

The Construction and Uses of Heffter Arrays

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Abstract

A Heffter array $H(m, n; s, t)$ is an $m \times n$ matrix with nonzero entries from \mathbb{Z}_{2ms+1} such that *i*) each row contains s filled cells and each column contains t filled cells, *ii*) every row and column sum to $0 \pmod{2ms+1}$, and *iii*) no element from $\{x, -x\}$ appears twice. In this talk we will discuss existence of these arrays in the case where $s = n$ (every cell is filled) and in the case where $m = n$ (square arrays). Heffter arrays are interesting in their own right and are also useful in embedding the complete graph K_{2ms+1} on an orientable surface where the embedding has the property that each edge borders exactly one s -cycle and one t -cycle. Such an embedding is termed a biembedding of an s -cycle system and a t -cycle system. We will demonstrate this use by showing that for every n , there exists a biembedding of the complete graph on $6n + 1$ points on an orientable surface such that each edge borders a 3-cycle and an n -cycle (a biembedding of a Steiner triple system and an n -cycle system).