A generalization of Petersen's matching theorem

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One of the earliest results in graph theory is Petersen's matching theorem from 1891 which states that every bridgeless cubic graph contains a perfect matching. Since the vertex-connectivity and edge-connectivity in a connected cubic graph are equal, Petersen's theorem can be stated as follows: If G is a 2-connected 3-regular graph of order n, then $\alpha'(G) = \frac{1}{2}n$, where $\alpha'(G)$ denotes the matching number of G. We generalize Petersen's theorem and prove that for $k \geq 3$ an odd integer, if G is a 2-connected k-regular graph of order n, then $\alpha'(G) \geq \left(\frac{k^2+k+6}{k^2+2k+3}\right) \times \frac{n}{2}$. The case when k is even behaves differently. In this case, for $k \geq 4$ even, if G is a 2-connected k-regular graph of order n, then $\alpha'(G) \geq \left(\frac{k^2+4}{k^2+k+2}\right) \times \frac{n}{2}$. For all $k \geq 3$, if G is a 2-connected non-regular graph of order n and maximum degree k, then we show that $\alpha'(G) \geq \frac{2n}{k+2}$. In all the above bounds, the extremal graphs are characterized.

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