Melding Combinatorics and Group Theory to Confirm The Automorphism Conjecture for Ordered Sets?

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The Automorphism Conjecture for ordered sets states that, for finite ordered sets, the ratio of the number of automorphisms to the number of endomorphisms goes to zero as the size of the underlying ordered set goes to infinity. This talk will present new insights into the automorphic structure of ordered sets, which rely on the following idea: An automorphism's action on an orbit's partition $\{C_1, \ldots, C_n\}$ into pairwise congruent blocks, together with the image of a block B in another orbit, controls a primitive action on a partition $\{S_1, \ldots, S_m\}$ of B into pairwise congruent subblocks iff there are i, j, k such that there is no automorphism that fixes C_i and that maps S_j to S_k. Together with deep results on finite primitive permutation groups, which rely on the classification of finite simple groups, this idea proves the Automorphism Conjecture for ordered sets in which factorial actions on orbits only occur in "large mutually controlling clusters." A more concrete result is the confirmation of the Automorphism Conjecture for ordered sets of width up to 10. Further work will, naturally, focus on small mutually controlling clusters, and the hope is that this will eventually confirm the Automorphism Conjecture.

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