Forcing Sets with Connected Complements

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In connected zero forcing, forcing sets are required to induce a connected subgraph. One natural question that can be asked is: Does there always exist a minimum zero forcing set that is connected? The answer is no; for instance, for any tree that is not a path, all minimum zero forcing sets are disconnected. We pose, in a sense, an opposite question: Is there always a minimum forcing set whose complement induces a connected subgraph?

Recently, Davila et al. answered this question in the affirmative for positive semidefinite (PSD) zero forcing. They showed that for a minimum PSD forcing set whose complement is not connected, one can "enlarge" a chosen component of its complement through a technique involving the reversal of certain forces in the forcing process of the original set.

In this talk, we show that standard zero forcing and skew zero forcing are amenable to a similar technique, thus answering our question affirmatively for those variants as well.

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