

\pm Skolem-type Difference Sets

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For a positive integer t , a Skolem-type 3-tuple difference set of order t is a partition of the set $\{1, 2, \dots, 3t\}$ into triples $\{a_i, b_i, c_i\}$ such that $a_i + b_i = c_i$. Skolem-type difference sets and their many generalizations are well studied, and necessary and sufficient conditions for Skolem difference sets are known. A practical application of Skolem-type difference sets involves rewriting the triple $\{a_i, b_i, c_i\}$ as $a_i + b_i + c_i = 0$ where c_i is necessarily negative. Hence, we study \pm Skolem-type 3-tuple difference sets of order $2t$ which are partitions of the set $\{\pm 1, \pm 2, \dots, \pm 3t\}$ into triples $\{a_i, b_i, c_i\}$ such that $a_i + b_i + c_i = 0$. We also study a Langford-type generalization of this concept, namely, partitions of the set $\{\pm d, \pm(d + 1), \dots, \pm(d + 3t - 1)\}$ into $2t$ triples $\{a_i, b_i, c_i\}$ such that $a_i + b_i + c_i = 0$ for $d = 2$ and $d = 3$. These concepts are also generalized to m -tuples and used to find cyclic decompositions of circulant digraphs into cycles.

Keywords: Skolem generalization, set partition, cyclic directed cycle decomposition