An FPRAS for k-edge connected unreliability

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In 2001, Karger proposed the first fully polynomial random approximation scheme to approximate the network unreliability, i.e. the probability that a graph whose edges fail stochastically independent with equal probability q becomes disconnected. It is based on the observation that the number of cuts up to a given size αc – where c denotes the size of the min-cut – is at most $n^{2\alpha}$ and thus grows polynomially. On the other hand, the chance that a cut of size αc fails decreases exponentially with α (namely $(q^c)^\alpha$). Karger could show that there is some treshhold-value α^* such that the probability that some cut with size at most $\alpha^* c$ is sufficient to approximate the probability that the graph becomes disconnected. Karger stated that this procedure can be easily extended to k-edge connected unreliablity (the probability that the surviving subgraph has edge-connectivity less than k) because – as he states – the probability that a cut has less than k surviving edges also decreases exponentially with the size of the cut. However, the probability that a cut of size αc has less than k surviving edges is $\sum_{i=0}^{k-1} \binom{\alpha c}{i} q^{\alpha c - i}(1-q)^i$ and thus also contains a factor which grows with the size of αc – which Karger does not account for. In this talk, we show that it is still possible to obtain an FPRAS using the approach of Karger whenever k is fixed. However, the runtime of our approximation scheme scales with n^k.

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