**Gallai-Ramsey numbers of $C_9$ and $C_{11}$ with multiple colors**

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We study Ramsey-type problems in Gallai-colorings. Given a graph $G$ and an integer $k \geq 1$, the Gallai-Ramsey number $gr_k(K_3, G)$ is the least positive integer $n$ such that every $k$-coloring of the edges of the complete graph on $n$ vertices contains either a rainbow triangle or a monochromatic copy of $G$. It turns out that $gr_k(K_3, G)$ behaves more nicely than the classical Ramsey number $r_k(G)$. However, finding exact values of $gr_k(K_3, G)$ is far from trivial. In this talk, I will present our new result that $gr_k(K_3, C_{2n+1}) = n \cdot 2^k + 1$ for $n \in \{4, 5\}$ and all integers $k \geq 1$. This new result provides partial evidence for the first two open cases of the Triple Odd Cycle Conjecture of Bondy and Erdős from 1973. Our technique relies heavily on the structural result of Gallai on edge-colorings of complete graphs without rainbow triangles. We believe the method we developed can be used to determine the exact values of $gr_k(K_3, C_n)$ for odd integers $n \geq 13$.

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