THE DECOMPOSITION OF KD+V INTO K4-E: A PARTICULAR CASE

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INTRODUCTION

In the field of design theory, the most well-known design is a Steiner Triple System. In general, a G-design on H is an edge-disjoint decomposition of H into isomorphic copies of G. In a Steiner Triple System, a complete graph is decomposed into triangles. In the designs studied, we let H be a complete graph with a hole, (Kd+v), and G be a complete graph on four vertices minus one edge, K4-e. Solutions are known when $v<4$ and for a few other specific cases. We focus on designs for larger values of v and when d is even. By restricting d and v we are able to resolve the case for a subset of Kd+v, using one-factors and difference methods.

SIGNIFICANCE

Graph decompositions are a significant part of design theory. Historically, investigators focused on decomposing complete graphs into copies of other complete graphs – like the Steiner Triple System. However, when Hanani (Hanani, 1975) wrote a paper explaining the importance of incomplete block designs, investigators began decomposing complete graphs with other graphs, which has opened doors to many more types of graph decompositions, including the most common type of combinatorial designs. Furthermore, there have been many papers published on decomposing some other graph H (not necessarily a complete graph) into copies of G. For example, since Hanani's paper, complete graphs with a hole have been decomposed with incomplete graphs, like a four-cycle with a pendant edge. Similarly, we decompose a complete graph with a hole with an incomplete graph, K4-e – a complete graph on four vertices minus one edge.

STUDENT INVOLVEMENT

I began learning about graph theory during my first semester of college, Fall 2016, and began this graph decomposition as part of a directed study in the Spring of 2017. I was able to continue working on this decomposition through the Summer of 2017 and under the guidance of Dr. Back wrote a paper for publication, which I submitted to Pi Mu Epsilon at the end of January. I have continued to work on the decomposition of Kd+v as part of directed studies and had the opportunity to present the decomposition for a particular case in February 2018 at the ASPiRE Conference in Fort Meyers.

REFERENCES

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