The Number of k-Nearly Independent Subsets

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Let G = (V, E) be a graph with vertex set V and edge set E. A subset I of V is an *independent vertex subset* if for every $u, v \in I$, $uv \notin E$. The number of vertex independent subset of a graph is a popular graph invariant with rich literature. See the survey in [S. Wagner, I. Gutman, Maxima and Minima of the Hosoya Index and the Merrifield-Simmons Index. Acta Appl Math 112, 323 – 346 (2010)], where it is called Hosoya index. In this paper we generalise the notion of vertex independent subsets, and we wish to understand the behaviour of the number $\sigma_k(G)$ of all subsets of the set of vertices of G that contain k edges. We call those sets k-nearly independent vertex subsets. We prove a lower (and upper) bound on $\sigma_1(G)$, where G is a graph of order n and size m. The families of graphs attaining these bounds are also characterised.

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