Extremal Collections of k-Uniform Vectors

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Extremal combinatorics is a field where we wish to optimize a discrete parameter for some family of large combinatorial objects. Most such questions have parallels in probabilistic combinatorics, where one is more concerned with the statistical behaviour of a randomly chosen object in this family.

Consider the particular context of a representable matroid (i.e. the columns of a matrix) over F_2 of rank $\leq n$. Here, one well-studied distribution is to fix a small k and large m and randomly generate m columns with k 1's. Indeed, when $k = 2$, this is the graphic matroid of the Erdős-Rényi random graph $G_{n,m}$.

We explore the corresponding extremal question. What is the maximum number of weight-k columns a matrix of rank $\leq n$ can have?