Survival Probabilities in a Probabilistic Counting-Out Game

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Our research focuses on the survival probability of a novel problem where there are n people in a line at positions $1, 2, \cdots, n$. Each position is either vulnerable or safe, and in each round of the counting-out game, one person in a vulnerable position is selected uniformly at random to be eliminated. When this person i is eliminated, we shift the remaining people to fill in position i by setting every person in position k, such that $k > i$, to position $k - 1$. We continue to select people until there is only one person left, who then becomes the survivor. This determines their survival probability, where $p_n(k)$ denotes the probability that the person initially in position k will be the last person remaining out of a line of n people. We are particularly interested in finding this survival probability for each initial position. By partitioning the line based on which positions are vulnerable and safe, we inductively derive a formula for the survival probability of any initial position for any possible ordering of the line. This formula proves that survival probabilities of people on the line are nondecreasing, so we present various orderings of the vulnerable and safe positions on the line that produce a sequence of survival probabilities which asymptotically approximate linear, exponential, and square root functions.

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