Gaining clustered trees on grid hypergraph intersection graphs

Michal Stern* and Nili Beck, Academic College of Tel Aviv - Yaffo

The Clustered Spanning Tree problem is given a hypergraph $H=<V,\boldsymbol{S}>$, where V is a set of vertices and $\boldsymbol{S}$ is a set of clusters, not necessarily disjoint. In a complete graph induced on the vertices of V, the problem is to find whether there exists a spanning tree, such that each cluster induces a subtree. Consider hypergraphs where no feasible solution trees exist. In order to gain feasibility, we find minimum cardinality feasible vertices insertion lists, that insert chosen vertices to the clusters. We address hypergraphs whose intersection graphs are n*m grid graphs, constructed of chordless cycles, each cycle contains four nodes, and every two cycles intersect in at most one edge. We present the Convert to Clique method, that inserts vertices to ensure that one vertex is contained in every cluster of the graph, thus changing the intersection graph into a clique. This method creates a feasible vertices insertion list for any given hypergraph. Surprisingly, this method yields a minimum cardinality feasible vertices insertion list, for given hypergraphs whose intersection graphs are grids. Our problem is applicable to many areas, such as organizational networks and security of information.