Distinguishing wheel graphs by the alliance polynomial

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The alliance polynomial of a graph G with order n and maximum degree Δ is the polynomial $A(G, x) = \sum_{k=-\Delta}^{\Delta} A_k(G)\, x^{n+k}$, where $A_k(G)$ is the number of exact defensive k-alliances in G. The distinctive power of the alliance polynomial has been studied in previous works, for instance, has been proved that the empty, path, cycle, star, complete, complete without one edge and star graphs are characterized by their alliance polynomials. Moreover, has been proved that the family of alliance polynomials of Δ-regular graphs with small degree is a very special one, since it does not contain alliance polynomials which are not Δ-regular as well. In this work we continue this study and prove that the alliance polynomial determines uniquely each wheel graph. Furthermore, we show that alliance polynomial of some quasi-regular graphs is unimodal.

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