Binding Number and Wiener Index

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Let G be a finite simple graph with vertex set $V(G)$. The *Wiener index* of G is defined as $W(G) = \sum_{u,v \in V(G)} \mathrm{d}(u, v)$, where $\mathrm{d}(u, v)$ is the distance between u and v. This topological index was defined by Wiener in 1947 for studying the structural graphs of molecules. Since that time, many theoretical results have been derived. The results we present concern the *binding number* of a graph G, denoted $\mathrm{bind}(G)$. Let $\mathcal{S} = \{S \subseteq V(G) \mid S \neq \emptyset \text{ and } N(S) \neq V(G)\}$, where $N(S)$ is the neighbor set of S. Then $\mathrm{bind}(G) := \min_{S \in \mathcal{S}} |N(S)|/|S|$. Lower bounds on the binding number have been used extensively to determine properties of graphs. Woodall provided the earliest such result in 1973 when he proved that $\mathrm{bind}(G) \geq 3/2$ implies G is hamiltonian. Our main result provides conditions on the Wiener index of G that guarantee $\mathrm{bind}(G) \geq b$ for some specified $b > 0$.

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