On Copnumbers of Periodic Temporal Graphs

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A *periodic temporal graph* $\mathcal{G} = (G_0, G_1, \ldots, G_{p-1})^*$ is an infinite periodic sequence of graphs $G_i = (V, E_i)$ where $G = (V, \cup_i E_i)$ is called the footprint. Recently, the arena where the Cops and Robber game is played has been extended from a graph to a periodic graph; in this case, the *copnumber* is also the minimum number of cops sufficient for capturing the robber. We study the connections and distinctions between the copnumber $c(\mathcal{G})$ of a periodic graph $\mathcal{G}$ and the copnumber $c(G)$ of a graph G (e.g. the footprint), and establish several facts. For instance, we show that the smallest periodic graph with $c(\mathcal{G}) = 3$ has at most 8 nodes; in contrast, the smallest graph G with $c(G) = 3$ has 10 nodes. On the other hand, we provide an example of a periodic graph with $c(\mathcal{G}) = 3$ whose footprint G is outerplanar and thus $c(G) = 2$. Based on these results, we derive upper bounds on the copnumber of a periodic graph from properties of its footprint, such as its treewidth and the existence of separators, block decompositions and cop-monotone strategies. In particular, let H be a subgraph of G and $\mathcal{G}[H] = (G_0[H], G_1[H], \ldots, G_{p-1}[H])^*$ where $G_i[H]$ is the subgraph of G_i induced by the nodes of H. We generalize a classic result and prove that $c(\mathcal{G}[H]) \leq c(\mathcal{G})$ under some conditions. We also disprove an extension of a result on isometric paths, so it cannot be used to say that planarity of G implies $c(\mathcal{G}) \leq 3$.

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