Stack-sorting simplices: geometry and lattice-point enumeration

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We study the subpolytopes of the permutahedron that arise as the convex hulls of stacksorting on particular permutations. We primarily focus on Ln1 permutations, i.e., permutations of length n whose penultimate and last entries are n and 1, respectively. First, we present some enumerative results on Ln1 permutations. Then we show that the polytopes that arise from stack-sorting on Ln1 permutations are simplices and proceed to study their geometry and lattice-point enumeration. Particular focus is taken on the Ln1 permutation $23 \cdots n1$. We show that the convex hull of all its iterations through the stack-sorting algorithm shares the same lattice-point enumerator as that of the (n-1)-dimensional unit cube and lecture-hall simplex. Lastly, we detail some results on the real lattice-point enumerator for variations of the simplices arising from stack-sorting on the permutation $23 \cdots n1$. This then allows us to show that those simplices are Gorenstein of index 2.

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