMS OF GRAPHS

by

(52)

J.A. Zimmer
Univ. of Calgary.

ABSTRACT: A function φ on the vertex set of a finite graph without loops or multiple edges is defined. Determination of $\varphi(x)$ can be made in $o(n^3)$ steps for any graph G of n vertices and any $x \in V(G)$. The determination uses information from the entire graph structure. It is conjectured that $\varphi(x) = \varphi(y)$ if and only if there exists an automorphism α of G such that $\alpha(x) = y$. The conjecture is made plausible with a computer test of 500 graphs of 15-30 vertices each. Based on this conjecture a polynomial time algorithm for deciding whether two graphs are isomorphic is presented. Elements of the codomain of φ have a structure somewhat more complex than G itself.

(53) **Isomorphism testing and symmetry of graphs**

László BABAI, Vanderbilt University, Nashville, Tenn.

The algorithmic problem of graph isomorphism testing is closely related to the effective solution of problems on the automorphism group of graphs. The determination of the order of the group Aut X (X a graph) is polynomial time equivalent to isomorphism testing. Asymmetry makes isomorphism testing easier. There exists a canonical labelling algorithm with linear average running time (L.B.&L.Kučera). The high asymmetry of random graphs plays a role here. On the other end, strongly regular graphs admit canonical labeling in $\exp(2\sqrt{n}\log^2 n)$ worst case time. This follows from an effective elimination of all automorphisms by fixing a subset of size $2\sqrt{n}\log n$.

Coincidence of eigenvalues can be regarded as one kind of symmetry. Therefore, the following result is no surprise. (L.B.&D.Grigoriev) Isomorphism testing of graphs with at most m-tuple eigenvalues can be performed in n^{2mtc} time. (c is a constant, n is the number of vertices.)

We define, in terms of permutation groups, a class of graphs which seems to be hard to test for isomorphism.

REFINEMENT TECHNIQUES FOR GRAPH ISOMORPHISM

Charles J. Colbourn, University of Toronto

(54) Many efficiently computable necessary conditions for two graphs to be isomorphic are known. The graph isomorphism problem is to find an efficiently computable necessary condition which is also sufficient. One approach to this problem is to consider invariants (necessary conditions for isomorphism) both alone and together, with the goal of determining the power of the invariant - its ability to distinguish among graphs.

A different approach is to consider an invariant and to attempt to show that the invariant is sufficient, or complete, for a particular class of graphs. As an example, one invariant of a graph is its degree sequence; this is quite a weak invariant for graphs, but it is complete for threshold graphs.

We consider a method known as refinement for enhancing the power of an invariant; a surprising result is that degree sequence with refinement is complete for trees, whereas degree sequence alone is not. This was first shown by Cornell in 1968. Since that time, the search for further classes characterized by refinement has been fruitless.

In this paper we introduce refinement techniques and describe their application to trees. We then demonstrate that distributive lattices are characterized by refinement; this answers a question of Babai on isomorphism of distributive lattices. Finally, we describe a generalization of the refinement technique and show that k-trees are characterized by generalized refinement.

ON THE NUMBER OF POLYNOMIAL EQUATIONS OF GIVEN HEIGHT

D.L. Bramwell, University of the West Indies, Jamaica

(55)

In a well known proof that the set of algebraic numbers is countable, the positive integer

$$h = \alpha_n + |\alpha_{n-1}| + \ldots\ldots + |\alpha_0| + n,$$

called the "height" of the polynomial equation of order $n \geq 1$

$$\alpha_n x^n + \ldots\ldots + \alpha_1 x + \alpha_0 = 0$$

where $\alpha_i (i = 0, \ldots n)$ are integers and $\alpha_n \geq 1$ plays a significant role. (See, for example, Courant & Robbins "What is Mathematics" p. 103). If a_h denotes the number of polynomial equations of height h and

$$F(t) = \sum_{h=1}^{\infty} a_h t^h$$

is the generating function of the sequence (a_h), then we show very quickly that

$$F(t) = 2t^2(1+t)/(1-t)(1-2t-t^2)$$

and deduce the surprising result that a_h satisfies the linear recurrence relation

$$a_h - 2a_{h-1} - a_{h-2} = 2 \quad a_1 = 0, \quad a_2 = 1.$$

Finally, noting the form of the results, we indicate an even simpler solution to the problem of finding the number of polynomial equations of given height.

Quadratic Properties of a Linearly Recurrent Sequence

Julio R. Bastida, Florida Atlantic University

(56)

Given nonnegative integers u,v,w such that $u > 1$, let $(x_n)_{n>0}$ be the sequence of integers satisfying the linear recurrence $x_{n+2} - 2ux_{n+1} + x_n = 0$ for every $n \geq 0$ and the initial conditions $x_0 = w$ and $x_1 = uw + v$. It is shown that $(x_n)_{n>0}$ is an increasing sequence of nonnegative integers such that, for every $n \geq 0$, the quadratic equalities $(x_{n+1} - ux_n)^2 = (u^2 - 1)(x_n^2 - w^2) + v^2$ and $(ux_{n+1} - x_n)^2 = (u^2 - 1)(x_{n+1}^2 - w^2) + v^2$ obtain.

Group Divisible Difference Sets and Families From s-Flats of Finite Geometries

(61)

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Let d,s,q be positive integers and q be a power of a prime. For $s \leq d-1$, let $A_s(d,q)$ be the set of all s-flats of AFF(d,q) and $P_s(d,q)$ be the set of all s-dim subspaces of PG(d,q). Let $v = q^d-1$ and $v^* = (q^{d+1}-1)/(q-1)$. Consider $A_s(d,q)$ as a set of q^s-element subsets of Z_v and $P_s(d,q)$ as a set of $(q^{s+1}-1)/(q-1)$-element subsets of Z_{v^*}. Then $A_s(d,q)$ always contains a group divisible difference family (GDDF) of Z_v. Furthermore, if $d \geq 6$ and $2 \leq s \leq d-3$, then $A_s(d,q)$ contains at least $2+(q^d-q)/2$ linearly independent irreducible GDDF's of Z_v. $A_s(d,q)$ contains a group divisible difference set of Z_v with $\lambda_1=0$ iff $(d-s)|d$. In projective case, $P_s(d,q)$ contains a difference family of Z_{v^*} if $(s+1,d+1)=1$. If in addition, $d \geq 7$ and $2 \leq s \leq d-3$ then for any subgroup H of Z_{v^*} of index m, $P_s(d,q)$ contains at least $3+\delta(m,v^*)+v^*/2$ linearly independent GDDF's relative to H in Z_{v^*}, where $\delta(m,n)=-1$ if m=1 or n and =0 otherwise. $P_s(d,q)$ contains a difference set of Z_{v^*} iff $s=d-1$.

Symmetric Designs with Bruck Subdesigns

R. D. Baker, University of Delaware

(62)

If P is a finite projective plane of order n with a proper subplane Q of order m which is not a Baer subplane, then a theorem of Bruck [Trans. AMS 78(1955), 464-481] asserts that $n \geq m^2 + m$. If the equality $n = m^2 + m$ were to occur then P would be of composite order and Q should be called a Bruck subplane. It can be shown that if a projective plane P contains a Bruck subplane Q, then in fact P contains a three dimensional projective geometry Q' of order m. A well known scheme of Bruck suggests using such a Q' to construct P. Bruck's theorem readily extends to symmetric designs [Kantor, Trans. AMS 146(1969), 1-28], hence the concept of a Bruck subdesign. This paper develops the analogue of Q' and shows (by example) that the analogous construction scheme can be used to find symmetric designs.

EXPECTED LENGTH OF SHORTEST PATHS AND ALGORITHM BEHAVIOR

(67)

Henry K. DeWitt* and Michael M. Krieger
University of California at Los Angeles

Two kinds of results are presented in this paper. First, we extend the theory of random graphs (a la Erdos and Renyi) to the expected length of shortest paths in random graphs. Second, we consider the algorithmic implications of these results, in particular analyzing Nicholson's shortest path algorithm. This method attempts to reduce the computation time to find the shortest path between vertices u and v in a graph by simultaneously working outward from both u and v. In the worst case, however, this algorithm may require twice as much computation as Dijkstra's algorithm. For this reason Nicholson's algorithm is not generally used.

We show that in fact the expected behavior of Nicholson's algorithm is superior in certain applications.

MULTIPLE-ORIGIN SINGLE-DESTINATION TRANSIT ROUTING

(68)

R.S.D. Thomas* and J.M. Wells, University of Manitoba

The paper describes the use of a computer to test the desirability of potential routes in the University of Manitoba's staff-student bus system.

COMPUTATIONS AND GENERALIZATIONS ON A REMARK OF RAMANUJAN

57 Ronald Alter, University of Kentucky

In a conversation with Ramanujan, G. H. Hardy mentioned that 1729 seemed to be a dull number. Ramanujan answered, "No, it is a very interesting number; it is the smallest number expressible as a sum of two cubes in two different ways". He also stated that the answer to the corresponding problem for fourth powers "....must be very large". In this paper, this problem and other generalizations to higher powers and larger sums is examined.

A GRAPH THEORETIC INTERPRETATION OF FRACTIONS, CONTINUED FRACTIONS AND THE GCD ALGORITHM

58 David W. Matula,* Southern Methodist University
Peter Kornerup, Aarhus University

A Fraction Graph has fractions as vertices where i/j and k/ℓ are adjacent if $|i\ell - kj| = 1$. We interpret properties of fractions in terms of properties of these graphs yielding an interesting pedagogic treatment of these elementary mathematical structures. The GCD algorithm and continued fraction convergents are shown to be intimately related to shortest paths in these fraction graphs.

The Farey Fraction Graph F_n, $n \geq 1$, has as vertices all irreducible fractions i/j with $0 \leq i/j \leq 1$, $j \leq n$. For every $n \geq 2$, we show that F_n is: (i) uniquely and minimally 3-colorable, (ii) uniquely Hamiltonian and (iii) perfect.

ON SEQUENCES OF CONSECUTIVE INTEGERS

59 David H. Eberly Bloomsburg State College

Certain integers can be expressed as the sum of two squares, whereas others cannot be. Such examples for the former are $65 = 4^2 + 7^2$ and $90 = 9^2 + 3^2$. Examples for the latter are 19, 21, and 91. I discovered that none of 75, 76, 77, 78, and 79 can be expressed as the sum of two squares. Similarly, none of 91, 92, 93, 94, 95, and 96 are a sum of two squares. This paper deals with proving that one may construct a sequence of consecutive integers which is of arbitrary length and where each and every integer is not the sum of two squares.

ON THE LINEAR INDEPENDENCE OF CERTAIN NUMBERS OVER THE FIELD OF RATIONALS

60 by Paul Hartung
Bloomsburg State College

It is familiar that right-angled triangles with integer sides exist, for example, (3,4,5), (5,12,13), and (8,15,17). Let us take such triangles (a,b,p) where p is a prime. Let $\theta_p = \mathrm{Tan}^{-1}(b/a)$. It is shown that the set $\{\theta_p : p \text{ is a prime}\}$ is linearly independent over the rationals.

Some Covering Numbers for Graphs

63 William R. Hare* and Renu Laskar
Clemson University

The cyclicity, linear arboricity, and other related concepts are studied for some special classes of finite, loopless, undirected graphs with no multiple edges.

DOMINANCE IN TOURNAMENTS : KINGS AND SERFS

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A vertex x in an n-tournament T is called a king if for every vertex y, y ≠ x, either x dominates y or there is a vertex z such that x dominates z and z dominates y. A vertex x is called a serf if x is a king in the dual of T. These definitions are due to S. Maurer. It is well known that if T has no transmitter (respectively, no receiver) then T has at least three kings (respectively, serfs). We will characterize those 4-tuples of integers (n,k,s,c) for which there exists an n-tournament with exactly k kings and s serfs, c of which are both kings and serfs. One can show that every n-tournament is a subtournament of an m-tournament in which every vertex is a king (and hence every vertex is a serf); we will determine the smallest such m. Also, it is easy to show that every n-tournament T without a transmitter is a subtournament of an m-tournament, $m \leq 2n$, in which the kings are exactly the vertices of T; the smallest such m is unknown in general, but we will discuss an upper bound due to K. Wayland.

SIMILARITY RELATIONS AND SEMIORDERS

K.H. Kim and F.W. Roush, Alabama State University
65 D.G. Rogers*, University of Western Australia

Two types of relation have recently been introduced in abstract statistical theory: one, similarity relations, in discussing the proximity of data; the other, semiorders for non-transitive preferences. We show here that they are logically converse when defined on finite totally ordered sets. We also establish a one-to-one correspondence between them and restricted lattice paths which enables us to enumerate easily various sets of these relations. The well known Catalan numbers occur prominently in this enumerative work.

Topologies on Finite Sets, II

66 H. Levinson and R. Silverman*
Rutgers University and Southern Connecticut State College

The length of the shortest maximal chain in the lattice of all topologies on a finite set S_n with n elements is shown to be 2n-1. The number of self-dual topologies on S_n is the number B_n of partitions of S_n, and the number of isomorphism classes of self-dual topologies on S_n is the number p(n) of partitions of n. An asymptotic formula for the number t_n of topologies on S_n is also exhibited.

ON GENERATING A SYSTEM OF CURCUITOUS ROUTES

69 F. Hadlock* and L. Yuan, Transportation Systems Center

Given a transit network with a station-station trip demand matrix, a problem is formulated of generating a minimal collection of routes providing a level of service which is based on demand. The level of service is taken to be the excess intermediate stations and a heuristic method is presented for generating a minimal system.

THE GENERALIZED PETERSEN GRAPHS G(n,4) ARE HAMILTONIAN FOR ALL n ≠ 8*

G.J. Simmons* and P.J. Slater, Sandia Laboratories, Albuquerque NM †

70 The generalized Petersen graph G(n,k) is defined to be the graph with vertex set $\{u_0, u_1, \cdots, u_{n-1}, v_0, v_1, \cdots, v_{n-1}\}$ and edge set $\{[u_i, u_{i+1}], [u_i, v_i], [v_i, v_{i+k}]\}$ where i is an integer modulo n. Obviously G(n,1) is Hamiltonian for all $n \geq 2$. In 1968 Robertson proved that G(n,2) is Hamiltonian if and only if $n \not\equiv 5 \bmod 6$. In 1972 Bondy proved that G(n,3) is Hamiltonian for all $n \neq 5$. In 1978 Bannai showed that G(n,k) is Hamiltonian for (n,k) = 1 except in the case where G(n,k) is isomorphic to G(n,2) and $n \equiv 5 \bmod 6$. More recently Alspach, Robinson and Rosenfeld have proven that for every $k > 2$, there exists an n(k) such that G(n,k) is Hamiltonian for all $n \geq n(k)$. For k = 4 this latter result leaves open the question of whether many of the cases $n < 255$ are Hamiltonian or not. In this paper we resolve this question and show that G(n,4) is Hamiltonian for all $n \neq 8$.

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†A U.S. Department of Energy Facility.

Hamiltonian cycles in vertex-transitive graphs of order 2p

71 Brian Alspach
Simon Fraser University and University of Arizona

The following result is proved: If G is a connected vertex-transitive graph with 2p vertices, p a prime, then G has a Hamiltonian cycle unless G is the Petersen graph.

Hamiltonian Cycles in Cayley Color Graphs of Some Special Groups

J.B. Klerlein & A.G. Starling*, Western Carolina University

72 Let $D_\Delta(Z_n \times_\phi Z_m)$ denote the Cayley color graph of the semi-direct product of two cyclic groups, using the standard presentation. These graphs are not always hamiltonian and generally, when they are, there seems to be a sparsity of hamiltonian cycles. In this paper we present a special class of these graphs which have numerous hamiltonian cycles.

(73)

Pincherle's Formula as a Taylor Theorem

J.M. Freeman, Florida Atlantic University

Let D and X be operators on polynomials such that D is shift-like (i.e., D1 = 0 and $\deg Dx^n = n - 1$ for $n \geq 1$) and X is an associated Sheffer-variable (i.e., $\delta(X) = DX - XD = I$). Let $\mathcal{C}(D)$ be the centralizer of D, and $\mathcal{A}(D)$ the algebra of operators generated infinitely by D, and finitely by X. It follows that $\mathcal{A}(D) = \mathcal{C}(D) \oplus X\mathcal{C}(D) \oplus X^2\mathcal{C}(D) \oplus \ldots$. The connection constant problem of Rota-Mullin then becomes that of projecting operators of the form $(A\ X\ B)^n$ on the summands $X^k\mathcal{C}(D)$ above, where A and B are in $\mathcal{C}(D)$. A key tool in solving this problem is the "Taylor theorem"

$$T(A) = \sum \delta^k(A) \frac{[T(X^k)]_{X=0}}{k!}$$

holding for operators T on $\mathcal{A}(D)$ which are $\mathcal{C}(D)$-linear. (Here $[\ .\ ]_{X=0}$ is symbolic notation for projection onto $\mathcal{C}(D)$). It is, in turn, not hard to recognize this formula as a generalization of Pincherle's formula.

(74)

A q-Umbral Calculus

Edwin C. Ihrig* and Mourad E.-H. Ismail, McMaster University

We develop a q-analogue of the Roman-Rota umbral calculus using bialgebras. The product of functionals is

$$\langle LM|x^n\rangle = \langle L|x^n\rangle\langle M|x^n\rangle,\ n = 0, 1, \ldots,$$

and the model polynomials are $\theta_n(x) = (x-1)(x-q)\ldots(x-q^{n-1})$. Earlier results of George Andrews immediately follow from the fact that the diagonal map is an algebra map. This includes characterizations of polynomials of q-binomial type in terms of generating functions and functional relationships. Expansion theorems will also be indicated.

SIMPLY SEQUENTIAL AND GRACEFUL GRAPHS*

D.W. Bange and A.E. Barkauskas, University of Wisconsin
(79) Peter J. Slater,* Sandia Laboratories†

The concept of a simply sequential graph is introduced as follows. A graph G with $|V(G) \cup E(G)| = k$ is called simply sequential if there is a bijection $h: V(G) \cup E(G) \to \{1, 2, \cdots, k\}$ such that for each edge $e = xy$ in $E(G)$ one has $h(e) = |h(x) - h(y)|$. Several problems concerning graceful and simply sequential graphs are discussed. In particular, it is conjectured that all trees are simply sequential.

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DEGREE PARTITION CLASSES FOR 4-TREES HAVING SPECIFIED PROPERTIES

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(80) In this paper we obtain for certain 4-trees the number of 4-trees in the degree partition class (q,t,s,p), where q, t, s, and p are the number of points of degree 4, 3, 2, and 1 respectively, and simple formulas for the numbers C(n) of nonempty such classes for trees with n points.

The 4-trees considered are rooted identity 4-trees with root degree j (j = 1, 2, 3). These trees are related to structural formulas for a variety of chemical compounds and the numbers obtained correspond to the numbers of constitutional valence isomers of these compounds. For these special cases, a simple observation enables us to make some statements pertaining to the stereoisomers of these compounds.

Suggestions for potentially interesting and applicable graph theoretical problems are discussed.

(85)

NEOFIELDS AND ORTHOGONAL LATIN SQUARES

Peter Tannenbaum, University of Arizona

Let $N = \{x_0 = 0, x_1 = 1, x_2, \ldots, x_{n-1}\}$ be a finite neofield. For $a \neq 0$ in N let L_a be the latin square having $(L_a)_{i,j} = ax_i + x_j$. If the neofield has the inverse property (IP), then the latin squares L_a and L_b $(a \neq -b)$ are orthogonal if and only if the map $\rho_{ab}(x) = (1 + ax) + bx$ is a bijection on N. For special types of IP neofields (not necessarily fields) maps ρ_{ab} which are bijections can be obtained.

(86)

OPTIMUM BRANCHING SYSTEMS

Gilberto Calvillo* and Jack Edmonds
Unidad de Investigacion y Desarrollo

A branching is a forest such that each one of its edges is directed towards a different node. The set of nodes of the branching towards which no edge is directed is called the root of the branching. A branching system of size k is a collection of edge-disjoint branchings $B_1, \ldots, B_k$. If R_i is the root of B_i, then the system is said to be rooted at $(R_1, \ldots, R_k)$. An efficient algorithm to solve the following problem is presented:

Given a directed graph G, a positive integer k a weight function c for the edges of G and a collection $R_1, \ldots, R_k$ of subsets of nodes, find in G a branching system $(B_1, \ldots, B_k)$ rooted at $(R_1, \ldots, R_k)$ such that $\sum_{j \in \cup B_i} c_j$ is maximum.

LATTICE PATHS WITH THREE STEP DIRECTIONS

H. Niederhausen, Massachusetts Institute of Technology

(75) We consider paths in a plane lattice, which take with equal probability one of the following three step vectors: $(0,1),(1,0),(c,\gamma)$, γ and c integer, $c > 0$. In the enumeration theory a "closed form" for the number of paths which reach a point (i,j) staying above a line usually means a polynomial $p_i(j)$. For piecewise affine lower boundaries, p_i can be expanded by well-known polynomials using the "Finite Operator Calculus" of G.-C. Rota (1973). For more general lower boundaries, together with a "roof", p_i has to be replaced by a piecewise polynomial function. The expansion of these functions is done by a simple generalization of Sheffer polynomials and results in a determinantal expression.

A DOUBLING CONSTRUCTION FOR CERTAIN HOWELL DESIGNS

B.A. Anderson, Arizona State University

(76) A method for building Howell Designs of side 2s from certain designs of side s is described. It appears that this method will be useful in finding designs of type $H^*(6p,6p+2)$, p prime and $p \equiv 1 \pmod 4$. Recent results of the author and F. A. Leonard have shown that if $E > 2$ is an even integer, $E \neq 24,48,54$ and $E \neq 6p$, p prime, then there is a Howell Design of type $H^*(E,E+2)$.

ROOM RECTANGLES

E.S. Kramer, S.S. Magliveras, and D.M. Mesner*
University of Nebraska - Lincoln

(77) A t-design t-(v,k,λ) is defined in the usual way as a set of blocks (k-subsets, here taken as distinct) from a v-set X, where each t-subset of X appears in λ blocks. In a Room square the blocks of a 2-$(v,2,1)$ design are placed in (some of the) distinct cells of an $n \times n$ square so that each row (or column) contains the $v/2$ blocks of a 1-$(v,2,1)$ design. A more general structure which we call a Room Rectangle is an assignment of the blocks of a t-(v,k,λ) design on a set X into distinct cells of an $m \times n$ array so that each row (column) contains the blocks of a t_1-design (t_2-design). We allow the i-th row to contain a t_1-(v_1,k,λ_1) on a subset X_i, $|X_i| = v_1 < v$, in which case $X_1,\ldots,X_m$ are themselves the blocks of a t_1-design. A Room rectangle shares the property with a Room square that interesting subdesigns are selected in a natural way from a big design, where each block of the big design is in at least two of the subdesigns. Further generalizations in the same spirit are suggested, admissible parameters are surveyed, and assorted Room-type designs are constructed.

SOME REMARKS CONCERNING THE MIT PUBLIC-KEY CRYPTOSYSTEM

H.C. Williams, University of Manitoba

(78) Let a message M be encrypted by raising M to a power e modulo R, where R and e are integers which are made public. The recipient of this encrypted form of M can decipher it by raising the cipher text to a power d modulo R. Only the recipient knows the values of the two large primes p_1,p_2 such that $R = p_1p_2$; consequently, only he knows d, as e is preselected such that $(e,(p_1-1)(p_2-1)) = 1$ and $ed \equiv 1 \pmod{(p_1-1)(p_2-1)}$.

Recently several attacks have been made on the proposed security of this cryptosystem. These attacks focus on a possible lack of security of the system under iteration of the encryption procedure. In this paper we discuss methods of selecting the primes p_1,p_2 and the encryption exponent e such that the possibility of breaking this cryptosystem by using an iteration procedure is minimized. Several numerical results are also presented.

THE CHROMATIC DIFFERENCE SEQUENCE OF A GRAPH

(81) Michael O. Albertson, Smith College
David M. Berman*, University of New Orleans

Let $\alpha_k(G)$ denote the maximum number of vertices in a k-colorable subgraph of G. Set $a_k(G) = \alpha_k(G) - \alpha_{(k-1)}(G)$. The sequence $a_1(G), a_2(G), \ldots$ is called the chromatic difference sequence of the graph G. We present necessary and sufficient conditions for a sequence to be the chromatic difference sequence of some four-colorable graph. We also discuss possible structures for three-colorable graphs having particular chromatic difference sequences.

ON HAMILTON DICIRCUITS

(82) Howard Kleiman, Queensborough Community College (CUNY)

We define the concepts of H-cycle, d-set of H-cycle and admissible permutation. These concepts are used to prove theorems on the number of hamilton dicircuits in a digraph. Furthermore, they lead naturally to a computer program in MACSYMA for obtaining all of the hamilton circuits of a graph. This algorithm will be presented in a separate paper.

"Some results on path numbers of some digraphs"

(83) Renu Laskar and William G. Frye*, Clemson University

Let $D = (V,U)$ denote a digraph with no loops and multiple edges. A path-factorization of D is an edge-disjoint collection of paths which cover the edges of D. The path number of D, denoted $\Pi(D)$, is the minimum cardinality of a path-factorization of D. In this paper path numbers of complete symmetric r-partite digraphs and complete cyclic r-partite digraphs are studied.

REMOVAL-COSPECTRAL SETS OF VERTICES IN A GRAPH

Allen J. Schwenk, U.S. Naval Academy, Annapolis, MD 21402

(84) The concept of cospectrally rooted graphs (also called graphs with cospectral points) was introduced by the author in an earlier work. We now extend this concept to allow subsets of vertices containing more than one vertex. In this new view, cospectrally rooted graphs are graphs containing removal-cospectral sets of vertices which happen to be singletons - namely, the roots of the respective graphs.

This extension yields numerous examples of cospectral graphs, including a construction discovered by Godsil and McKay. It also provides an explanation of the phenomenon of unrestricted substitution vertices developed by Herndon and Ellzey. Finally, there are some interesting examples using graphs first studied in connection with the reconstruction conjecture.

CONSECUTIVE ONES MATRICES AND THE GAP MINIMIZATION PROBLEM

(87) A. K. Dewdney, University of Western Ontario

Given an nxm binary matrix M, it is desired to find an nxn permutation matrix P such that PM has the minimum number of gaps over all nxn permutation matrix. A "gap" in a matrix refers to a set of consecutive zeros lying in the same column and bounded by 1's. If it is required that PM has no gaps, the resulting problem is called "the consecutive ones problem": several efficient algorithms for finding such a P, if it exists, are known. The more general problem which we here call "the gap minimization problem" is known to be NP-complete.

There is an efficient algorithm, based on Gray codes, which solves the general problem approximately; it is evaluated in the light of many large-scale computer experiments on random nxm binary matrices.

COMBINATORIAL PROPERTIES OF A FUZZY MATRIX SEMIGROUP

(88) Jin Bai Kim, West Virginia University
Yong M. Lee,* Trenton State College

Let $K = \{r_i \in [0,1]: i=0,1,2,\ldots,m\}$ be a finite subset of the unit interval $[0,1] = I$ with $r_0 = 0 < r_1 < r_2 < \cdots < r_m = 1$. $M_n(K)$ denotes the set of all $n \times n$ fuzzy matrices over K. $M_n(K)$ forms a multiplicative semigroup (see [1] or [2] for the sum and the product of fuzzy matrices) and we call it a fuzzy matrix semigroup. ($M_n(K)$ is called a Boolean matrix semigroup when K has just two elements). Let $a \in M_n(K)$. $D_a = \{b \in M_n(K)$: $ax = c$, $cy = a$, $uc = b$, $vb = c$ for some elements x, y, u, v, c in $M_n(K)\}$ is called the D-class containing a. The number of all D-classes of $M_n(K)$ is unknown for a positive integer $n \geq 2$. We count the number $|D(M_2(K))|$ of all D-classes of $M_2(K)$ by the combinatorial method and show that $|D(M_2(K))| = (m+1)(m+2)(m^2+3)/6$. (We announced this result in NOTICES of Amer. Math. Soc., August 1978, A-520).

1. J. B. Kim, A certain Matrix semigroup, Math. Japonica 22-5 (1978), 519-522.
2. M. G. Thomason, Convergence of powers of a fuzzy matrix, J. Math. Analysis and Applications 57(1977), 476-480.

Seymour's Theorem and Good Algorithms for Totally Unimodular Matrices

(89) Jack Edmonds, University of Waterloo

Good algorithms for recognizing a totally unimodular matrix and for solving a linear program having a totally unimodular cofficient matrix will be presented. I will discuss in this connection recent work of P. Seymour, J.F. Maurras, W. Cunningham, R. Bland, K. Truemper, M. Akgul, and myself.

COMPARISON OF SEARCH TECHNIQUES FOR ORDERED LISTS

L. Garey* and C. Pottle, University of New Brunswick.

(90) The binary search technique is a common method used for locating particular entries in an ordered list. Asymptotically, the method requires $\log_2 n$ probes for a list of length n. In this article, two methods for continuous functions will be shown to require $K\log_2\log_2 n$ probes asymptotically for $1 < k < 2$.

The continuous methods considered are those known as the Pegasus and Illinois methods. Both of these techniques are straight line methods requiring one probe per iterate.

Some numerical results are presented.

Some sets of pairwise orthogonal row complete Latin squares

(91) Katherine Heinrich, University of Arizona

A Latin square of order n based on an n-set S is called row complete if for any ordered pair of distinct elements (α, β) in S there is exactly one row of the square in which they occur as adjacent elements, i.e. as $\alpha\beta$. It is well known that there exist row complete Latin squares of orders n=2m, 21, 27, 39, 55 and 57. We shall show that there exist pairs of orthogonal row complete Latin squares of orders 21, 27, 39, 55 and 57. Also, in each of these pairs one of the squares is also column complete; thus answering the question "Do there exist row complete Latin squares with column complete orthogonal mates?"(Keedwell, 1973) in the affirmative.

Maximal permutation anticodes.

(92) Deza M., C.N.R.S and Université de Paris VII, FRANCE

For any two permutations $a,b \in S_n$ define distance $d(a,b) = n-f(a^{-1}b)$ where $f(c)$ is permutation character of $c \in S_n$. A set $A \subset S_n$ is ℓ-anticode if $d(a,b) \leq \ell$ for any $a,b \in A$. A ℓ-anticode A is maximal if $A \cup (c)$ is not a ℓ-anticode for any $c \in S_n \setminus A$. Maximal ℓ-anticodes are studied using special Sperner families of subsets and coverings.

AN ANALYSIS TECHNIQUE FOR STEINER TRIPLE SYSTEMS

(93) Marlene J. Colbourn, University of Toronto

One major difficulty in generating a catalogue of Steiner triple systems (STS) is the removal of isomorphic systems. Invariants of the design are typically used to assist in this process. One particular invariant, which we term cycle structure, has enjoyed great success in determining designs. Cole and Cummings examined this invariant under the name of graph of interlacing, and used it to characterize small STS. Recently Petrenyuk and Petrenyuk considered T-tables, an equivalent invariant. They pose the question of determining how effective (sensitive) the invariant is for larger v. In this paper, we consider the efficacy of cycle structure in distinguishing cyclic STS. Using cycle structure as a primary tool in isomorph rejection, we have catalogued all cyclic STS for v at most 45. The use of cycle structure in this generation is two-fold. Many designs had unique cycle structure and hence were not duplicated elsewhere in the list. The remainder are canonized using a subexponential algorithm based on cycle structure.

We summarize our findings on the sensitivity of cycle structure as an invariant. We further present the details of a more general algorithm which canonizes t-(v,t+1,1) designs in $O(v^{\log v})$ time. This extends Miller's algorithm for STS.

ON THE EXISTENCE OF HOWELL DESIGNS H(2n+1,4n)

(93.5) Paul J. Schellenberg, University of Waterloo

A Howell design H(s,2n) based on the elements of a 2n-set X consists of a square array of side s such that (1) each cell either is empty or contains an unordered pair of elements from X, (2) each element of X appears exactly once in each row and each column of the array and (3) each unordered pair of X is contained in at most one cell of the array. It is easily seen that for a Howell design to exist, $n \leq s \leq 2n-1$.

S.H.Y. Hung and N.S. Mendelsohn have shown that there is an H(n+1,2n) for all odd positive integers n. For n even, they have shown there is no H(3,4) nor H(5,8) but there does exist an H(7,12) and an H(9,16).

This paper contains descriptions of a direct product type of construction and a direct singular product type of construction for H(n+1,2n)'s. Using these constructions it is shown there is an H(2n+1,4n) for $2n+1 \equiv 1,7,9,13,17$, or 19 (mod 24) with the possible exceptions of $2n+1 \in \{13,17,19,37\}$.

STRUCTURAL RIGIDITY OF BAR & JOINT AND CABLED FRAMEWORKS

(94) Ivo G. Rosenberg, Universite de Montreal

Constructions like bridges or industrial trusses are formed by relatively rigid bars joined to one another at their ends by joints that are at least locally free to change in angles. The desire for nondeformability leads to the following model. A framework is a set of points $P_1,\ldots,P_n$ in space together with a graph on $\{1,\ldots,n\}$. It is rigid if there is no continuous deformation $P_1(t),\ldots,P_n(t)$ keeping invariant the euclidean distance of $P_i(t)$ and $P_j(t)$ for each edge ij of the graph. For the more general cable or tensegrity structures (allowing also cables between points) the distance conditions are equations or inequalities. This problem area, well over 150 years old, flourished in the last century but became neglected until a recent rebirth of interest. The first part of the present paper studies rigidity in the general case using special equation systems while the remainder of the paper is devoted to the more traditional infinitesimal rigidity of bar & joint frameworks. A determinant like characteristic function is derived. This function, vanishing exactly if the framework is not infinitesimally rigid, is the sum of products of volumes of tetrahydra in which each term corresponds to a special orientation of the underlying graph. This combinatorial part brings forth several interesting graph-theoretical problems.

A WEIGHTED DIGRAPH MODEL FOR PREDICTING THE EFFECTS OF COMBINED STRESSORS ON HUMAN PERFORMANCE

(95) R. D. Ringeisen[1]* and C. A. Shingledecker[2]
Aerospace Medical Research Laboratory, Wright-Patterson Air Force Base

An important problem for psychologists concerned with the optimization of human performance has been predicting the effects of various environmental, situational, and personal stressors on behavior. Because such stressors rarely occur in isolation, a useful model for predicting human performance must take into account both the individual effects of various stressors and the effects (usually nonadditive) they may have when present in combination with one another.

In this paper we employ a weighted digraph model with pulse analysis similar to those used for a variety of problems by F. Roberts and others. Using data obtained from a five choice serial reaction task where the pairwise combinations of stressors have been experimentally measured, the model predicts the possible effects on various performance measures should three or more stressors be combined in a single experiment. The methods are applicable to a variety of tasks involving larger numbers of stressors.

ON THE NUMBER OF BALANCED ORIENTATIONS OF EVEN GRAPHS

(96) Kenneth A. Berman, University of Waterloo

A balanced orientation of an even graph G is an orientation of the edges such that the indegree equals the outdegree at each vertex. In this paper, a formula for the number of balanced orientations of G is given in terms of the circuit partitions of G. This leads to a similar formula for the number of face 3-colourings of a planar 4-valent map.

Nearly Cycle Complete and Nearly Cocycle Complete Graphs

(97) Margaret Cozzens, Math. Dept., Rutgers Univ., New Brunswick NJ 08903, and Stephen B Maurer,* Math. Dept., Princeton Univ., Princeton NJ 08544

Graph G is cycle complete if no two cycles are edge disjoint. G is nearly cycle complete if no three cycles are edge disjoint. Analogous definitions apply to cocycles. Cycle complete and cocycle complete graphs have been characterized previously. Here we characterize planar nearly cycle complete graphs and planar nearly cocycle complete graphs. The characterizations are explicit in that we list all graphs of these types which are minimal in certain natural senses. We also find all graphs which are both nearly cycle and nearly cocycle complete; all graphs with this property are planar.

Data Directed Graphs as Models of Relational Algorithms

(98) William R. Edwards* and Edward Paul Katz
Computer Science Dept., U. of Southwestern Louisiana

It is well known that algorithms may be specified in terms of relations or assertions describing the goal state of the system in question, as well as the more common sequential, procedural statements of conventional programming languages. There is also considerable recent interest in data-driven, graph-theoretic models of computation, in which the execution of a module or node is determined by the availability of data (for example, data flow graphs and Petri nets). A graph model of a relational algorithm is developed, in which each node represents a relation, and direction as well as timing of value production is determined by data availability. This is therefore a model of parallel, reconfigurable computation by an undirected or partially directed graph.

THE WORD PROBLEM FOR INCIDENCE STRUCTURES

(99) Stéphane Foldes, CNRS, Grenoble, France

Evans proved that the word problem is recursively solvable for some variety V of universal algebras if and only if the embeddability problem is solvable for V (i.e. there is an algorithm to decide for any finite partial algebra A if it can be embedded in some algebra B of the variety V). We take a similar approach to the problem of deciding the validity of certain generalized configuration theorems in a given class C of incidence structures. For any class C, this validity problem is equivalent to the problem of deciding for any finite incidence structure A if A is embeddable in some incidence structure B belonging to C.

A NOTE ON INFINITE GRAPHS WITH ACTIONS

(100) Ernst Leiss, University of Kentucky

We define a graph with actions $\underset{\sim}{G} = (V, E, a)$ to be a graph (V, E) together with an action function $a: E \to A^*$ which associates with each edge e a language a(e) over the finite alphabet A. Given a vertex $v_0 \in V$ and a subset $Q \subseteq V$ we define the global action $AC(G, v_0, Q)$ to be the language $\bigcup \alpha(p)$ where p ranges over all finite paths from v_0 to some vertex in Q and $\alpha(p) = a(e_1)\cdot\ldots\cdot a(e_n)$ if $p=(e_1,\ldots,e_n)$. We are interested in operations on $\underset{\sim}{G}$ which leave the global action unchanged. We show how under certain conditions a particular operation, namely removing an edge, can be applied in order to replace an infinite graph by a finite graph with the same global action. - This problem arose in the study of formal languages which are solutions of certain equations.

Minimum Cost Partitions of a Rectangle

(101) Michelle Wachs, University of Miami

The problem of partitioning a rectangle into n regions of equal area so that the total length of the boundaries is a minimum arose in the work of S. Fuller on multiprocessor solutions of PDE's. T. C. Hu posed a modified version of the problem. The regions are required to be rectangles and the partitions are obtained by dividing existing rectangles into two rectangles. We present a closed form solution of the modified problem. By using a variational technique we first reduce the problem to that of finding a partition of minimum cost within a much smaller class of partitions. A minimum cost partition within this extremal class can be neatly described.

(102)

On Maximal Partial 2-spreads of PG(5,q)

J.W. Freeman*, Virginia Commonwealth University
A.A. Bruen, University of Western Ontario

The purpose of this note is to exhibit in the five-dimensional projective space over GF(q), PG(5,q), a set, W, of q^3-q^2+1 projective planes no two of which share a point, called a partial 2-spread, that is not properly contained in a partial 2-spread. Such a set W is termed a maximal partial 2-spread. If W is a maximal partial 2-spread of PG(5,2), then $/W/ = 5$, which sharpens the known bound. This information is then used to discuss the nonexistence of certain switching sets of PG(5,q) as well as nets of order 8 and degree 6.

Translation Planes of Order 25 with Non-trivial X-OY Perspectivities

(103)

Elwyn H. Davis, Pittsburg State University

As is well-known any translation plane can be coordinatized by a quasifield. Quasifields can be constructed from special sets of matrices called t-spread sets. A computer generated list of 1-spread sets sufficient to yield all translation planes of order 25 with non-trivial X-OY perspectivities is given. The left nucleus of each associated quasifield is given, and this information is used to determine isomorphisms of some of the resulting planes with previously known planes. The list yields 15 non-isomorphic planes. Two have left nucleus of order 2, three of order 3, two of order 4. The others are isomorphic to known planes.

GRAPH VERTEX COLORING ALGORITHMS

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(107)

A naive branch and bound algorithm for optimal vertex coloring is presented as a standard of comparison. Two approximating algorithms are presented: one a sequential vertex algorithm and the other a sequential color algorithm. These two algorithms are shown to produce the same coloring for any given ordering of vertices thus showing that two apparently different classes of coloring algorithms are the same. Several other types of coloring algorithms are surveyed.

(108)

Hadwiger's Conjecture for graphs on the Klein bottle

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Hadwiger's Conjecture states that an r-chromatic graph contracts to K_r, the complete graph on r vertices. We prove that every six-chromatic graph which embeds on the Klein bottle contracts to K_6. Together with previously known results this shows that Hadwiger's Conjecture is true for all graphs on the Klein bottle and, more generally, for all graphs which embed on a surface with nonnegative Euler characteristic.

SOME CHARACTERISTIC PATTERNS ON PASCAL'S TRIANGLE

(112)

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Consider the binomial coefficients along the slopes of Pascal's triangle. We are interested in finding the location and the value of the greatest binomial coefficient along a given slope.

The solution of this problem involves relations between the entropy function, binomial coefficients and generalized Fibonacci numbers.

In this paper, we exhibit the patterns of such binomial coefficients for different families of slopes, and we present some of their properties.

(113)

Conjugate Orthogonal Latin Square Graphs

F.E. Bennett* and N.S. Mendelsohn, University of Manitoba

An orthogonal latin square graph (OLSG) is one in which the vertices are latin squares based on the same set of symbols and two vertices are joined if and only if the latin squares are orthogonal. From any given latin square L, we may construct six (not necessarily distinct) conjugate latin squares based on the same set as L. If C(L) denotes the set of conjugates of a latin square L, then it is known that $|C(L)| = 1,2,3$ or 6. If G is an arbitrary finite graph, we say that G is realizable if there is an OLSG isomorphic to G. It is known that every finite graph is realizable as an OLSG. In this paper it is shown that, with the possible exception of a few values of $n \le 42$, there exists an idempotent latin square of order n such that the graph of its conjugates realizes a 6-cycle.

(104) On some properties of 1-factorizations of complete graphs

Eric Mendelsohn,* University of Toronto
Alexander Rosa, McMaster University

Given a 1-factorization F of the complete graph K_{2n} and a class of quadratic graphs Q, we define Q(F), the Q-index of F, to be the largest integer n such that there exists a partition of the 1-factors of F into classes each having at least n 1-factors, with the property that the union of any two 1-factors from the same class is isomorphic to a member of Q. We investigate the Q-index in the two "extremal" cases, namely when the graphs in Q are connected or when they have as many components as possible; in these two cases the Q-index is called the Dundas index, and the tightness index, respectively. We consider the indices of several old and new families of 1-factorizations of complete graphs, and indicate an application to a problem concerning tight families of Steiner triple systems.

(105) Orthogonal 1-factorizations of K_n

Jeff Dinitz, The Ohio State University

Two 1-factorizations F_1, F_2 of the complete graph K_n are orthogonal if for any 1-factors f_1 of F, and f_2 of F_2, f_1 and f_2 have at most one edge in common. Let $q = 2^k t + 1$ be a prime power with $t > 1$ odd. Mullin and Nemeth have shown that there exists a set of three pairwise orthogonal 1-factorizations of K_n where $n = q + 1$. If $k = 1$ (i.e. $q \equiv 3 \mod 4$), Wallis gives a construction for $t = \frac{1}{2}(q-1)$ pairwise orthogonal 1-factorizations of K_n. We generalize both these previous results to obtain a set of $t = (q-1)/2^k$ pairwise orthogonal 1-factorizations of K_{q+1} for any $k \geq 1$.

(106) MAXIMAL SETS OF DEFICIENCY THREE

L. Caccetta*, Queen's University and W.D. Wallis, University of Newcastle

A set S of edge-disjoint one-factors in a graph G is called maximal if there is no one-factor of G which is edge disjoint from all members of S, and if the union of S is not all of G. A set of one-factors is called Hamiltonian if it contains two one-factors whose union is a Hamiltonian cycle. The object of this paper is to prove that the complete graph K_{2n} has a maximal set of $2n - 3$ one-factors which is Hamiltonian whenever $2n \geq 16$.

(109) ON UNIMODAL SUBSEQUENCES

F.R.K. Chung, Bell Laboratories, Murray Hill, NJ

Let $S = (a_1, a_2, \ldots, a_n)$ denote a sequence of real numbers. A subsequence of S, denoted by $(a_{i_1}, a_{i_2}, \ldots, a_{i_t})$ where $i_1 < \ldots < i_t$, is said to be monotone if either $a_{i_j} \leq a_{i_{j+1}}$ for all j or $a_{i_j} \geq a_{i_{j+1}}$ for all j. A subsequence $(a_{i_1}, a_{i_2}, \ldots, a_{i_t})$ is said to be unimodal if there exists an integer k such that $(a_{i_1}, \ldots, a_{i_k})$ and $(a_{i_k}, \ldots, a_{i_t})$ are monotone subsequences. Let $\rho(n)$ denote the largest integer m with the property that any sequence of n real numbers contains a unimodal subsequence of m real numbers. In this paper we prove that $\rho(n) = \lceil \sqrt{3n - 3/4} - 1/2 \rceil$. We also mention many related problems

(110) UPPER BOUND FOR THE NUMBER OF TERMS OF ARITHMETIC PROGRESSIONS IN VAN DER WAERDEN THEOREM

Victor G. Tupitsyn, Southern Illinois University

In this paper we present the following result:

Theorem I. For any $N \geq 2$, for $Q \geq N(N-1)2^{(N-1)(N+2+[\log_2(N-1)])} + 2$ and for any partition $[1, 2, \ldots, Q] = C_1 \cup C_2$ there is a C_j ($j = 1$ or 2) containing arithmetic progression of length N.

We also estimate $Q(N,L)$ such that L arithmetic progressions of length N are contained in C_j.

(111) SOME APPLICATIONS OF FINITE FOURIER SERIES IN THE CONSTRUCTION OF CERTAIN GEOMETRIC DESIGNS

K. Nageswara Rao, North Dakota State University

For any odd prime p and an arbitrary integer $n > 0$, let $GF(p^n)$ denote a fixed Galois field of order p^n. For any α and $0 \neq \mu \in GF(q)$, where $q = p^n$, let

$$\tau(\alpha) = \alpha + \alpha^p + \ldots + \alpha^{p^{n-1}}$$

$$e(\alpha) = \exp\left(\frac{2\pi i \tau(\alpha)}{p}\right)$$

and

$$e_\mu(\alpha) = e(\mu\alpha).$$

Also, let $R(\mu, a)$ stand for the set of those elements γ of $GF(q)$ which are such that

$$\sum_\beta e_\mu(a^{-1}\beta\gamma) = 0$$

where $0 \neq a \in GF(p)$, $\mu \in GF(q)$ and the summation runs over all elements β of $GF(q)$.

The aim of the paper is to construct certain affine geometric designs with the sets $R(\mu, a)$ and study their related properties using finite Fourier series and character sums.

(114) A GRAPH THEORETIC ALGORITHM FOR EVALUATING ARRAYS OF EXTENDED SURFACE

A.D. Snider* and A.D. Kraus, USF

In the consideration of single longitudinal fins of rectangular, trapezoidal, and triangular profile and in arrays of extended surface composed of these fins, it is shown that conditions of heat flow and temperature excess at the fin or array tip induce conditions of heat flow and temperature excess at the fin or array base. In particular, there is a linear transformation between the aforementioned data at the fin tip and the fin base. The conventional fin efficiency is abandoned and single fins or arrays of extended surface are instead characterized by a single important parameter, the heat flow to temperature excess ratio, which is a function only of fin geometry and heat transfer parameters. Algorithms are provided for combining the effects of individual fins in arrays of extended surface.

(115) REALIZABILITY OF AN ECCENTRICITY SEQUENCE

Narsingh Deo, Washington State University, Pullman, WA

This paper is concerned with the realizability of a sequence S of n positive integers as the eccentricities of a graph. Necessary and sufficient conditions for S to be realizable as eccentricities of an n-order tree are obtained. A recursive algorithm based on these realizability conditions is presented. Attempts are made at generalizing the results to include unicyclic graphs and other connected undirected graphs. Other related problems, such as, enumeration of nonisomorphic trees realizing a given eccentricity sequence, are also discussed. Applications to computer networks and other resource sharing networks are demonstrated.

(116) Generating permutations of a multiset in lexicographical order

(Wolfgang [illegible]enkönig, Fachhochschule des Saarlandes, Saarbrücken, West Germany)

Abstract. Based on a simple algorithm for generating permutations of distinct objects in lexicographical order, a procedure is developed for the more general case that the object array is a multiset and, hence, the repeated generation of identical permutations must be prevented. The objects which may be of any nature are not examined, and they will not be moved if, instead of the object array, an index array is transmitted to the procedure. If this option is used, the objects are accessible by the index array.

The procedure operates on two additional integer arrays:

(1) An array whose elements can be interpreted as digits of a mixed based integer representing the lexicographical order of the current permutation. The changes of objects (or elements of the index array) are directly controlled by this array.

(2) An array representing the structure of the current permutation with respect to the multiple occurance of indistinguishable elements. It is used for skipping identical permutations.

(117) DECOMPOSITION OF TOTALLY UNIMODULAR MATRICES

P.D. Seymour: Oxford and Waterloo

A matrix is totally unimodular (t.m.) if every square submatrix has determinant ± 1 or 0. The vertex-edge incidence matrix of a directed network is t.m., and so is its transpose. Conversely, every t.m. matrix may be constructed from matrices of these two sorts and copies of one special t.m. 5×5 matrix, by piecing them together in a certain simple way and pivoting. (Both these operations preserve being t.m.) The proof is lengthy and matroid-theoretic, but some aspects are described.