

## The Hadwiger-Nelson Problem with Two Forbidden Distances

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In 1950, Edward Nelson asked the following simple-sounding question:

*How many colors are needed to color the plane such that no two points distance 1 apart are colored the same?*

We say that 1 is a *forbidden* distance. Despite the passage of time, and the work of many mathematicians, we only know that the answer is 4, 5, 6, or 7. All efforts to reduce this list were unsuccessful.

In this paper we consider a related problem in which we require *two* forbidden distances, 1 and  $d$ . In other words, for a given positive number  $d \neq 1$ , how many colors are needed to color the plane such that no two points distance 1 or  $d$  apart are assigned the same color?

We find several values of  $d$ , for which the answer to the previous question is at least 5. We also provide evidence that this version of the problem can be used to attack Nelson's original question.

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