Recognizing Relating Edges in Graphs without Cycles of Length 6

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A graph G is *well-covered* if all its maximal independent sets are of the same cardinality. Let $w : V(G) \longrightarrow R$ be a weight function. Then G is *w-well-covered* if all its maximal independent sets are of the same weight. An edge $xy \in E(G)$ is *relating* if there exists $S \subseteq V(G)$ such that both $S \cup \{x\}$ and $S \cup \{y\}$ are maximal independent sets. If xy is relating then $w(x) = w(y)$ for every weight function w such that G is w-well-covered. Relating edges are of crucial importance for investigating w-well-covered graphs. The problem whether an edge is relating is **NP-**complete. We prove that this problem remains **NP-**complete even for graphs without cycles of length 6.
A graph G belongs to the class $\mathbf{W_2}$ if every two pairwise disjoint independent sets in G are included in two pairwise disjoint maximum independent sets. A vertex $v \in V(G)$ is *shedding* if for every independent set $S \subseteq V(G) \setminus N[v]$ there exists $u \in N(v)$ such that $S \cup \{u\}$ is independent. Shedding vertices play an important role in studying the class $\mathbf{W_2}$. Recognizing shedding vertices is co-**NP-**complete. We prove that this problem is co-**NP-**complete even for graphs without cycles of length 6.

Keywords: w-well-covered, relating edge, shedding vertex, class $\mathbf{W_2}$