G-Designs for the Connected Tripartite Unicyclic Graphs with Seven Edges

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For a subgraph G of the complete graph K_n, a G-design of order n is a partition of the edge set of K_n into edge-disjoint copies of G. The spectrum problem in this context asks for which n a G-design of order n exists. This problem has been completely solved for every graph with 6 or less edges and most of the graphs with 7 or 8 edges. We solve the spectrum problem for one of the few remaining families of small graphs by proving that if G is a connected tripartite unicyclic graph with 7 edges, then a G-design of order n exists if and only if $n \equiv 0$ or $1 \pmod 7$, with a few exceptions when $n \in \{7, 8\}$.

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