Some New Necessary Conditions for the Existence of Balanced Arrays with Applications

D.V. Chopra* and Richard M. Low; Wichita State University

Balanced (B-arrays) arrays are generalizations of orthogonal arrays (O-arrays), connected to several areas of combinatorial mathematics and have been useful in constructing balanced fractional factorial designs which are "optimal" in some sense. A B-array T with m factors (rows, constraints), N columns (treatment-combinations, runs), s symbols (levels; say $0, 1, 2, \ldots, s-1$) and of strength t is merely a matrix T of size $(m \times N)$ such that in every $(t \times N, t \leq m)$ submatrix T^* of T, the following condition is satisfied: $\lambda(\underline{\alpha}; T^*) = \lambda(P(\underline{\alpha}); T^*)$, where $P(\underline{\alpha})$ denotes a vector obtained by permuting the elements of $(t \times 1)$ vector $\underline{\alpha}$ of T^* and $\lambda(\underline{\alpha}; T^*)$ denotes the frequency of $\underline{\alpha}$ in T^*. In this paper, we restrict ourselves to arrays with two symbols (say, 0 and 1). For this special case, the above combinatorial constraints can be stated in terms of the weights of $\underline{\alpha}$ (denoted by $w(\underline{\alpha})$, which is the number of 1s in $\underline{\alpha}$). The parameters of the array T are m and $\underline{\mu}' = (\mu_0, \mu_1, \ldots, \mu_t)$. It is quite clear that the total number of columns in T are known once we know $\underline{\mu}'$. In this paper, we obtain some inequalities which are necessary for the existence of balanced arrays involving m and the elements of $\underline{\mu}'$. Given $\underline{\mu}'$, the inequalities will be a polynomial function in m. We use these results to obtain further information on the number of constraints m.

Key words: balanced arrays, orthogonal arrays, strength of an array, runs, optimal, fractional factorial designs, constraints.