Totally non-negativity of a family of change-of-basis matrices

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A matrix is *totally non-negative* if all its minors are nonnegative. Let $\mathbf{a} = (a_1, a_2, \ldots, a_n)$ and $\mathbf{e} = (e_1, e_2, \ldots, e_n)$ be real sequences. We let $M_{e\to a}$ denote the $(n+1)\times(n+1)$ change-of-basis matrix whose (m, k)th entry is the coefficient of the polynomial $(x - a_1)(x - a_2)\ldots(x - a_k)$ in the expansion of $(x - e_1)(x - e_2)\ldots(x - e_m)$ as a linear combination of the polynomials $1, x - a_1, \ldots, (x - a_1)\ldots(x - a_m)$.
In this talk, we will see that $M_{e\to a}$ encodes many combinatorial objects. Further, we fully characterize $\mathbf{a}$ and $\mathbf{e}$ sequences such that $M_{e\to a}$ is totally non-negative.
Keywords: total non-negativity, change-of-basis matrices, algorithm