Symmetric Tensor Power of Graphs and Permutations

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Given a matrix A defined on an n-dim vector space V with orthonormal basis $v_1, v_2, \cdots, v_n$, one can define its kth symmetric tensor power to be the matrix $A^{\odot k}$ acting on the symmetric tensor power of V, denoted as $V^{\odot k}$, as

$$A^{\odot k}(v_{i_1} \odot v_{i_2} \odot \cdots \odot v_{i_k}) = \frac{1}{k!} \sum_{\sigma \in \mathfrak{S}_k} \left(A v_{i_{\sigma(1)}} \right) \otimes \cdots \otimes \left(A v_{i_{\sigma(k)}} \right).$$

In [1], we studied some combinatorial properties of certain families of graphs, like the path graph, the cycle graph and the complete graph, by applying the kth symmetric tensor power of their corresponding adjacency matrices. In this talk we will present some of those properties and we will discuss the case in which the matrix A is a permutation matrix. This is current joint work with Sebastian Caballero.

Keywords: graphs, permutations, matrices, tensor product, symmetric tensor product

References

[1] W. Astaiza et al. "Symmetric tensor powers of graphs". In: *Series A: Mathematical Sciences* 36.3 (2026), pp. 13–35.