Vertex Fault Tolerant Zero Forcing

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Zero forcing is an iterative graph coloring process studied for its wide array of applications. In this process, the vertices of the graph are initially designated as blue or white, and a zero forcing set is a set of initially blue vertices that results in all vertices becoming blue after repeated application of a color change rule. The zero forcing number of a graph is the minimum cardinality of a zero forcing set. The zero forcing number has motivated the introduction of a host of variants motivated by linear-algebraic or graph-theoretic contexts. We define a variant we term the k-fault tolerant zero forcing number, which is the minimum cardinality of a set B such that every subset of B of cardinality $|B| - k$ is a zero forcing set. We study the values of of this parameter on various graph families and behavior under several graph operations. For tree graphs, we develop a theoretical framework based on compatible path cover collections that provides an exact characterization of the 1-fault tolerant zero forcing number, allowing us to completely characterize this parameter for tree graphs. Our compatible path cover collections also provide a lower bound on the 1-fault tolerant zero forcing number for all graphs for which the parameter exists.

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