On Coloring Complete Binary Trees

Maryam Khosravi*, Shahid Bahonar University of Kerman, Iran, Eddie Cheng, Oakland University, Ke Qiu, Brock University, Zhizhang Shen, Plymouth State University

In parallel processing, efficient mapping schemes are required to distribute data in such a way that regular patterns called templates of various data structures such as trees can be retrieved in parallel without memory conflicts. Previously, Das and Pinotti studied the problem when the underlying structures are general q-ary trees and binomial trees and templates are leaf-to-root paths and subtrees. In particular, for complete binary trees where templates are leaf-to-root paths, an optimal and balanced scheme was proposed. In this scheme, nodes of a complete binary tree are recursively colored (assigned to different memory banks) such that for any path from a leaf to the root, nodes on the path are assigned different colors (to guarantee conflict-free access) in such a way that the load is balanced among all the memory banks. Denoting $f_{h,j}$ as the frequency of the color j used in a complete binary tree of height h, let $x_h = \max_{0\leq j\leq h-1}\{f_{h,j}\}$, and $y_h = \min_{0\leq j\leq h-1}\{f_{h,j}\}$, where x_h and y_h are respectively the maximum and minimum load on the memory banks (the root and only the root is assigned color h, thus $f_{h,h} = 1$). An open problem raised is whether $x_h/y_h \geq 3/2$. The answer to this question is yes and provided earlier. Numerical evidence shows that this x_h/y_h ratio is arbitrarily close to 2 as h tends to infinity. In this talk, we will discuss this observation theoretically. In addition, our study reveals some interesting combinatorial properties of the coloring sequence $\{f_{h,j}\}$ and colored complete binary trees.