On the dual of Erdős' circuit spectrum problem

Bing Wei, Haidong Wu, Shunzhe Zhang* University of Mississippi

The cycle spectrum of a graph is defined to be the set of cycle sizes. Erdős (1992) conjectured that every connected graph G with girth g and average degree d has a cycle spectrum of size at least $\Omega(d^{\lfloor (g-1)/2 \rfloor})$. Sudakov and Verstraë (2008) proved this conjecture in a stronger form. In this paper, we consider the dual question of Erdős: Can we find an exponential function $f(\delta, g)$ such that every graph G with girth g and minimum degree δ has a bond spectrum of size at least $f(\delta, g)$? We solve this problem by giving such an exponential function $f(\delta, g)$ when the girth of the graph is at least seven. We also show that the bond spectrum of any graph with sufficiently large number of vertices and large girth contains an arithmetic progression.

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