The Unicyclic Random Graph Process Abstract

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Let $\mathcal{U}(n)$ denote the set of unlabeled unicyclic graphs of order n. A graph G_0 in $\mathcal{U}(n)$ is called an *initial graph* in $\mathcal{U}(n)$, if the deletion of any edge of G_0 produces a graph not in $\mathcal{U}(n)$.

Next, starting at any initial graph G_0 in $\mathcal{U}(n)$, randomly, that is, with uniform probability, add an edge $\{u, v\}$ to start a random walk (G_i) such that at each step

$$G_{i+1} = G_i \cup \{u, v\} \text{ is in } \mathcal{U}(n) \text{ for all } i \geq 0.$$

The probability that edge $\{u, v\}$ is selected is 1/N, where $N \neq 0$ is the number of edges such that $G_{i+1} = G_i \cup \{u, v\}$ is in $\mathcal{U}(n)$. Such edges are called *admissible edges*. If N = 0, then the graph G_i is called a *terminal graph.* Note that the n-cycle is both an initial graph and a terminal graph.

The *transition digraph* for this random process is the union of all random walks described above. Namely, the node set for the transition digraph is $\mathcal{U}(n)$ and the arc set consists of the arcs (G_i, G_{i+1}) weighted with the probability of going from G_i, to G_{i+1}. This probability is called the *transition probability* for (G_i, G_{i+1}).

The properties of this random process and in particular the properties and structure of its associated transition digraph are discussed and some open problems are presented.

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