The Pinnacle Sets of a Graph

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We introduce and study the pinnacle sets of a simple graph G with n vertices. Given a bijective vertex labeling $\lambda : V(G) \to [n]$, the label $\lambda(v)$ of vertex v is a *pinnacle* of (G, λ) if $\lambda(v) > \lambda(w)$ for all vertices w in the neighborhood of v. The *pinnacle set of* (G, λ) contains all the pinnacles of the labeled graph. A subset $S \subseteq [n]$ is a *pinnacle set of* G if there exists a labeling λ such that S is the pinnacle set of (G, λ). We show that when G is connected, G has a size-k pinnacle set if and only if G has an independent set of the same size. We identify all the pinnacle sets of complete graphs, complete bipartite graphs, cycles and paths. We also present a technique for deriving new pinnacle sets from old ones that imply a typical graph has many pinnacle sets.

Keywords: Graph labeling, pinnacle, pinnacle set, independent set, poset, path, cycle